Modelling Dynamic PHB through Machine Learning Techniques

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SAPIENZA UNIVERSITÀ DI ROMA

ORDINE NAZIONALE DEGLI ATTUARI

GENERALI

Agenda

1. Some Analytics in Insurance
2. Dynamic PHB
3. Modelling PHB
4. Case Studies

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1. Some Analytics in Insurance
2. Dynamic PHB
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Some Analytics in Insurance

- *Market Basket Analysis*
- *Churn Rate Prediction*
- *Non-Life Pricing*
- *Individual Claim Reserving*
- *Cat Bond Parametric Triggers*
- *Dynamic Policyholder Behaviour*

Some Analytics in Insurance

- *Market Basket Analysis*
- *Churn Rate Prediction*
- *Non-Life Pricing*
- *Individual Claim Reserving*
- *Cat Bond Parametric Triggers*
- *Dynamic Policyholder Behaviour*

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Dynamic PHB – *Regulatory reasons (SII)*

QIS5, SCR.7.44 → *Lapse risk is the risk of loss or change in liabilities due to a change in the expected exercise rates of policyholder options*

QIS5, TP.2.46 → *The valuation of premium provisions should take into account the future policyholder behaviour such as likelihood of policy lapse during the remaining period*

QIS5, TP.2.107 → *Assumptions about the likelihood that policyholders will exercise contractual options should be based on analysis of past policyholder behaviour*

Dynamic PHB – *Regulatory reasons (IFRS)*

IFRS 17, B62 → *The measurement of a group of insurance contracts shall reflect, on an expected value basis, the entity’s current estimates of how the policyholders in the group will exercise the options available, and the risk adjustment for non-financial risk shall reflect the entity’s current estimates of how the actual behaviour of the policyholders may differ from the expected behaviour. This requirement to determine the expected value applies regardless of the number of contracts in a group; for example it applies even if the group comprises a single contract.*

Dynamic PHB – *Empirical reasons*

Emergency Fund Hypothesis	*Interest Rate Hypothesis*
Lapse risk is mainly driven by a natural – and irrational to some extent – response to the need of money due to personal conditions in time of distress	Lapse risk is mainly driven by rational reasonings (e.g., interest rate arbitrage and preference for different products) due to the policyholder‘s risk appetite

Policyholder-related factors
- Age
- Family
- Salary
- …

Product-related factors
- Duration
- Premium
- Guaranteed rate
- …

Macroeconomic factors
- Market yields
- Unemployment rate
- Gross domestic product
- …

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Modelling PHB – *Lapse rate estimation*

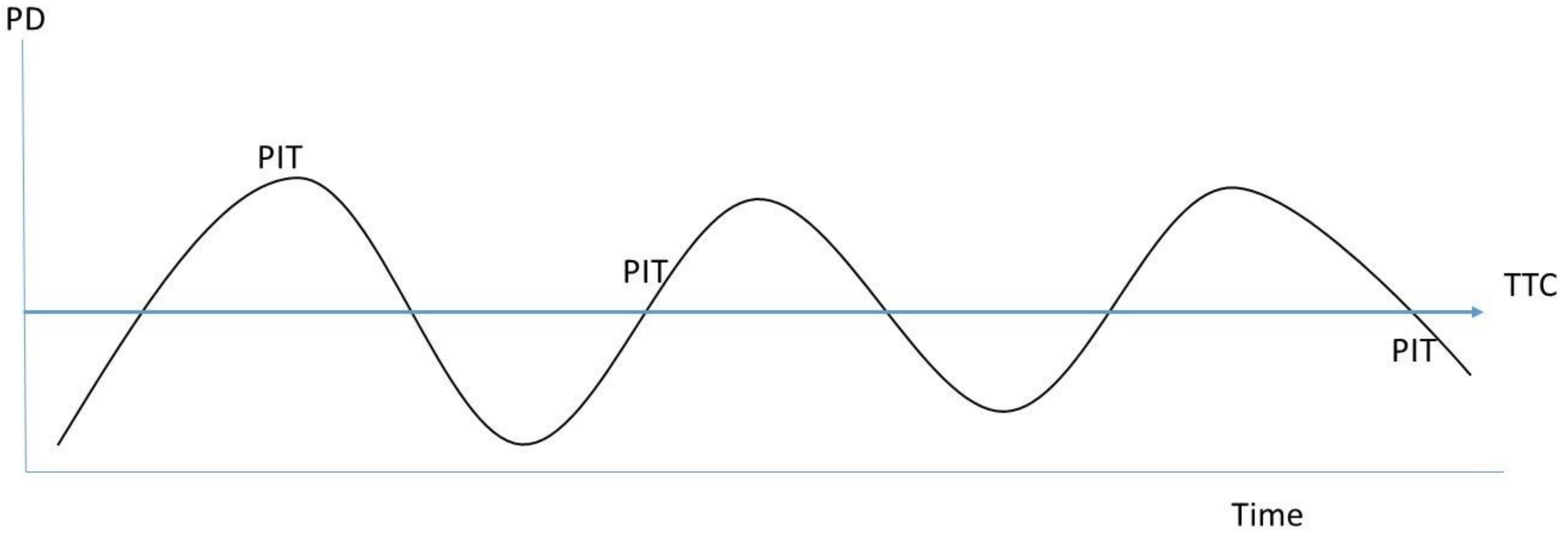
Point-In-Time (PIT)	*Through-The-Cycle (TTC)*
PIT models explicitly control for the state of the specific contract as well as the overall economy by estimating time-dependent probabilities	TTC models generally abstract from any variable condition by including only static risk drivers known at the beginning of the analysis

Parametric techniques
- **Logit model**
- Probit model
- Discriminant analysis
- …

Machine learning methods
- Nearest neighbours
- **Classification trees**
- Neural networks
- …

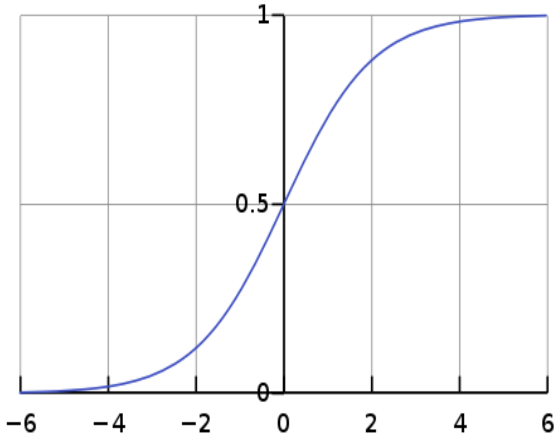
Survival analysis models
- Kaplan-Meier estimator
- **AFT models**
- CPH models
- …

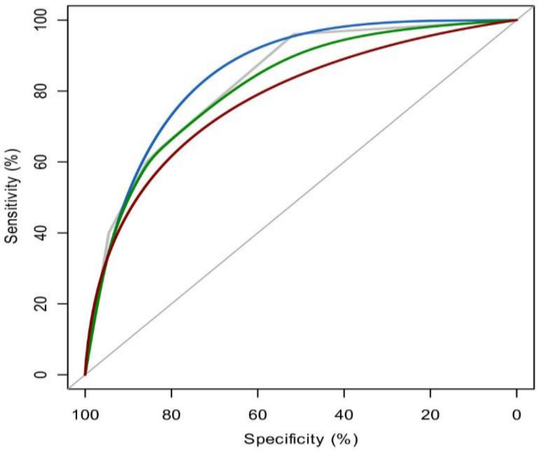
Modelling PHB – *Lapse rate estimation*

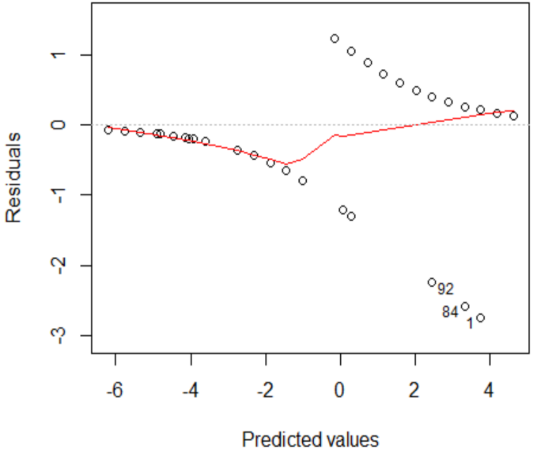


Modelling PHB – *Logit model*

$$\operatorname{logit} p(x) = \ln\left(\frac{p(x)}{1-p(x)}\right) = \beta_0 + \beta_1 x \quad \Leftrightarrow \quad \frac{p(x)}{1-p(x)} = e^{\beta_0+\beta_1 x} \quad \Leftrightarrow \quad p(x) = \frac{1}{1+e^{-(\beta_0+\beta_1 x)}}$$



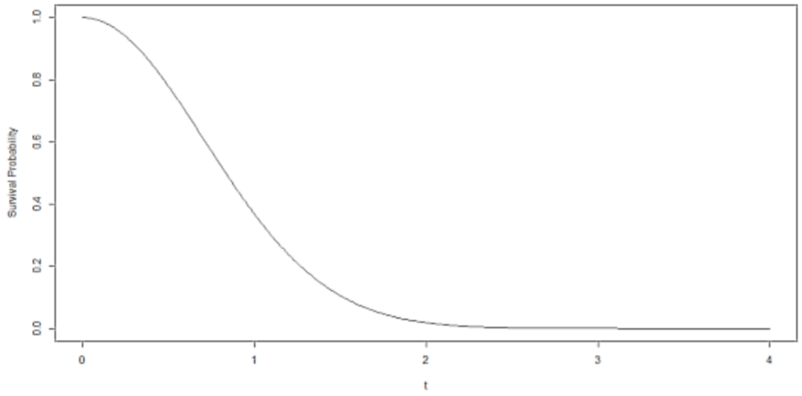




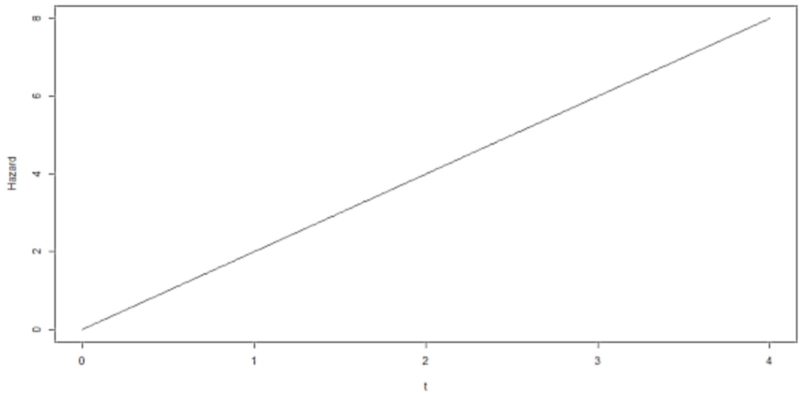
Modelling PHB – *AFT models*

$$\log(T) = \beta' \boldsymbol{x} + \sigma \epsilon$$

$$S(t \mid \boldsymbol{x}) = S_0\left(t \cdot \exp(-\beta' \boldsymbol{x})\right)$$



$$h(t \mid \boldsymbol{x}) = h_0\left(t \cdot \exp(-\beta' \boldsymbol{x})\right) \exp(-\beta' \boldsymbol{x})$$



Modelling PHB – *AFT models*

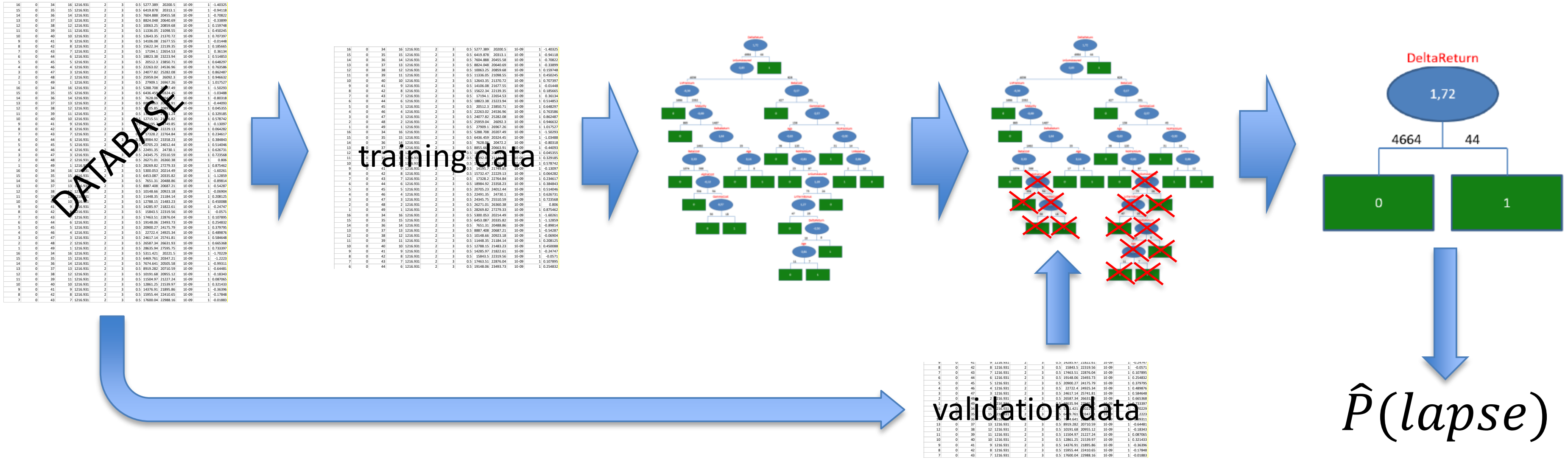
$$\log(T) = \beta' \boldsymbol{x} + \sigma \epsilon$$

$S(t) = \exp(-\lambda t)$ ⟺ *Exponential* ⟺ $h(t) = \lambda$

$S(t) = \exp(-\lambda t^p)$ ⟺ *Weibull* ⟺ $h(t) = \lambda p t^{p-1}$

$S(t) = \dfrac{1}{1 + \exp(\theta) t^\kappa}$ ⟺ *Log-Logistic* ⟺ $h(t) = \dfrac{\exp(\theta) \kappa t^{\kappa-1}}{1 + \exp(\theta) t^\kappa}$

Modelling PHB – *Classification trees*



Modelling PHB – *Classification trees*

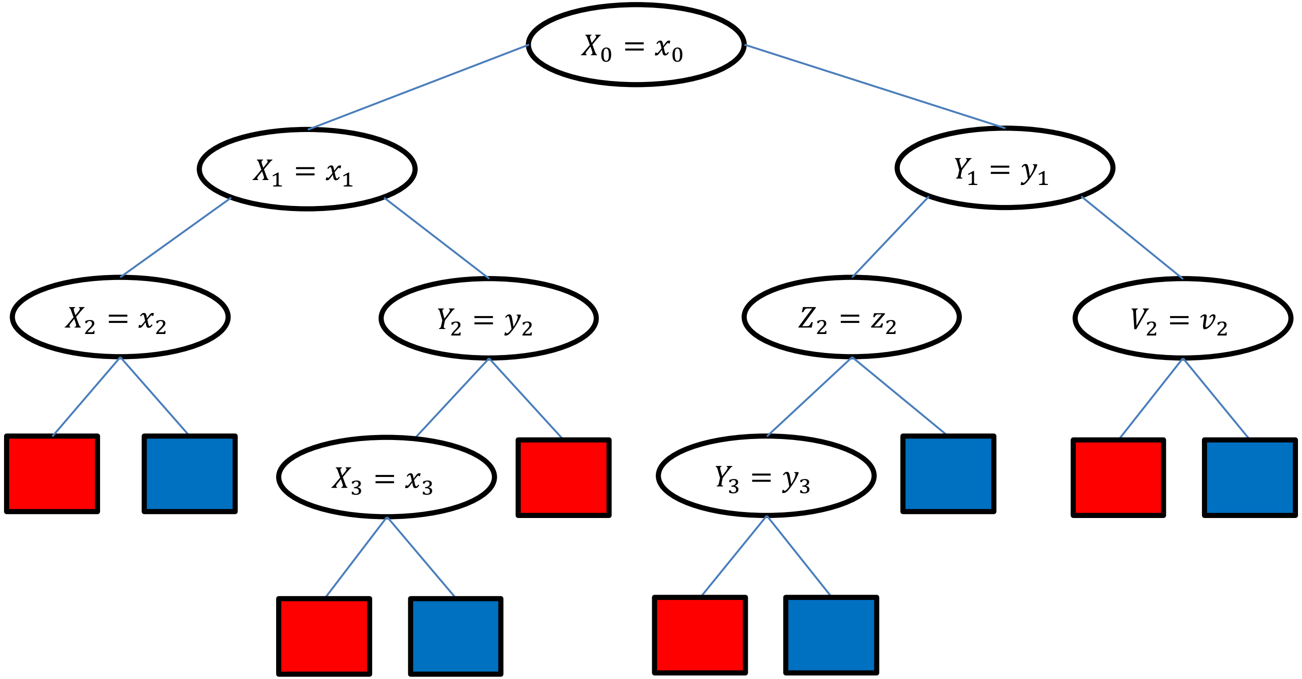
- *Recursive partitioning*

1. Uno degli n predittori viene selezionato assieme ad uno dei suoi possibili valori come candidati per splittare A in A_L and A_R.
2. Viene calcolata la riduzione di impurità data dallo split:

$$\Delta I = p(A)I(A) - p(A_L)I(A_L) - p(A_R)I(A_R)$$

3. Se lo split massimizza la riduzione di impurità, i dati vengono splittati e si va al passo 4, altrimenti si torna al passo 1.
4. Si ripetono i passi 1-3 finché non è più possibile eseguire split, oppure una certa condizione di tolleranza si verifica.

Modelling PHB – *Classification trees*

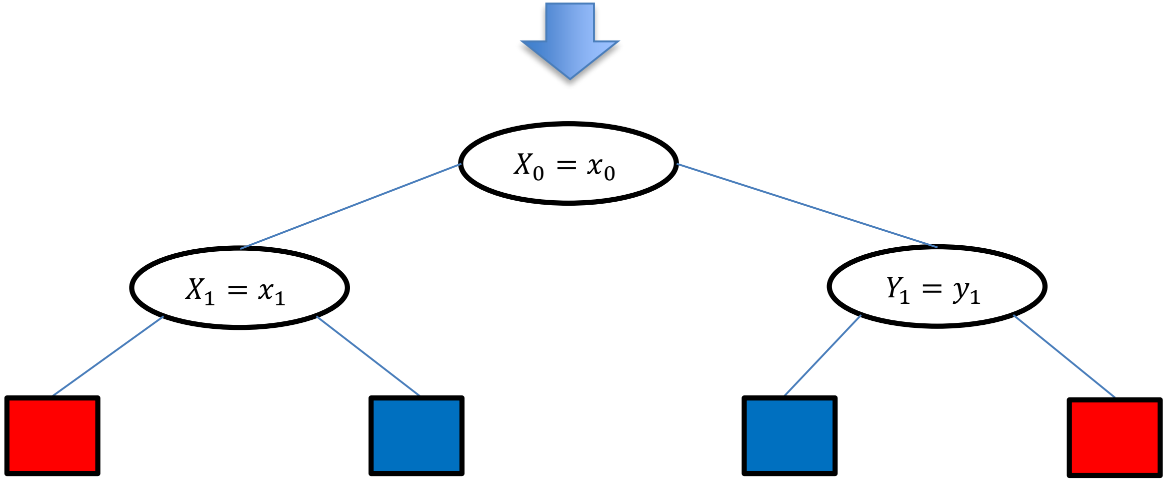


Modelling PHB – *Classification trees*

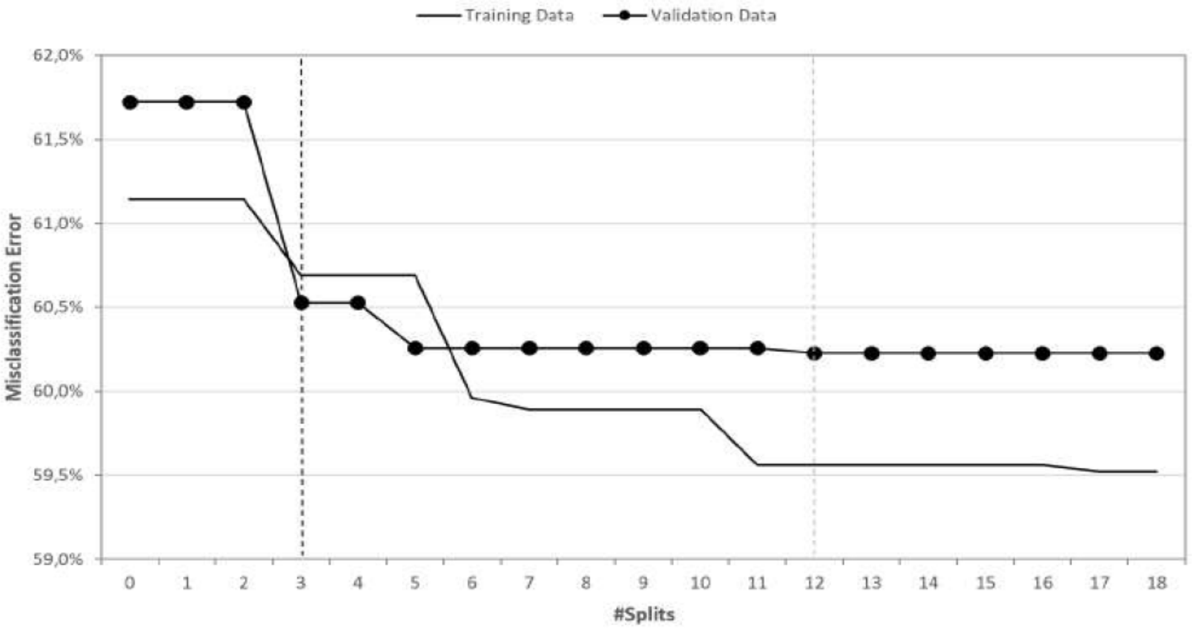
- *Pruning the tree*
  - La partizione ricorsiva appena vista rende un albero il più *puro* possibile, poiché ad ogni regione appartengono esclusivamente record della stessa classe, e l’errore commesso è nullo
    - L’albero costruito a partire dal training dataset è puro per definizione (*full-grown tree*), poiché utilizza i suoi stessi nodi decisori
  - Quando questo non si verifica, l’albero si dice *spurio*
    - Dato che gli ultimi livelli dell’albero puro causano overfitting (ossia fittano il «rumore» piuttosto che l’informazione), il validation dataset viene usato per eliminare i rami superflui in modo da minimizzare il **complexity cost**

Modelling PHB – *Classification trees*

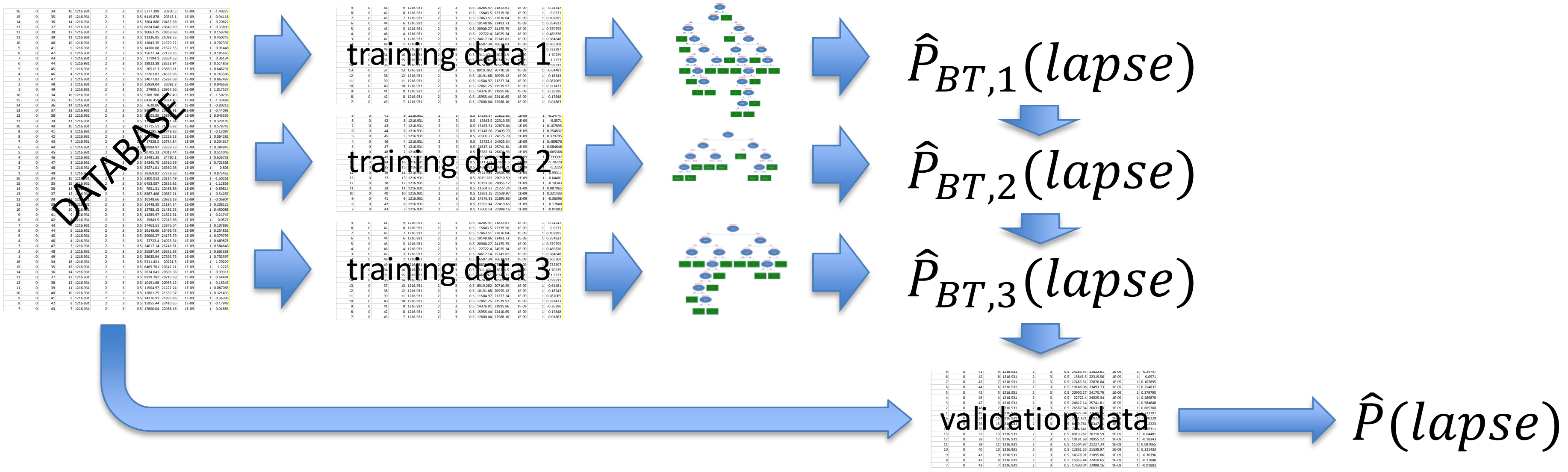
$$CC(n, n-1) := \frac{Error(T_{n-1}) - Error(T_n)}{\#Leaves(T_n) - \#Leaves(T_{n-1})}$$



Modelling PHB – *Classification trees*



Modelling PHB – *Bagging trees*



Modelling PHB – *Bagging trees*

- *Bagging algorithm*
  1. Estrarre un campione dal training dataset
  2. Generare l'albero (puro) relativo al campione
  3. Aggiornare la stime di ogni record come media delle stime ottenute da tutti gli alberi a disposizione:

$$\hat{\theta}_n^{bag}(record) \coloneqq \frac{1}{n}\sum_{k=1}^{n} \hat{\theta}_k(record)$$

  4. Calcolare il training error e il validation error utilizzando tutti gli alberi a disposizione.
  5. Si ripetono i passi 1-4 finché una certa condizione di tolleranza si verifica.

Modelling PHB – *Bagging trees*

- *Random forest*
  1. Estrarre un campione dal training dataset e un insieme casuale di predittori
  2. Generare l'albero (puro) relativo al campione
  3. Aggiornare la stime di ogni record come media delle stime ottenute da tutti gli alberi a disposizione:

$$\hat{\theta}_n^{bag}(record) := \frac{1}{n}\sum_{k=1}^{n} \hat{\theta}_k(record)$$

  4. Calcolare il training error e il validation error utilizzando tutti gli alberi a disposizione.
  5. Si ripetono i passi 1-4 finché una certa condizione di tolleranza si verifica.

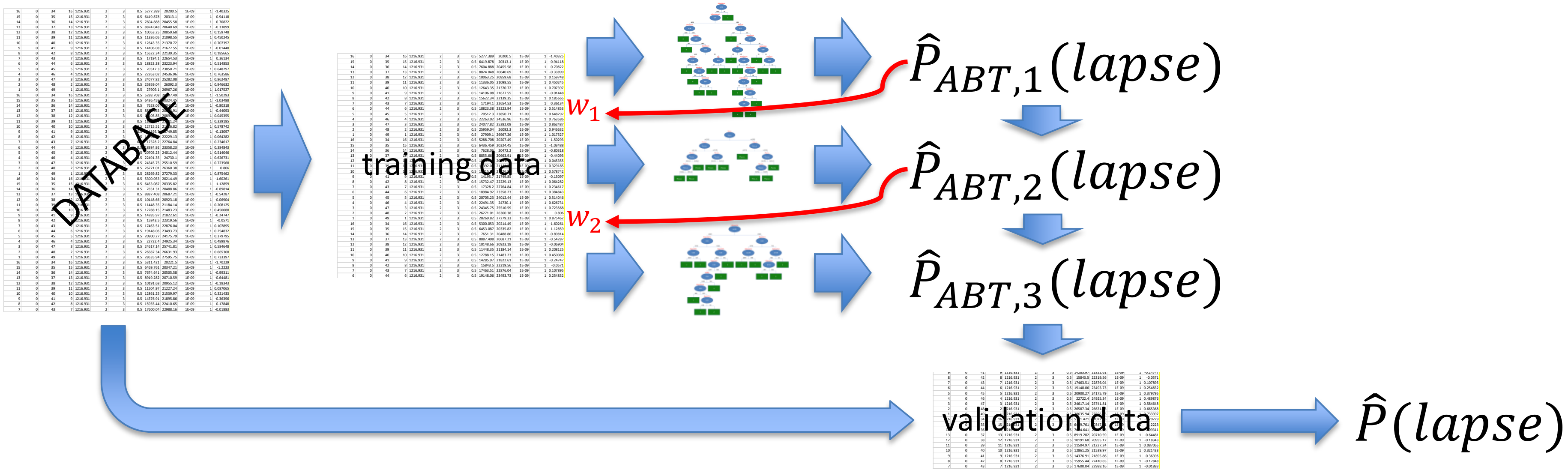
Modelling PHB – *Bagging trees*

- *More generally…*
  1. Estrarre un campione dal training dataset **e introdurre varianza**
  2. Generare l'albero (puro) relativo al campione
  3. Aggiornare la stime di ogni record come media delle stime ottenute da tutti gli alberi a disposizione:

$$\hat{\theta}_n^{bag}(record) := \frac{1}{n}\sum_{k=1}^{n}\hat{\theta}_k(record)$$

  4. Calcolare il training error e il validation error utilizzando tutti gli alberi a disposizione.
  5. Si ripetono i passi 1-4 finché una certa condizione di tolleranza si verifica.

Modelling PHB – *Boosting trees*



Modelling PHB – *Boosting trees*

Informally, ***hypothesis boosting problem*** *asks whether an efficient learning algorithm* […] *that outputs an hypothesis whose performance is only slightly better than random guessing implies the existence of an efficient algorithm that outputs an hypothesis of arbitrary accuracy**

* M. Kearns, *Thoughts on Hypothesis Boosting*, unpublished manuscript, 1988

Modelling PHB – *Boosting trees*

*A model of learnability in which the learner is only required to perform slightly better than guessing is as strong as a model in which the learner's error can be made arbitrarily small**

* R. E. Schapire, The Strength of Weak Learnability, Machine Learning, Kluwer Academic Publishers 5 (2), pp. 197-227, 1990

Modelling PHB – *Boosting trees*

- *Boosting algorithm*
  1. Scegliere un'intero *d* per l'*interaction depth* e un λ per il *learning rate*
  2. Definire un valore iniziale $\hat{\theta}_0^{boost}$ per la stima
  3. Calcolare gli errori $\varepsilon_k(record)$ sul training dataset tramite le stime correnti
  4. Generare un albero a *d* livelli con il training dataset per stimare gli errori $\varepsilon_k(record)$
  5. Aggiornare le stime di ogni record:

$$\hat{\theta}_n^{boost}(record) \coloneqq \hat{\theta}_0^{boost} + \sum_{k=1}^{n} \lambda \hat{\varepsilon}_k(record)$$

  6. Si ripetono i passi 3-5 finché una certa condizione di tolleranza si verifica.

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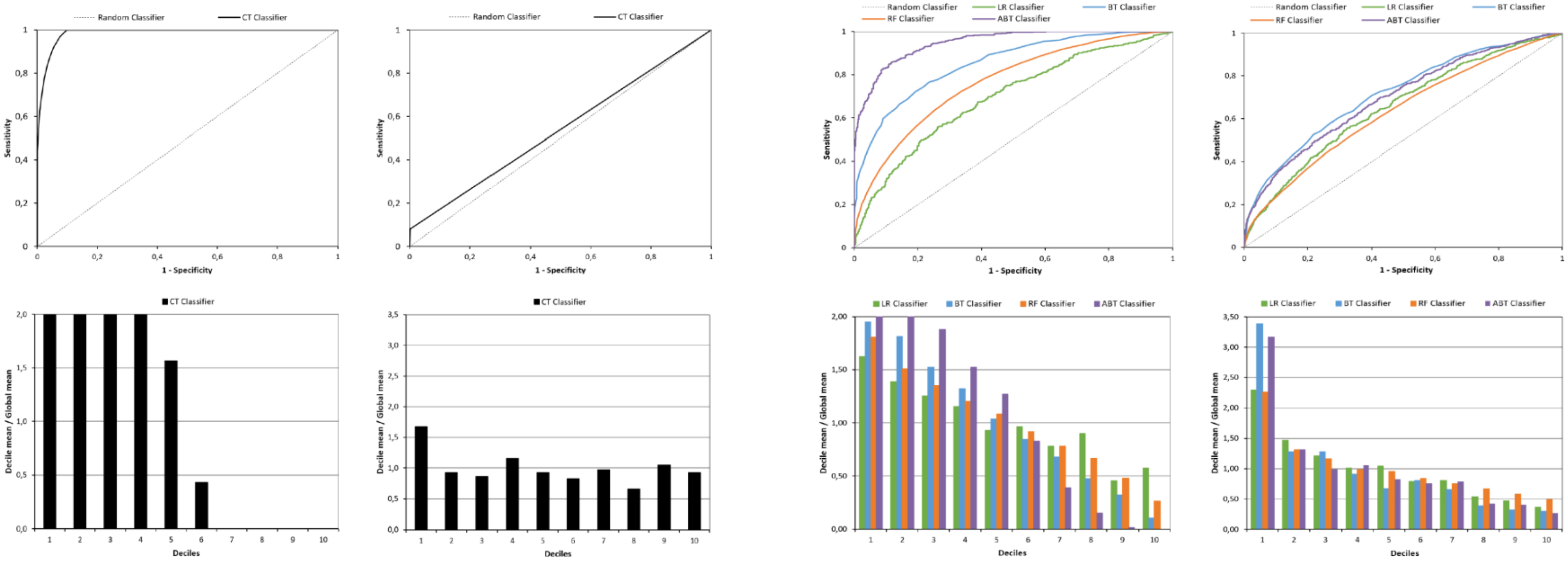
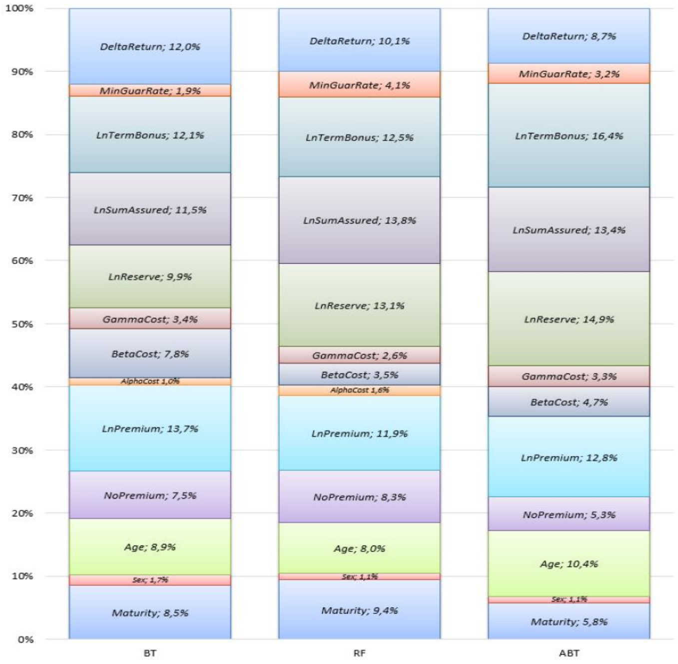
Case Study 1 – *Database*

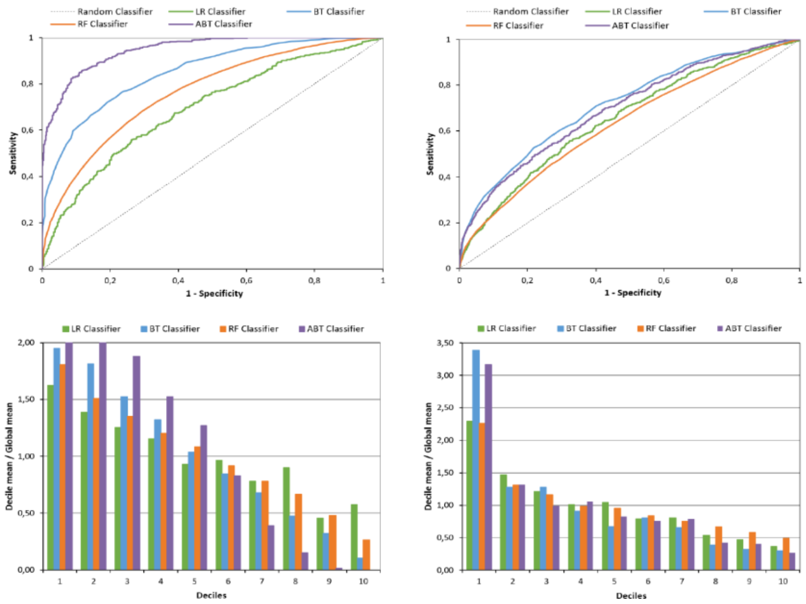
- 11.700 endowment products sold in Italy
- Portfolio at 31.12.2005
- Surrenders observed by 31.12.2006
- Policyholder-related fields (sex and age)
- Product-related fields
  - tariff (premium, costs, guarantee)
  - status (maturity, reserve, sum assured, terminal bonus)
- Macroeconomic fields (delta return)

Case Study 1 – *Lapse prediction*



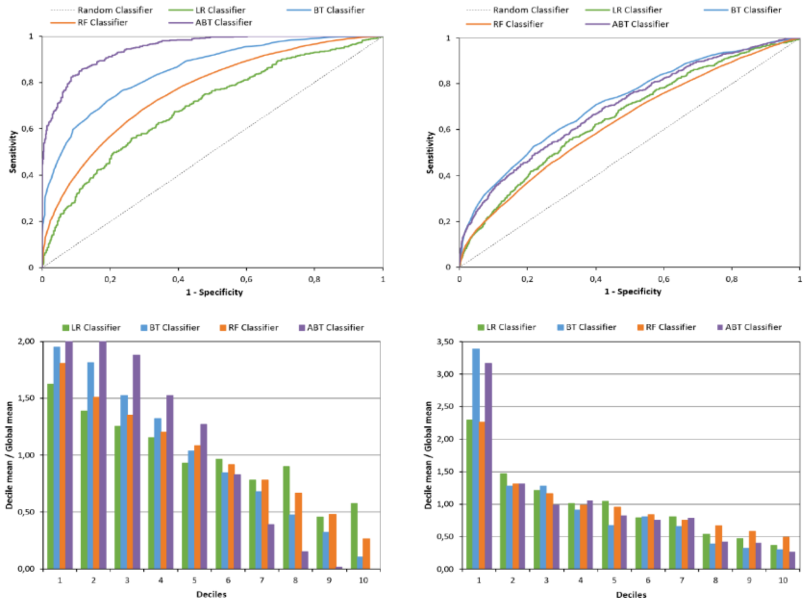
Case Study 1 – *Lapse prediction*





Case Study 1 – *Lapse prediction*

METHOD	TRAINING AUC	VALIDATION AUC
Logistic reg.	69%	65%
Bagging trees	80%	72%
Random forest	77%	64%
Boosting trees	92%	70%



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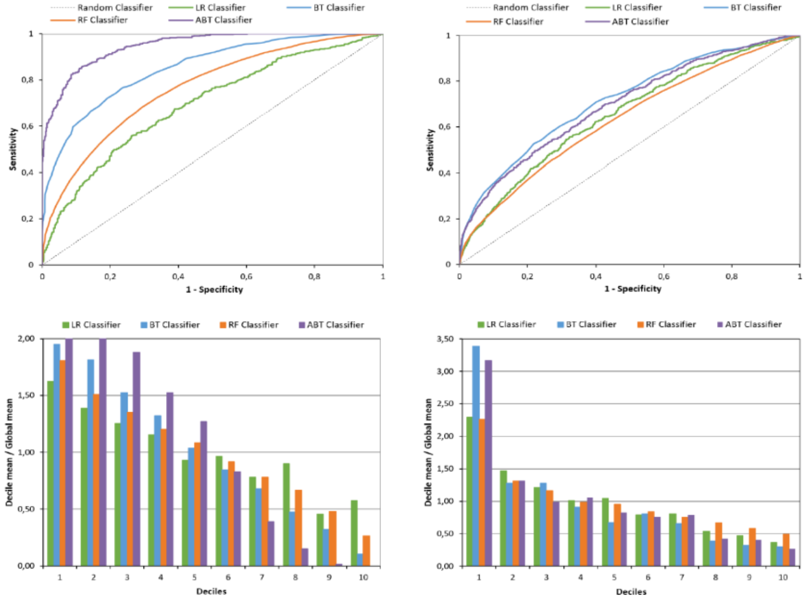

Case Study 1 – *Segregated fund modelling*

- Interest rate dynamic (G2++)

$$dX_t = -\mu_x X_t dt + \sigma_x dZ_t^x$$
$$dY_t = -\mu_y Y_t dt + \sigma_y dZ_t^y$$
$$dZ_t^x dZ_t^y = \rho dt$$

$$i_t := \overbrace{X_t + Y_t}^{} + \underbrace{\phi(t)}_{}$$

$$f(t) + \frac{\sigma_x^2}{2\mu_x^2}(1 - e^{-\mu_x t})^2 + \frac{\sigma_y^2}{2\mu_y^2}(1 - e^{-\mu_y t})^2 + \frac{\rho\sigma_x\sigma_y}{\mu_x\mu_y}(1 - e^{-\mu_x t})(1 - e^{-\mu_y t})$$

Case Study 1 – *Segregated fund modelling*

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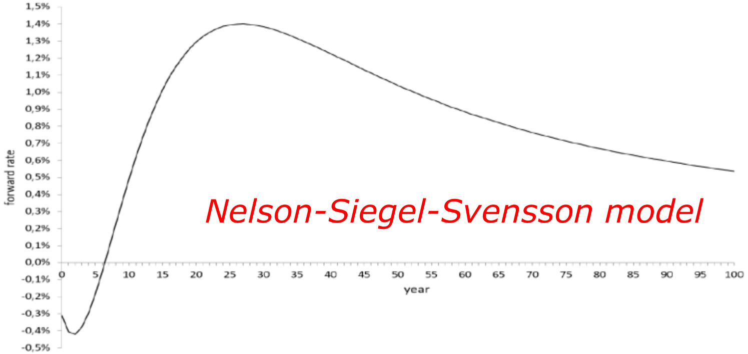

Nelson-Siegel-Svensson model

Case Study 1 – *Segregated fund modelling*

- Bond yield dynamic (G2++ with deterministic spread)

$$dX_t = -\mu_x X_t dt + \sigma_x dZ_t^x$$
$$dY_t = -\mu_y Y_t dt + \sigma_y dZ_t^y$$
$$dZ_t^x dZ_t^y = \rho dt$$

$$r_t := X_t + Y_t + \phi^*(t)$$

$$f(t) + \left(\frac{\sigma_x^2}{2\mu_x^2} + d_x\right)(1 - e^{-\mu_x t})^2 + \left(\frac{\sigma_y^2}{2\mu_y^2} + d_y\right)(1 - e^{-\mu_y t})^2 + \frac{\rho \sigma_x \sigma_y}{\mu_x \mu_y}(1 - e^{-\mu_x t})(1 - e^{-\mu_y t})$$

parameter	BTP1	BTP2	BTP3
b_i	30%	30%	30%
c_i	1,0%	3,0%	5,0%
t_i	0 yrs	10 yrs	23 yrs
T_i	10 yrs	15 yrs	30 yrs

Case Study 1 – *Segregated fund modelling*

- Equity dynamic (geometric brownian motion)

G2++ → r_τ

risk premium → μ_S

volatility → σ_S

$$S_t = S_0 e^{\int_0^t r_\tau d\tau + \left(\mu_S - \frac{\sigma_S^2}{2}\right)t + \sigma_S Z_t^S}$$

parameter	FTSE	EURO	S&P
$1-b$	10%		
μ_S	-1,96%	2,46%	9,35%
σ_S	20,22%	12,15%	10,83%

Case Study 1 – *Segregated fund modelling*

- Segregated fund dynamic
  - Segregated fund rate → $g(t) := bR_C(t) + (1-b)R_S(t)$
  - Crediting rate → $R(t) := \max\{\min\{\eta g(t), g(t) - k\}, \varrho\}$
  - Premium amount → $\Pi = \dfrac{P(1+l)}{1-\alpha-\beta-\gamma}$
  - Expenses → $E_0 = \alpha\Pi \qquad E_t = (\beta+\gamma)\Pi$
  - Sum assured → $S_0 = P\dfrac{\ddot{a}_{\overline{x:n}|}}{{}_nE_x} \qquad S_t = \dfrac{V_{t-1} + P\ddot{a}_{\overline{t+x:n-t}|}}{{}_{n-t}E_{x+t}}$
  - Mathematical reserve → $V_0 = P \qquad V_t = V_{t-1}[1+R(t)] + P$

parameter	value
η	90%
ϱ	1,0%
k	0,2%
n	20 yrs
x	30 yrs
P	1000
l	15%
α	2,0%
β	3,0%
γ	0,5%

Case Study 1 – *Segregated fund modelling*

- Segregated fund dynamic
  - Profit & Loss → $$P\&L_t = \begin{cases} \Pi - E_0 & t = 0 \\ \overline{\Pi}_{t-1} + \overline{V}_{t-1}R(t) - \overline{E}_t - \Delta\overline{V}_{t-1,t} & \forall t = 1,\dots,n-1 \\ \overline{V}_{n-1}R(n) - \overline{E}_n - (\overline{S}_n - \overline{V}_{n-1}) & t = n. \end{cases}$$
  - Discounted P&L → $$D_t(\cdot) := \sum_{\tau=1}^{t} P\&L_\tau(\cdot)v_\tau(\cdot)$$
  - Mean discounted P&L → $$D(\cdot) := \sum_{t=1}^{n} p_t(\cdot)D_t(\cdot) = \sum_{t=1}^{n} p_t(\cdot)\sum_{\tau=1}^{t} P\&L_\tau(\cdot)v_\tau(\cdot)$$

Case Study 1 – *Segregated fund modelling*

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$$P_{t-1}(L_t)\prod_{\tau=1}^{t-1}(1 - P_{\tau-1}(L_\tau))$$

Case Study 1 – *Segregated fund modelling*

- Segregated fund dynamic
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$$P_{t-1}(L_t)\prod_{\tau=1}^{t-1}(1 - P_{\tau-1}(L_\tau))$$

Case Study 1 – *Profit metrics*

- P&L
  - Discounted P&L

$$D[ce, p(ce)] := \sum_{t=1}^{n} p_t(ce) D_t(ce)$$

$$D[k, p(ce)] := \sum_{t=1}^{n} p_t(ce) D_t(k) \longrightarrow \frac{1}{N}\sum_{k=1}^{N} D[k, p(ce)]$$

$$D[k, p(k)] := \sum_{t=1}^{n} p_t(k) D_t(k) \longrightarrow \frac{1}{N}\sum_{k=1}^{N} D[k, p(k)]$$

$$D[k, t^*(k)] := D_{t^*(k)}(k) \longrightarrow \frac{1}{N}\sum_{k=1}^{N} D[k, t^*(k)]$$

Case Study 1 – *Profit metrics*

- P&L and TVOG
    - Discounted P&L

$$D[ce, p(ce)] := \sum_{t=1}^{n} p_t(ce) D_t(ce)$$

$$D[k, p(ce)] := \sum_{t=1}^{n} p_t(ce) D_t(k) \longrightarrow \frac{1}{N}\sum_{k=1}^{N} D[k, p(ce)] \longrightarrow D[ce, p(ce)] - D[N, p(ce)]$$

$$D[k, p(k)] := \sum_{t=1}^{n} p_t(k) D_t(k) \longrightarrow \frac{1}{N}\sum_{k=1}^{N} D[k, p(k)] \longrightarrow D[ce, p(ce)] - D[N, p(N)]$$

$$D[k, t^*(k)] := D_{t^*(k)}(k) \longrightarrow \frac{1}{N}\sum_{k=1}^{N} D[k, t^*(k)] \longrightarrow D[ce, p(ce)] - D[N, t^*(N)]$$

Case Study 1 – *Profit metrics*

- P&L and TVOG

PHB-free TVOG → $D[ce, p(ce)] - D[N, p(ce)]$

lapse-rate TVOG → $D[ce, p(ce)] - D[N, p(N)]$

time-to-lapse TVOG → $D[ce, p(ce)] - D[N, t^*(N)]$

Case Study 1 – *Profit metrics*

- P&L and TVOG components

$$1 = \underbrace{\frac{TVOG[N,p(ce)]}{TVOG[N,p(N)]}}_{\text{guarantee}} + \underbrace{\frac{V_{PHB}(p)}{TVOG[N,p(N)]}}_{\text{PHB}}$$

$$1 = \frac{TVOG[N,p(ce)]}{TVOG[N,t^*(N)]} + \frac{V_{PHB}(t^*)}{TVOG[N,t^*(N)]}$$

PHB-free TVOG → $D[ce,p(ce)] - D[N,p(ce)]$

lapse-rate TVOG → $D[ce,p(ce)] - D[N,p(N)]$

time-to-lapse TVOG → $D[ce,p(ce)] - D[N,t^*(N)]$

Case Study 1 – *Results*

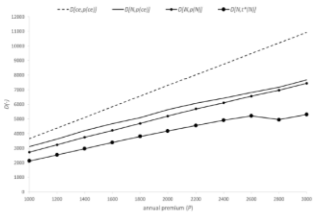


Fig. 4.36: Profit metrics by annual fair premium (EURO STOXX 50 case)

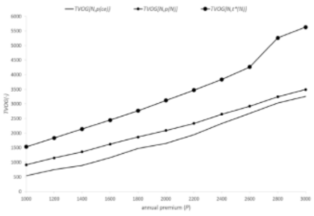


Fig. 4.37: TVOG metrics by annual fair premium (EURO STOXX 50 case)

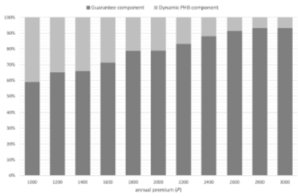


Fig. 4.64: $TVOG[N, p(N)]$ guarantee-PHB split by annual fair premium (EURO STOXX 50 case)

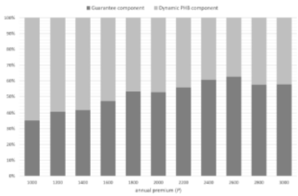


Fig. 4.65: $TVOG[N, t^*(N)]$ guarantee-PHB split by annual fair premium (EURO STOXX 50 case)

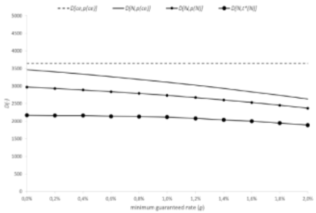


Fig. 4.38: Profit metrics by minimum guaranteed rate (EURO STOXX 50 case)

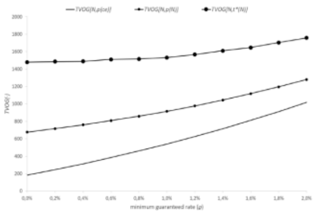


Fig. 4.39: TVOG metrics by minimum guaranteed rate (EURO STOXX 50 case)

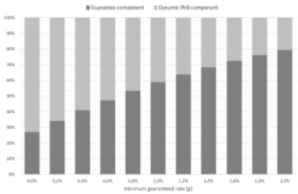


Fig. 4.66: $TVOG[N, p(N)]$ guarantee-PHB split by minimum guaranteed rate (EURO STOXX 50 case)

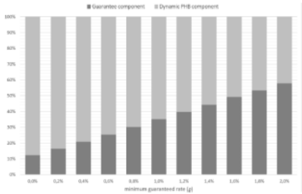


Fig. 4.67: $TVOG[N, t^*(N)]$ guarantee-PHB split by minimum guaranteed rate (EURO STOXX 50 case)

Case Study 1 – *Results*

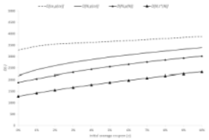


Fig. 4.40: Profit metrics by initial average coupon (FTSE MIB case)

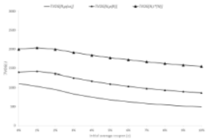


Fig. 4.41: TVOG metrics by initial average coupon (FTSE MIB case)

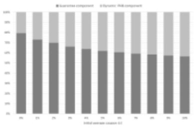


Fig. 4.68: $TVOG[N, p(N)]$ guarantee-PHB split by initial average coupon (FTSE MIB case)

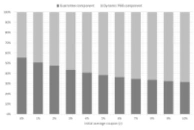


Fig. 4.69: $TVOG[N, t^*(N)]$ guarantee-PHB split by initial average coupon (FTSE MIB case)

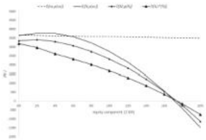


Fig. 4.46: Profit metrics by equity percentage (FTSE MIB case)

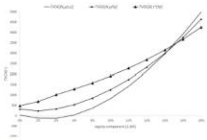


Fig. 4.47: TVOG metrics by equity percentage (FTSE MIB case)

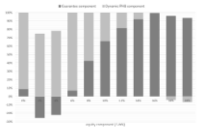


Fig. 4.74: $TVOG[N, p(N)]$ guarantee-PHB split by equity percentage (FTSE MIB case)

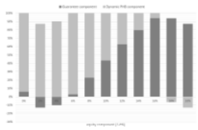


Fig. 4.75: $TVOG[N, t^*(N)]$ guarantee-PHB split by equity percentage (FTSE MIB case)

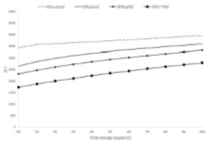


Fig. 4.42: Profit metrics by initial average coupon (EURO STOXX 50 case)

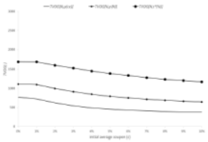


Fig. 4.43: TVOG metrics by initial average coupon (EURO STOXX 50 case)

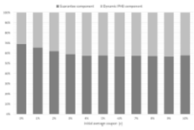


Fig. 4.70: $TVOG[N, p(N)]$ guarantee-PHB split by initial average coupon (EURO STOXX 50 case)

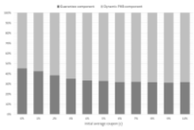


Fig. 4.71: $TVOG[N, t^*(N)]$ guarantee-PHB split by initial average coupon (EURO STOXX 50 case)

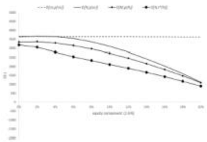


Fig. 4.48: Profit metrics by equity percentage (EURO STOXX 50 case)

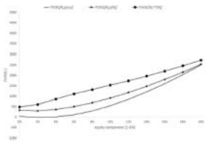


Fig. 4.49: TVOG metrics by equity percentage (EURO STOXX 50 case)

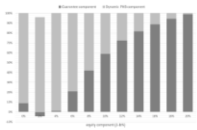


Fig. 4.76: $TVOG[N, p(N)]$ guarantee-PHB split by equity percentage (EURO STOXX 50 case)

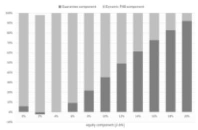


Fig. 4.77: $TVOG[N, t^*(N)]$ guarantee-PHB split by equity percentage (EURO STOXX 50 case)

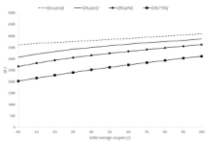


Fig. 4.44: Profit metrics by initial average coupon (S&P 500 case)

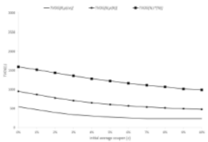


Fig. 4.45: TVOG metrics by initial average coupon (S&P 500 case)

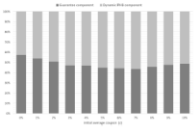


Fig. 4.72: $TVOG[N, p(N)]$ guarantee-PHB split by initial average coupon (S&P 500 case)

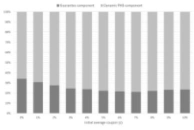


Fig. 4.73: $TVOG[N, t^*(N)]$ guarantee-PHB split by initial average coupon (S&P 500 case)

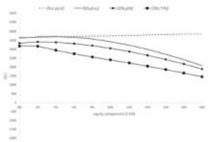


Fig. 4.50: Profit metrics by equity percentage (S&P 500 case)

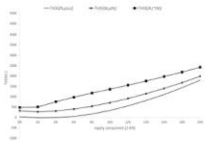


Fig. 4.51: TVOG metrics by equity percentage (S&P 500 case)

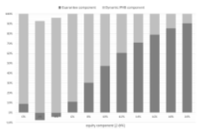


Fig. 4.78: $TVOG[N, p(N)]$ guarantee-PHB split by equity percentage (S&P 500 case)

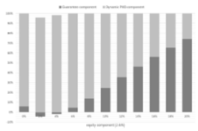


Fig. 4.79: $TVOG[N, t^*(N)]$ guarantee-PHB split by equity percentage (S&P 500 case)

Case Study 1 – *Results*

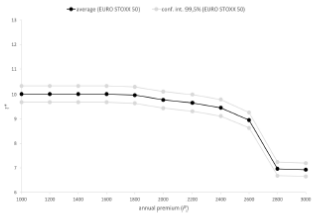


Fig. 4.60: Average lapse year by annual fair premium (EURO STOXX 50 case)

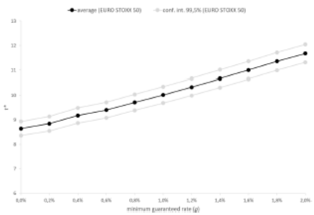


Fig. 4.61: Average lapse year by minimum guaranteed rate (EURO STOXX 50 case)

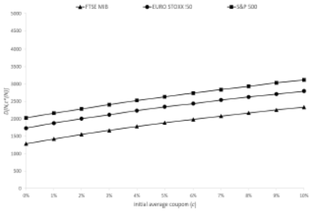


Fig. 4.56: $D[N, t^*(N)]$ by initial average coupon and equity investment

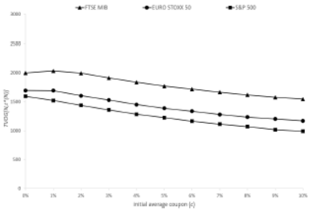


Fig. 4.57: $TVOG[N, t^*(N)]$ by initial average coupon and equity investment

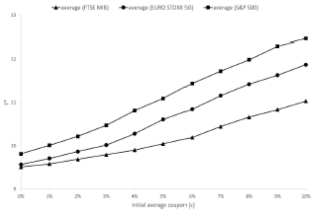


Fig. 4.62: Average lapse year by initial average coupon and equity investment

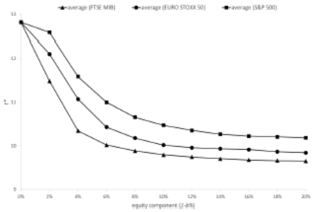


Fig. 4.63: Average lapse year by equity percentage and equity investment

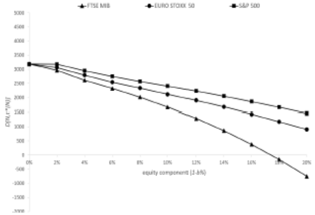


Fig. 4.58: $D[N, t^*(N)]$ by equity percentage and equity investment

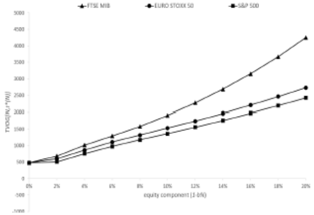
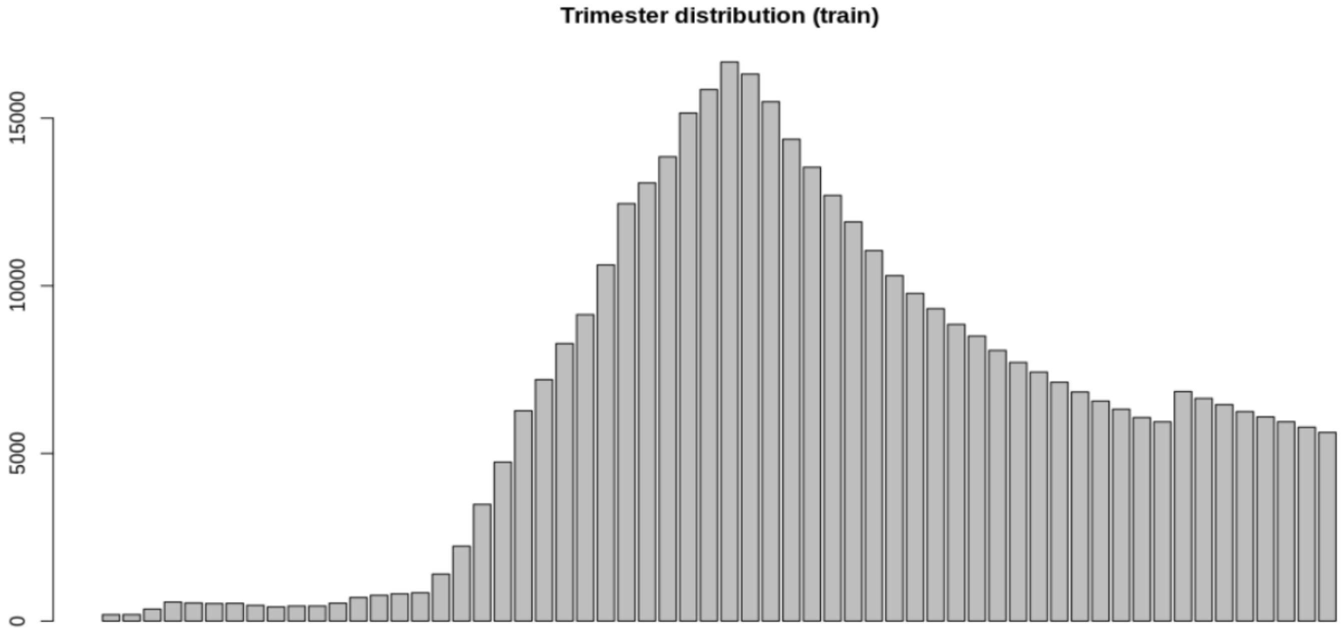


Fig. 4.59: $TVOG[N, t^*(N)]$ by equity percentage and equity investment

Case Study 2 – *Database*

- 50.000 loan borrowers in US
- Panel data synthesised in cross-section form
- Censored data with maximum loan observation time of 60 trimesters (i.e. 15 years)
- Policyholder-related fields (FICO score and investor type)
- Product-related fields
  - loan (sum borrowed and real estate type)
  - status (outstanding balance and instalment)
- Macroeconomic fields (unemployment rate and GDP)

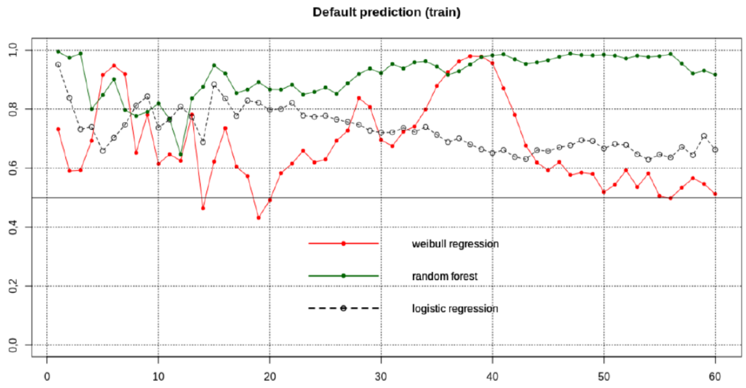
Case Study 2 – *Censored data*

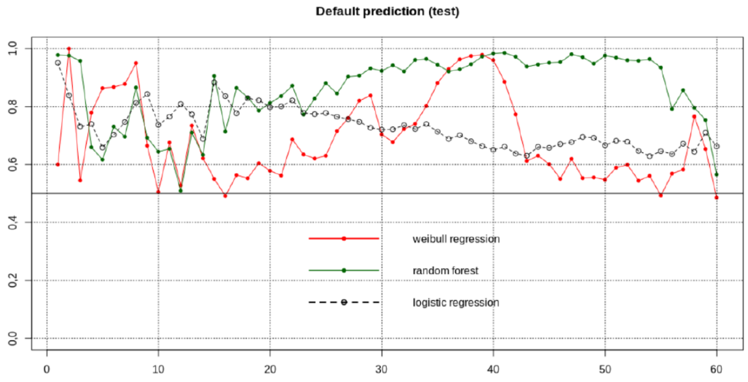


Case Study 2 – *Default prediction*

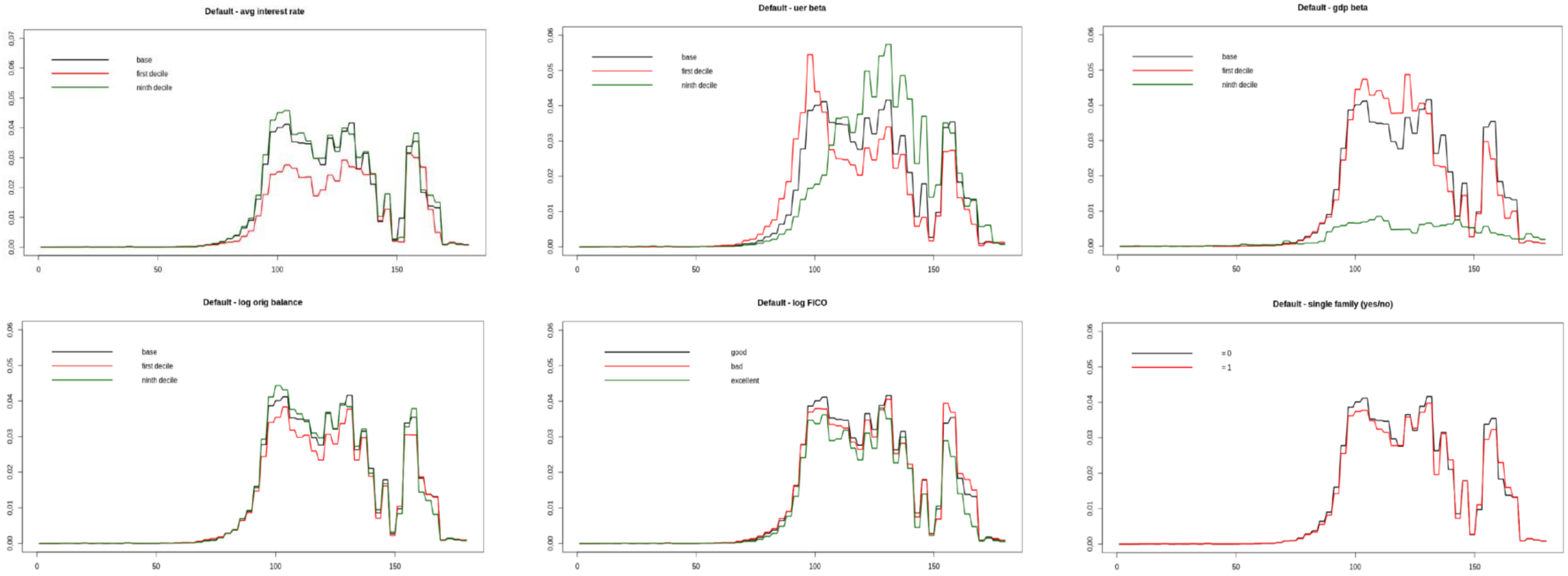
$$AUC = \frac{\sum_{t=1}^{60} n_t AUC_t}{\sum_{t=1}^{60} n_t}$$

METHOD	TRAINING AUC	VALIDATION AUC
Weibull regression	71%	72%
Random forest	**94%**	**91%**





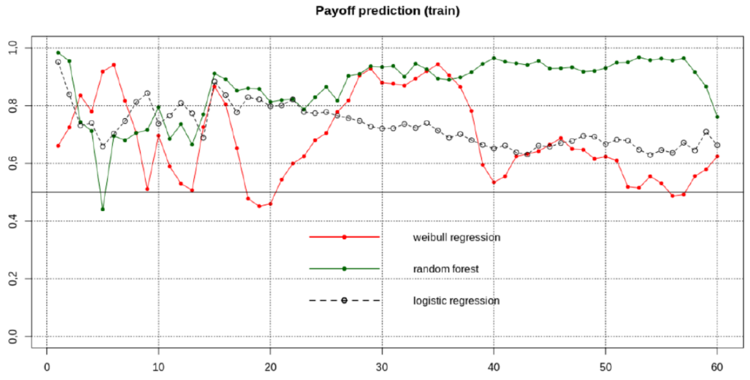
Case Study 2 – *Default prediction*

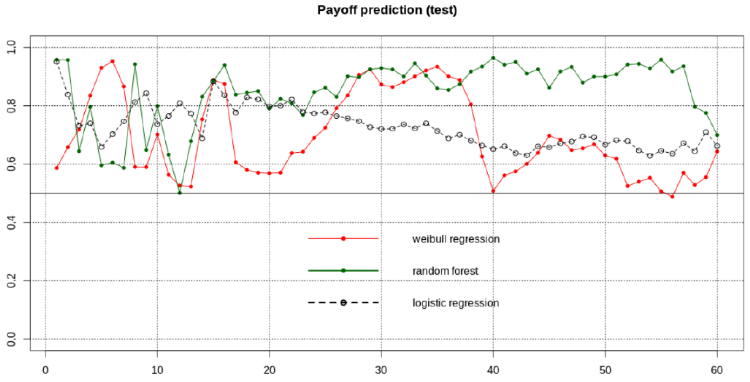


Case Study 2 – *Prepayment prediction*

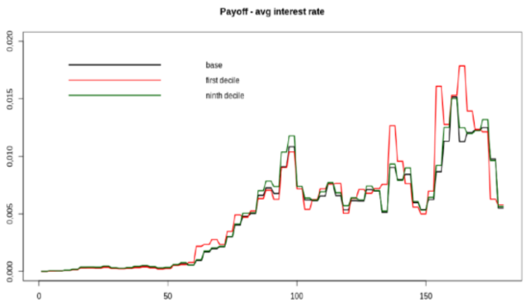
$$AUC = \frac{\sum_{t=1}^{60} n_t AUC_t}{\sum_{t=1}^{60} n_t}$$

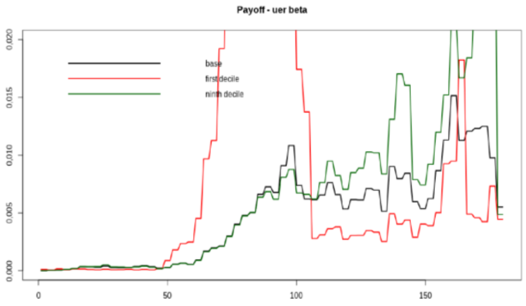
METHOD	TRAINING AUC	VALIDATION AUC
Weibull regression	72%	73%
Random forest	**91%**	**89%**

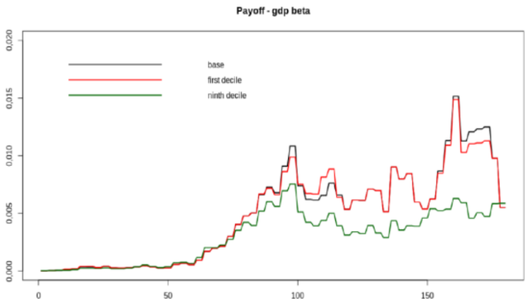


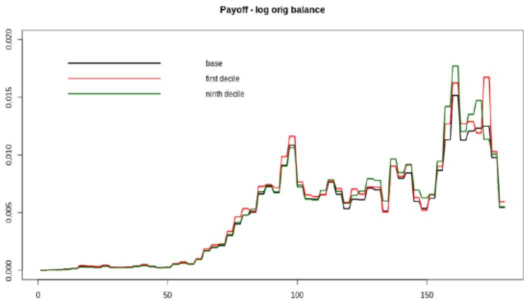


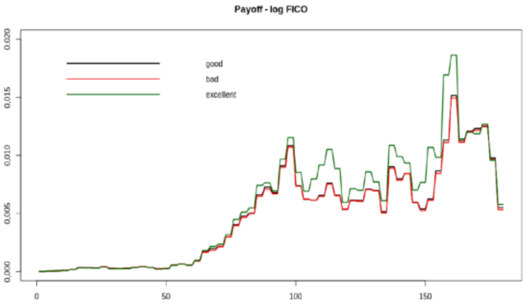
Case Study 2 – *Prepayment prediction*

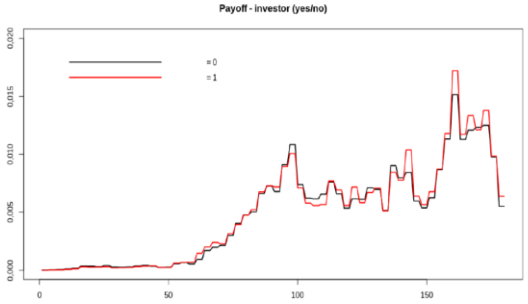












Case Study 2 – *CPI modelling*

- Loan / CPI dynamic
  - Overall lapse rate → $\hat{l}_r := 1 - \left[1 - \hat{Q}_P(r)\left(1 - \frac{\hat{Q}_D(r)}{2}\right)\right]\left[1 - \hat{Q}_D(r)\left(1 - \frac{\hat{Q}_P(r)}{2}\right)\right]$
  - Interest rate model (Vasicek) → $dr_t = a(b - r_t)dt + \sigma dZ_t$
  - Premium amount → $P = C_0 U_{\overline{x:n}|i} = C_0 \frac{IA_{\overline{x:n}|i}}{\ddot{a}_{\overline{x:n}|i}}$
  - Loan outstanding balance → $C_{t-1} = \frac{(1+i)^t}{(1+i)^n - 1} C_0$
  - Loan instalment → $I_t = (r_{t-1} + s)C_{t-1}$
  - Mathematical reserve → $V_t = C_t U_{\overline{x+t:n-t}|i} = C_t \frac{IA_{\overline{x+t:n-t}|i}}{\ddot{a}_{\overline{x+t:n-t}|i}}$

Case Study 2 – *Results*

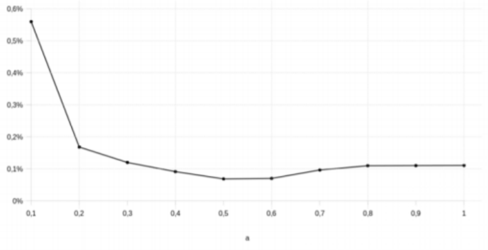


Fig. 32: TVOG shape by a.

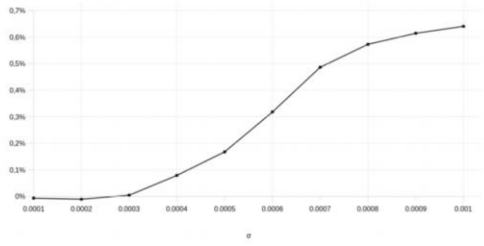


Fig. 33: TVOG shape by σ.

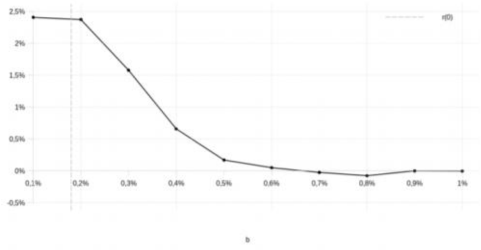


Fig. 34: TVOG shape by b.

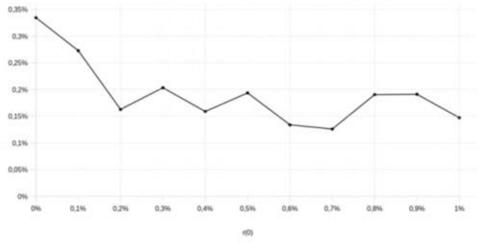


Fig. 35: TVOG shape by $r(0)$.

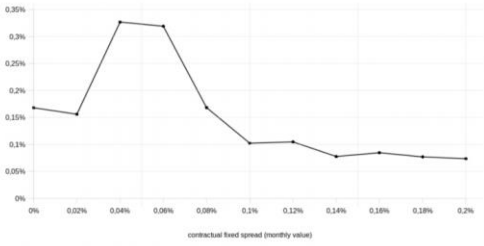


Fig. 36: TVOG shape by s.

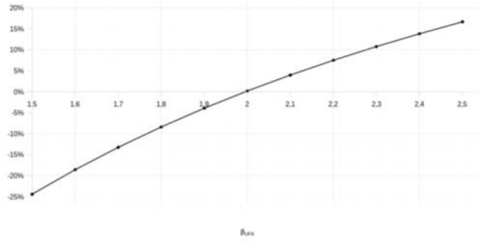
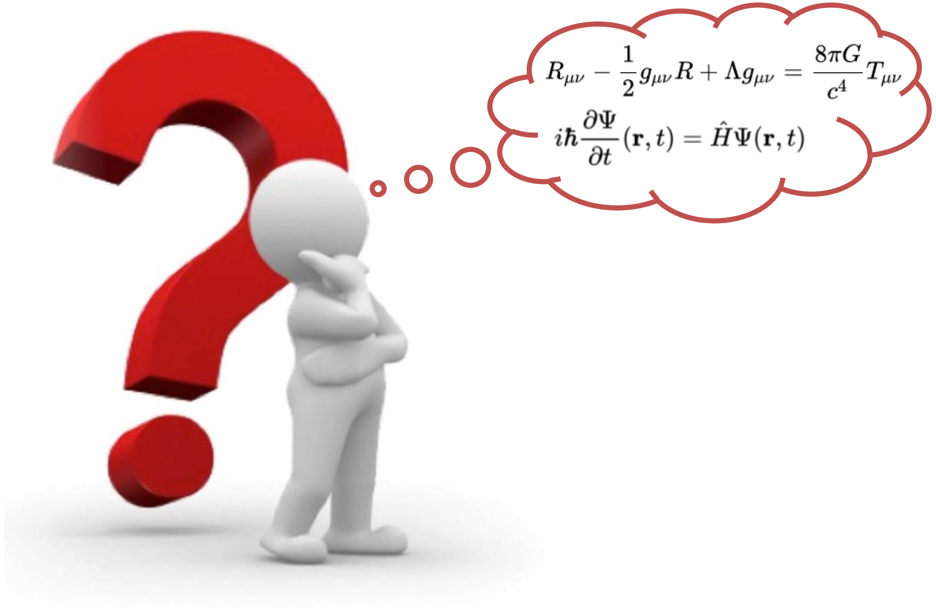


Fig. 37: TVOG shape by β.

Any Questions?



Thank you very much for your attention!

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