“Undertaking Specific Parameters or a Partial Internal Model under Solvency 2?”

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Agenda

- Introduction
- USPs within Non-Life Premium Risk
- Towards a Partial Internal Model for Premium Risk
- GLM, GAM or Mixture of them?
- Case study: USPs versus PIRM
- How about volatility?
- Conclusions
- References
Introduction

Solvency 2 directive represents a complex project for reforming the present vigilance system of solvency for European insurance companies.

- **What?**
  
  A definition of a **Solvency Capital Requirement** ("SCR") as an economic capital to reflects the **true risk profile** of the undertaking, taking account of **all quantifiable risks**, as well as the net impact of **risk mitigation techniques**.

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### Distribution of Total Losses

- **Expected Losses**
- **VaR\textsubscript{99.5}**
- **0.5%**
- **99.5%**

### SCR

- **Time Horizon**: 1 year
- **Risk Measure**: Value at Risk
- **Probability of Ruin**: 0.5%
Introduction

How?
In principle, Solvency 2 provides a range of methods to calculate the SCR which allows undertakings to choose a method that is **proportionate to the nature, scale and complexity of the risk** that are measured.
Introduction

The scope of this work is to compare the USPs’ methodologies proposed in QIS5 with a PIRM for premium risk.

In particular we introduce this approach:

- for Personal Line insurance and/or for each product priced using regression techniques
- in order to stress the value of the model used from Pricing Staff

The premium risk is defined in the TS of QIS5: [2] “Premium risk results from fluctuations in the timing, frequency and severity of insured events (…). Premium risk includes the risk that premium provisions turn out to be insufficient to compensate claims or need to be increased. Premium risk also includes the risk resulting from the volatility of expense payments.(…)”.

USP or PIRM under Solvency 2?
USPs within Non-Life Premium Risk

\[ \sigma_{(\text{prem }, \text{LoB})} = c \cdot \sigma_{(U, \text{prem }, \text{LoB})} + (1 - c) \cdot \sigma_{(M, \text{prem }, \text{LoB})} \]

Undertakings can replace a part of standard parameters with specific parameters (USP):

- According a criterion of credibility that depends on LoB and the length of the time series \( N_{\text{lob}} \) used for the estimation:

  For GTPL, MTPL, Credit and Suretyship:

  \[
  \begin{array}{cccccccccc}
  N_{\text{lob}} & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & \geq 15 \\
  C & 34\% & 43\% & 51\% & 59\% & 67\% & 74\% & 81\% & 87\% & 92\% & 96\% & 100\% \\
  \end{array}
  \]

  For the other LoBs:

  \[
  \begin{array}{cccccc}
  N_{\text{lob}} & 5 & 6 & 7 & 8 & \geq 10 \\
  C & 34\% & 51\% & 67\% & 81\% & 92\% & 100\% \\
  \end{array}
  \]

  - The data used for the calculation of undertaking-specific parameters should be complete, accurate and appropriate.
## Which USPs to choose?

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assumptions</strong></td>
<td><strong>In addition to the assumptions of Method 1:</strong></td>
<td><strong>A separate analysis of the random variables number of claims and cost per claims</strong></td>
</tr>
<tr>
<td>- The expected loss is proportional to the premium</td>
<td>- The distribution of the loss is lognormal</td>
<td>- Based on the Swiss Solvency Test approach (Gisler, 2009)</td>
</tr>
<tr>
<td>- The company has a different but constant expected loss ratio (“ELR”)</td>
<td>- The maximum likelihood fitting approach is appropriate</td>
<td></td>
</tr>
<tr>
<td>- The least squares fitting approach is appropriate</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Approach</strong></th>
<th><strong>Approach</strong></th>
<th><strong>Approach</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>- This method use the Ultimate Cost after one year by accident year</td>
<td>- It is a method similar to the previous</td>
<td>- The approach is significantly influenced by the variability in the exposure and in the number of claims</td>
</tr>
<tr>
<td>- The Volatility depends on volatility year by year of Earned Premium or ELR</td>
<td></td>
<td>- Requiring a greater number of information than the other two methods</td>
</tr>
<tr>
<td>- One year of adverse claim experience can produce material effects on the volatility</td>
<td></td>
<td>- If the company has reserved less prudently in the first development year, probably it has a volatility higher than the values obtained with Methods 1 and 2.</td>
</tr>
<tr>
<td>- The company tends to reserve prudently in the first accident year</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Towards a Partial Internal Model for Premium Risk

**Why?**

With a (Partial) Internal Risk Model, an Insurance Company can **calibrate** the volatility of its business and risk profiles.

\[
\sigma_{SF}^{(\text{prem}, \text{lob})} \geq \sigma_{(\text{prem}, \text{lob})} = c \cdot \sigma_{(U, \text{prem}, \text{lob})} + (1 - c) \cdot \sigma_{SF}^{(M, \text{prem}, \text{lob})} \geq \sigma_{\text{PIRM}}^{(\text{prem}, \text{lob})}
\]

**Premium Risk**

Undertakings, therefore, will have to evaluate the error in the assumptions, models or methods used to solve a pricing problems.

\[
P = E(X) = E(N) \cdot E(Y)
\]

**Aggregate Claim Amount («ACA»)**

\[
X = \sum_{i=1}^{N} Y_i
\]

**Pure Premium or Expected Value of ACA**

**Volatility or Premium Risk or standard deviation of ACA**

\[
\sigma_{\text{PIRM}}^{(\text{prem}, \text{lob})} \geq \sigma(X)
\]
Different prospective

The **new idea** of this presentation is represented by:

\[ \sigma_{\text{prem,lob}}^{\text{PIRM}} = \sigma(X) \]

This seems to be in contrast with the definition of SCR, but in a PIRM: “Insurance and reinsurance undertakings may use a different time period or risk measure (…) to calculate the Solvency Capital Requirement in a manner that provides policy holders and beneficiaries with a level of protection equivalent to that set out in Article 101” (art. 122(1) S2 directive)

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**Pricing**

- i. are usual to do a GLM pricing exercise to determine the risk cost (Pure Premium);
- ii. make a number of adjustments to reflect IBNR, inflation trends, etc, to produce a **Total Premium**.

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**Premium Risk**

- Using GLM for all risk profiles
- Updating the observed ACA at ultimate 1-year
- Integrated Methods to calculate Premium and its volatility.
GLM, GAM or a Mixture of these?

GLM is a benchmark within this technical framework:

- How to manage continuous rating variables?
- Is Cluster Analysis a good solution?
- An obvious disadvantage is that the premium for two policies with different but close values for the rating variable may have substantially different premiums if the values happen to belong to different intervals

This is what we propose:

Mixture

Data Analysis → Cluster Analysis

GLM

Frequency

Severity

$N \approx \text{Poi}(\lambda)$

$Y \approx \text{Gamma}(k, \theta)$

GAM
Case study: USPs versus PIRM

**Perimeter**

Hypothetical portfolio - Car

- **Size**: Medium (in term of Volume)
- **Lob**: Motor Third Party Liability solo
- **Nlob**: 15 years (*full credibility - USPs*)
- **Insurance Portfolio**: all risks which are associated claims and any (*ultimate 1-year*) costs incurred by year (2009-2011 - PIRM).

**Purposes**

- Estimating the impact of the use of USPs
- Defining the «Best Model» with goodness of fit analysis between a GLM or GLM after a GAM analysis («Mixture Model», «GLM(GAM)»)
- A comparison between the SF market parameters, USPs and the standard deviation of the model
Insurance Portfolio

With a one-way analysis we can appreciate the probabilistic assumptions:

### Distrib. Gamma

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treshold</td>
<td>0.9</td>
</tr>
<tr>
<td>Scale</td>
<td>2172</td>
</tr>
<tr>
<td>Shape</td>
<td>2</td>
</tr>
<tr>
<td>Means</td>
<td>4532</td>
</tr>
<tr>
<td>Std Dev</td>
<td>3076</td>
</tr>
</tbody>
</table>

### Moments

<table>
<thead>
<tr>
<th></th>
<th>Total Portfolio</th>
<th>Exposure</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>271,602</td>
<td>3,525,452</td>
</tr>
<tr>
<td>Mean</td>
<td>4,352</td>
<td>4,048</td>
</tr>
<tr>
<td>Std Dev</td>
<td>93</td>
<td>16,386,113</td>
</tr>
<tr>
<td>CoV (%)</td>
<td>93</td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

### Graphs

- Histogram of COSTO_1yr
- Percentiles of gamma (Alfa = 2)

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USP or PIRM under Solvency 2?

30th ICA - R.R. Cerchiara & V. Magatti
### GLM vs Mixture Model

#### Severity

<table>
<thead>
<tr>
<th>Rating Factors</th>
<th>GLM</th>
<th>GLM (GAM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deviance</td>
<td>444,633</td>
<td>-1.0%</td>
</tr>
<tr>
<td>Scaled Deviance</td>
<td>782,667</td>
<td>0.0%</td>
</tr>
<tr>
<td>Dev(s) / GdL</td>
<td>1</td>
<td>0.0%</td>
</tr>
<tr>
<td>Chi-Squared</td>
<td>995,642</td>
<td>-1.0%</td>
</tr>
<tr>
<td>Scaled Chi-Sq.</td>
<td>1,752,581</td>
<td>-1.3%</td>
</tr>
<tr>
<td>Chi-SQ(s)/ GdL</td>
<td>2</td>
<td>9.0%</td>
</tr>
<tr>
<td>AIC</td>
<td>13,338,237</td>
<td>-2.1%</td>
</tr>
<tr>
<td>AICC</td>
<td>13,338,237</td>
<td>-2.2%</td>
</tr>
<tr>
<td>BIC</td>
<td>13,338,977</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

#### Frequency

<table>
<thead>
<tr>
<th>Rating Factors</th>
<th>GLM</th>
<th>GLM (GAM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deviance</td>
<td>1,084,599</td>
<td>-1.5%</td>
</tr>
<tr>
<td>Scaled Deviance</td>
<td>866,789</td>
<td>0.0%</td>
</tr>
<tr>
<td>Dev(s) / GdL</td>
<td>1</td>
<td>0.0%</td>
</tr>
<tr>
<td>Chi-Squared</td>
<td>3,533,096</td>
<td>-1.3%</td>
</tr>
<tr>
<td>Scaled Chi-Sq.</td>
<td>2,823,576</td>
<td>-1.4%</td>
</tr>
<tr>
<td>Chi-SQ(s)/ GdL</td>
<td>3.3</td>
<td>-0.5%</td>
</tr>
<tr>
<td>AIC</td>
<td>1,196,920</td>
<td>-3.0%</td>
</tr>
<tr>
<td>AICC</td>
<td>1,196,920</td>
<td>-2.8%</td>
</tr>
<tr>
<td>BIC</td>
<td>1,197,586</td>
<td>-1.0%</td>
</tr>
</tbody>
</table>
Premium Model

Before a convolution of the frequency/severity model:

<table>
<thead>
<tr>
<th>Observed</th>
<th>GLM</th>
<th>GLM(GAM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Claim Cost (Y)</td>
<td>1,657,230,759</td>
<td>- 0.5%</td>
</tr>
<tr>
<td>Number of Claim (N)</td>
<td>782,724</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

- No difference for frequency
- Mixture Model is better than GLM

After the convolution between frequency and severity model in case of the Gamma distribution:

<table>
<thead>
<tr>
<th></th>
<th>GLM</th>
<th>GLM (GAM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating Factors</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Deviance</td>
<td>138,501</td>
<td>-0.5%</td>
</tr>
<tr>
<td>Scaled Deviance</td>
<td>866,806</td>
<td>0.0%</td>
</tr>
<tr>
<td>Dev(s) / GdL</td>
<td>1</td>
<td>0.0%</td>
</tr>
<tr>
<td>Chi-Squared</td>
<td>194,889</td>
<td>-1.1%</td>
</tr>
<tr>
<td>Scaled Chi-Sq.</td>
<td>1,219,712</td>
<td>-1.5%</td>
</tr>
<tr>
<td>Chi-SQ(s)/ GdL</td>
<td>1</td>
<td>-0.0%</td>
</tr>
<tr>
<td>AIC</td>
<td>10,683,615</td>
<td>-0.6%</td>
</tr>
<tr>
<td>AICC</td>
<td>10,683,615</td>
<td>-0.0%</td>
</tr>
<tr>
<td>BIC</td>
<td>10,684,081</td>
<td>-0.7%</td>
</tr>
</tbody>
</table>
How about volatility?

**Perimeter**

Case Study
- **Size**: Medium (in term of Volume)
- **Lob**: Motor Third Party Liability solo
- **Nlob**: 15 years

\[ \sigma^{SF}_{(M, prem, lob)} = 10\% \]
\[ c = 100\% \]

PIRMs allow a considerable saving in term of SCR for the *Premium Risk* thanks to a model already used by Pricing Staff.

<table>
<thead>
<tr>
<th>Market (SF)</th>
<th>USP Meth.1</th>
<th>USP Meth.2</th>
<th>USP Meth.3</th>
<th>GLM</th>
<th>GLM(GAM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0%</td>
<td>8.0%</td>
<td>7.8%</td>
<td>9.1%</td>
<td>4.0%</td>
<td>3.9%</td>
</tr>
</tbody>
</table>

In this case study → Small difference between the two PIRMs
## Conclusions

### Standard Formula
- Factor based or scenario based
- Quite simple to deploy

### USPs
- Risk based on the historical data
- The volatility could be lower than SF
- Data: complete, accurate and appropriate
- Supervisor Pre-Approval process

### PIRM
- Strength connection → Pricing/Premium Risk
- The volatility could be lower than USPs
- More detailed Pre-Application process than USPs

### Future Developments - PIRM
- Determine the 99.5% percentile of the ACA distribution
- Explore other statistical models to evaluate the random effects (e.g. GEE and GLMM)
- Evaluate different models for Attritional Losses and Large Losses (e.g. GLM within a Quasi-Likelihood approach)
- Use an (Ultimate 1-year) Aggregate Claim Amount net of the reinsurance
- Check the model for a total MTPL business (car, motorcycle, moped, etc.)
- Define a way to aggregate different LoBs and discover the correlation with Reserve/CAT risk

<table>
<thead>
<tr>
<th>Pro</th>
<th>Contro</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor based or scenario based</td>
<td>It couldn’t take into account the real risk profile</td>
</tr>
<tr>
<td>Quite simple to deploy</td>
<td></td>
</tr>
<tr>
<td>Risk based on the historical data</td>
<td>Data: complete, accurate and appropriate</td>
</tr>
<tr>
<td>The volatility could be lower than SF</td>
<td>Supervisor Pre-Approval process</td>
</tr>
<tr>
<td>Strength connection → Pricing/Premium Risk</td>
<td>More detailed Pre-Application process than USPs</td>
</tr>
<tr>
<td>The volatility could be lower than USPs</td>
<td></td>
</tr>
</tbody>
</table>
Thank you
Main References


