Stochastic Ordering of the Risks Affecting the Social Security Coverage in Africa

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Outline

1. Introduction
2. Risks affecting the social security coverage
3. Countries
4. Methodology
   - Risk and risk measures
   - Stochastic ordering
   - Stochastic prioritisation
5. Problem setting

Introduction

- The population covered by at least one social protection benefit is **45.2% worldwide**, while this rate is **17.8% in Africa** [1]
- Factors such as **limited formal economy**, **unemployment** and **high rates of inflation**
- **Low productivity**, **weak governance** mechanisms, **administrative problems** in some existing social security schemes pose serious challenges to efficient delivery and undermine trust and public support for social security
- **High** levels of **unemployment** and **underemployment**, as well as the inadequacy of current labour and social protection standards, prevent the delivery of social protection in many African countries [2].

Introduction

- Based on ILO's estimates, **29%** of the **global population** are covered by comprehensive social security systems - **full range of benefits**, from child and family benefits to old-age pensions.
- Yet the **large majority – 71%** are not, or are only partially, protected.
- **Coverage gaps** are associated with a significant underinvestment in social protection, particularly in Africa, Asia and the Arab States [1].

Introduction

- Social security coverage is a **top global priority**
- **Universal social security coverage** is achievable and affordable for countries at different levels of economic development.
- **87% of the African countries**, which are the members of International Social Security Association (ISSA), considers the extension of social security coverage to be a **priority challenge**.
- Coverage can be extended through a combination of **voluntary and mandatory contribution schemes**, **subsidised** and **tax-financed** programs [3].

Risks Affecting Social Security Coverage

- The **voluntary contribution scheme** in **Cameroon**, which extends social security coverage to the **informal sector**, and **community-based health insurance schemes** for informal employment in **Rwanda** are the examples of successful implementations.
- Risks affecting the social security coverage are closely related with the **social and economic conditions** of the countries and they are characterised by different **socio-economic indices**

Risks Affecting Social Security Coverage

- **Human Development Index (HDI)** as a composite index of **life expectancy**, **education** and **gross national income** per capita
- **Gender Development Index (GDI)** based on the sex-disaggregated HDI defined as a **ratio of the female to the male HDI**;
- **Gender Inequality Index (GII)**, which reflects gender-based inequalities in three dimensions – **reproductive health**, **empowerment** and **economic activity**.

Risks Affecting Social Security Coverage

- There is a strong relation between the key indicators of social security and the socio-economic indices [4, 5].
- We propose that this relation can be represented by stochastic risk prioritisation.

Aim: to analyse those risks defined as a **composite indicator** for selected **African countries** by using **stochastic ordering** within the framework of **partial order theory (POT)**. Considering the relation between the social security coverage and the HDI, GDI and GII we will investigate the stochastic dominance of the indicators as random variables.

Countries

- Three **sub-Saharan African countries** namely, **Cameroon**, **Ghana** and **Rwanda**.
  - **geographic** locations (centre, west and east sub-Sahara)
  - similar **poverty reduction strategies** that mainly addressed the social sectors such as health and education during early 2000s [6]
  - **rankings** in the socio-economic indices [7].
- Due to the effect of the **geography-related risks** on prioritisation, focusing on three different locations constitutes a good sample to represent sub-Saharan Africa
- These three countries have both **similarities and differences**, which will enable us to provide critical comments and compare our prioritisation results
- Collaboration with **African Institute for Mathematical Sciences (AIMS)** in all three countries

Methodology - Risk and Risk Measures

- **Risk** - a non-negative random variable which can be preferable to another for two reasons:
  1. the other risk is *larger*
  2. it is *thicker-tailed (riskier)* - probability of large values is larger
- **Risk measures** assign a real number to a risk, which is described by a random variable $X : \Omega \to \mathbb{R}$
- X - a potential loss but we allow X to assume negative values, which means that a gain occurs
- Axioms for risk measures
  1. **monotonocity:** if $X \leq Y$ then $\rho(X) \leq \rho(Y)$ - *the larger loss is more risky*
  2. **convexity:** $\rho(\lambda X + (1-\lambda)Y) \leq \lambda\rho(X) + (1-\lambda)\rho(Y)$ for all $\lambda \in [0, 1]$ - *diversification*

Methodology - Risk and Risk Measures

Various measures of risks

- **Value-at-risk** at probability level α is the α-quantile of X:

$$VaR_\alpha(X) = F_X^{-1}(\alpha) = \inf\{x \in [0, \infty) : F_X(x) \geq \alpha\}$$

- **Conditional Tail Expectation** at probability level α when X is continuous:

$$CTE_\alpha(X) = E(X|X > VaR_\alpha(X))$$

Methodology - Stochastic Ordering

Partial Order Theory (POT)

allows one to compare and order objects, characterised by multiple indicators, when there is an acceptable binary relation between two objects.

- multiple indicators $\rightarrow$ an index
- finding basic **indicators** and the **weights**

Consider a set $\mathcal{X} = (X_1, X_2, X_3, ...)$ of objects. To each object $X_i, i \geq 1$, we associate real-valued indicators $(I_i^1, I_i^2, I_i^3, ..., I_i^k)$ where k is an integer.

- An object X_i is intrinsicly *better* than X_j and we note $X_i \geq X_j$ iff $\mathrm{I}_i^k \geq I_j^k$ for all k

Methodology - Stochastic Ordering

- not every pair of elements needs to be comparable (**ambiguity**) $\rightarrow$ **partial order** $\rightarrow$ introduce the indicator as an index
$$\gamma(X) = \gamma(I^1, \ldots, I^k)$$

to solve this ambiguity

- a simple example of γ is given by the following linear **combination of the indicators**:
$$\gamma(I^1, \ldots, I^k) = w_1 I^1 + \ldots + w_k I^k,$$

where $w_i,\ 1 \leq i \leq k$ is a number and $\sum_{i=1}^{k} w_i = 1$.

Methodology - Stochastic Ordering

The index γ defines the linear ordering on the set of **objects** $\mathcal{X}$ by:

$$X_i \leq_\gamma X_j \text{ iff } \gamma(X_i) \leq \gamma(X_j), \quad (1)$$

that is, it defines a **stochastic ordering**.

- the index must be **monotone increasing** for each variable individually
- if the derivative of the index $\gamma(I^1, \ldots, I^k)$ exist, then $\frac{\gamma}{I_j} \geq 0, \forall j$ must be true
- if the index $\gamma(I^1, \ldots, I^k)$ is **linear**, i.e. $\gamma = w_1 I^1 + \ldots + w_k I^k$ then $w_j \geq 0, \forall j$

Methodology - Stochastic Ordering

Axioms for the Partial ordering of DFs

Consider that $F_{X_1}(s), F_{X_2}(s)$ and $F_{X_3}(s)$ with $s \geq 0$ are dfs of rvs X_1, X_2 and X_3, respectively. **The binary relation $\leq$ is a partial order** on a set $\boldsymbol{P = \{F_{X_1}, F_{X_2}, F_{X_3}, \ldots\}}$ if the axioms of POT are fulfilled as follows:

1. **Reflexivity:** $F_{X_1}(s) \leq F_{X_1}(s)$ for all $F_{X_1} \in P$ with $\forall s \geq 0$.
2. **Transitivity:** $F_{X_1}(s) \leq F_{X_2}(s)$ and $F_{X_2}(s) \leq F_{X_3}(s)$ implies $F_{X_1}(s) \leq F_{X_3}(s)$ for all $s \geq 0$.
3. **Antisymmetry:** $F_{X_1}(s) \leq F_{X_2}(s)$ and $F_{X_2}(s) \leq F_{X_1}(s)$ implies $F_{X_1} \equiv F_{X_2}$ $(F_{X_1}(s) = F_{X_2}(s),\ \forall s \geq 0)$.

Methodology - Stochastic Ordering

Assuming all mentioned random variables have a finite mean and are defined on a common probability space $(\Omega, \mathcal{F}, P)$

1. $X \leq_{\mathbf{st}} Y$, *if* $E[f(X)] \leq E[f(Y)]$ *for all increasing* f
2. $X \leq_{\mathbf{cx}} Y$, *if* $E[f(X)] \leq E[f(Y)]$ *for all convex* f
3. $X \leq_{\mathbf{icx}} Y$, *if* $E[f(X)] \leq E[f(Y)]$ *for all increasing convex* f
4. $X \leq_{\mathbf{icv}} Y$, *if* $E[f(X)] \leq E[f(Y)]$ *for all increasing concave* f

- the ordering $\leq_{\mathbf{st}}$ is called usual **stochastic ordering** or **first order stochastic dominance (FSD)**
- i.e. any rational decision maker prefers the loss X to the loss Y

Methodology - Stochastic Ordering

For any random variable X we denote by

$$F_X(t) := P(X \leq t),\ t \in \mathbb{R}$$

the **distribution function** and by

$$q_X(\alpha) := \inf\{x \in \mathbb{R} | P(X \leq x) \geq \alpha\},\ 0 < \alpha < 1,$$

the **quantile function** which is the generalised inverse of the distribution function.

Methodology - Stochastic Ordering

Theorem: *For random variables X and Y with distribution functions F_X and F_Y the following statements are equivalent:*

1. $X \leq_{st} Y$
2. There is a probability space $(\Omega', \mathcal{F}', P')$ and random variables X' and Y' on it with the distribution functions F_X and F_Y such that $X'(w') \leq Y'(w')$ *for all* $w' \in \Omega'$
3. $F_X(t) \geq F_Y(t)$ *for all* t
4. $q_X(\alpha) \leq q_Y(\alpha)$ *for all* $\alpha \in (0, 1)$

Methodology - Stochastic Ordering

Stop-loss order

The ordering $\leq_{\mathbf{icx}}$ is also known as **stop-loss order (second order stochastic dominance (SSD))** in actuarial sciences since $\leq_{icx}$ holds iff the corresponding stop-loss transforms are ordered. The stop-loss transform π_X of a random variable X is defined as

$$\pi_X(t) = E(X-t)_+ = E(\max\{X-t, 0\}) = \int_t^{\infty} (1 - F_X(s))ds, \ t \in \mathbb{R}$$

In decision theory $\mathbf{X} \leq_{\mathbf{icx}} \mathbf{Y}$ has the meaning that any risk averse decision maker prefers the loss X to the loss Y.

Methodology - Stochastic Prioritisation

Definition

There exists a non-decreasing real-valued utility function u on $\mathbb{R}$ with $u(0) = 0$ s.t.

$$X \leq Y \Leftrightarrow \mathbb{E}[u(X)] \leq \mathbb{E}[u(Y)]; \; X, Y \geq 0$$

where the expected utility is

$$\mathbb{E}[u(X)] = \int_0^\infty S_X(t)\,\mathrm{d}u(t) = \int_0^1 u[S_X^{-1}(q)]\,\mathrm{d}q$$

Problem Setting - Risks Affecting the Social Security Coverage

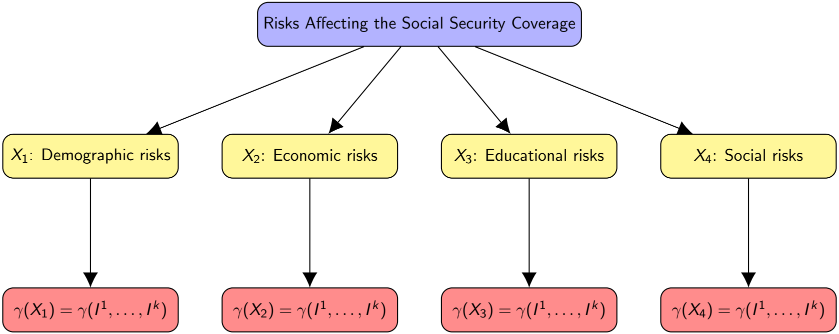

Figure: Stochastic prioritisation of the risks affecting the social security coverage

Problem Setting - Common Indicators

Indicator no	Indicator name
I^1	life expectancy at birth
I^2	infant mortality rates
I^3	life expectancy at age 65
I^4	unemployment
I^5	income
I^6	educational attainment
I^7	government expenditure on education
I^8	consumer price index
I^9	interest rates
I^{10}	labour force & labour force participation
$\vdots$	$\vdots$

What is next?

- Determining the **common indicators**
- Finding the historical **data** for each indicator and objects
- **Dependency** between the indicators and objects
- Constructing **composite indicators**
- Choosing the best **risk measure** to compare/order/prioritise the objects
- Proposing relevant **social security reforms** to target the prioritised risks

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