

**Pensioners and annuitants Working Group**

**Pensioners and annuitants  
in Italy:  
mortality from 1980 to 2009  
and projections to 2040**

*Executive summary*

17 October 2012





This study has been prepared by the Pensioners and annuitants Working Group, a working party set up with the help of the Italian Actuarial profession.

The Group is composed by experts working for the institutions and the organizations that joined the initiative or that have been directly designated by the Italian Actuarial Profession.

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## **PRESENTATION**

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We take great pleasure in presenting you an “Executive Summary” of a recent study\* on the mortality of pensioners in Italy and their life expectancy. The study is the result of a joint effort of the National Council of Actuaries and the Italian Institute of Actuaries.

For the first time in Italy mortality has been analyzed distinctly by selected groups according to the position individually held during active life by each single pensioner. This approach provided both pension funds and State schemes benefits with new detailed and precious information.

Life expectancy of all pensioners and annuitants during 1980 - 2009 marks the starting point of the survey. Evidence shows that in the last 30 years life expectancy of all pension recipients increased at an average rate of 1 percent. Projections of data to the next thirty years results in the survival of old-age pension recipients of 4 more years, at the end of the period, i.e. in 2040, for both men and women.

The actuarial committee that produced these projections was formed by actuaries working in Government agencies, insurance companies and pension funds. Working together, they confirmed once more the social value of the actuarial profession.

Riccardo Ottaviani

Giampaolo Crenca

\*The English version of the complete paper will be posted on the website ([www.ordineattuari.it](http://www.ordineattuari.it)) at the end of October 2012.

*The authors thank the institutions and organizations that joined the initiative (ANIA, ASSOFONDIPENSIONE, ASSOPREVIDENZA, CASSA FORENSE, ENPALS, ENPAM, INAIL INPDAP and INPS), for data and information provided and for the participation to the study of their experts. Special thanks go to Susanna Levantesi, Massimiliano Menziatti, Piero Cocevar and Carlo Conforti, members of the working group, for the analyses and calculations provided.*

*Thanks, also, to COVIP and ISVAP for having assisted to the whole working process.*

*Furthermore, the authors would like to thank Stephen Richards and Longevitas for their "graduation" method that has been used by the working group to extrapolate mortality rates at the oldest ages.*

*Ultimately, the authors would also like to thank the CMI - Continuous Mortality Investigation Bureau, in the person of Neil Robjohns, for information provided on CMI APC (Age-Period-Cohort) model. The CMI has not tested the processing of the APC model used for this study and the authors assume full responsibility for the results of the APC model. The authors of the study have also used the software R(\*) for preparing certain analysis and graphs.*

*The study has been patrocinised by Italian Actuarial Profession bodies (Consiglio dell'Ordine Nazionale degli Attuari e Consiglio Nazionale degli Attuari).*

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## INTRODUCTION

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Changes to life expectancy of the beneficiaries of public and private pension schemes can have an impact on the economy and the welfare system. It is desirable to monitor closely these trends and increase awareness in the management of longevity risk. This information can help improve risk management processes and contribute to the transparency and balance of provisions.

For mandatory public funded pensions analysing the mortality of specific pensioners' groups is important both to help the long-term balance of social security schemes and to understand the real cost of benefits paid out, supporting the information already available on general population data and the forecasts from public institutions.

The private sector in Italy over the next decades should see a gradual increase in demand for annuities. Due to the lack of specific data, the mortality tables used by life office annuities in Italy are based on general population forecasts adjusted with selection factors derived from other countries to take into account of the potential difference among general population and the specific group of annuitants as required by the regulator.

To respond to these challenges, the Italian Actuarial Profession decided to set up a Working Group starting from the group who carried out the first investigation into the pensioner data in 2008 (see ANIA-INPS-Ordine degli Attuari, [2]), widening participation and scope. The study prepared by the Working Group performs and updates analyses on changes in public pensioner and private annuitants' mortality rates in Italy and forecasts in life expectancy, with the aim to support the actuarial community, the pension institutions - including pension funds and insurance companies - and everyone else interested in mortality trends in pension framework.

This document is limited to the 1. *Executive summary*, giving an essential summary of analyses and results.

The report published by the Group includes the other main contents and results of the work that has been carried out, shown in:

2. *The collection data process*, briefly describing the process used to collect the data from the various institutions;
3. *Mortality of pensioners and other annuitants in 1980-2009*, which shows mortality trends over the observed period, split by gender and occupational group, including comparisons with the general population, differential mortality among groups and insights on cohort and income effect;
4. *Old-age pensioners mortality projections to 2040*, that contains results of projections carried out through different projection models (both stochastic and deterministic) on different group of pensioners.

Following that, the *Methodological appendix* describes in more detail, for those who are interested in getting a deeper understanding of the technical issues, the working process, the methods used in data gathering, the choice of the models, the tests done to check goodness of fit of the models and further insights on models features and forecasting process. Certain parts of the appendix have not been translated into English and readers are remanded to the Italian version of the study.

Finally, the section *References* comprise details of publications and data being referenced elsewhere in the study.

The *Annexes* attached to the study provide further data, information, graphics and numerical evidence on the analysis carried out.

The final version of this study has been published after a consultation process within the Italian actuarial community. A draft version was published on 2012, April 16th on the website [www.ordineattuari.it](http://www.ordineattuari.it). The publication was announced in CNA Prot n. 038/2012 Circ. N. 10/2012 and ONA Prot. 064/2012 Circ. No. 11/2012, requesting feedback by 2012, June 1st. A summary of the comments received together with feedback from the Working Group are reported as follows.

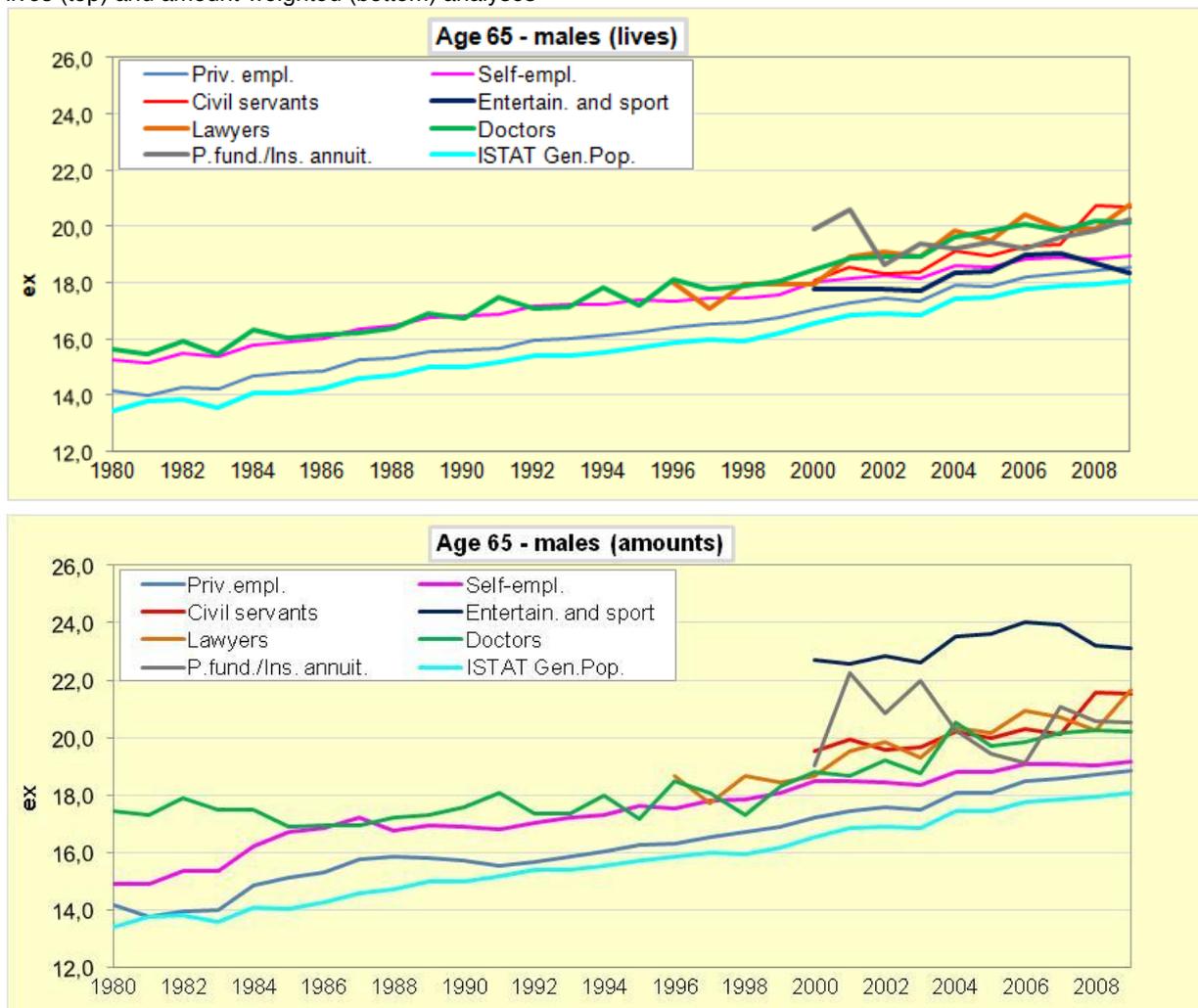
## 1. EXECUTIVE SUMMARY

This study comprises the following mortality analyses:

- mortality of pensioners on normal retirement (i.e. pensioners reaching either the retirement date or the maximum working age), both from first and second pillar institutions, with the first pillar institutions being by far the largest contributors;
- mortality for pensioners receiving disability pensions;
- widow and widower's pensions.

In the first section, the study includes an analysis of the mortality of pensioners and annuitants in the period 1980-2009: to give an indication of the large volume of available data, in 2009 there were about 10 million pension positions with an annual pension of over 142 billion Euro. The following figures illustrate life expectancy for normal retirement pensioners (males, age 65) by occupation (private employees, self-employed, civil servants, entertainment and sport workers, lawyers, medical doctors) and for pension funds / life office annuitants over the observation period. The top graph of figure 1 shows life expectancy calculated on lives, while in the bottom graph life expectancy are weighted by annuity amounts.

**FIG. 1 – LIFE EXPECTANCY AT 65 YEARS IN THE OBSERVATION PERIOD (\*) - MALES**  
lives (top) and amount-weighted (bottom) analyses



(\*) Missing data has been left-truncated.

There are some general trends that should be noticed:

- there is a general increase in life expectancy, and that is broadly in line, in relative terms, with the general population;
- life expectancy of pensioners, regardless of their occupation, is higher than life expectancy in the general population;

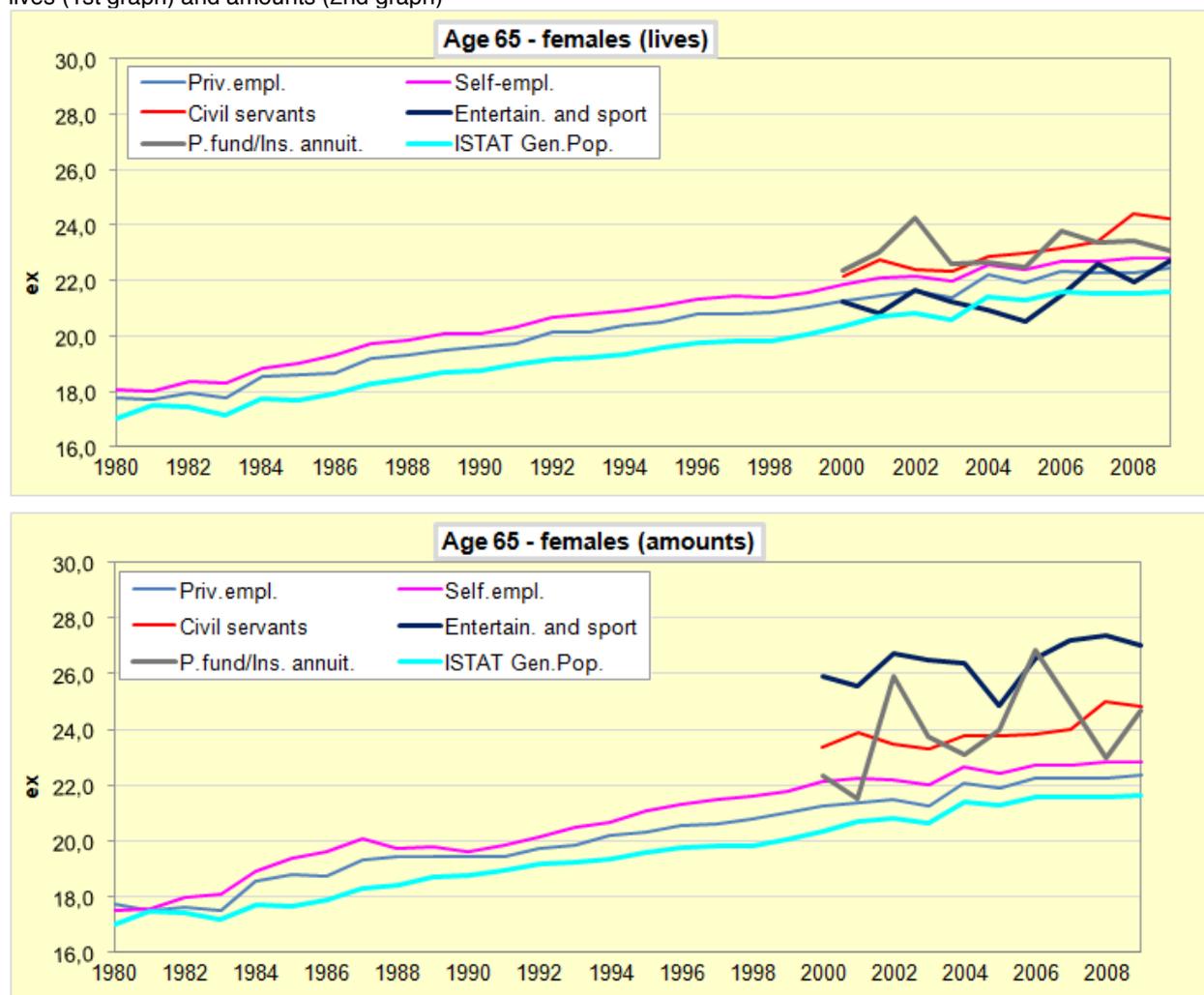
- life expectancy weighted by amounts is always higher than life expectancy that is not weighed by amounts.

There are the following comments in relation to trends observed for different occupational categories of pensioners:

- civil servants, doctors, lawyers and pension fund / life office annuitants have life expectancy that is significantly higher than the general population, 2 years or more in 2009;
- self-employed have higher life expectancy than the general population, but lower than the other afore-mentioned pensioners groups; the difference in life expectancy with general population was still positive in 2009, being less than 1 year;
- private employees show a lower level of life expectancy than the other groups, but still higher than the general population;
- there is a strong correlation between survival and pension income for workers in the arts and sports: life expectancy in the amount-weighted analysis is considerably higher than values according to life expectancy calculated by lives (in 2009 about 5 years more than general population); the values are slightly higher than for the private employees and there is a convergence to the data observed for the general population in more recent years.

For normal retirement females (see Fig. 2), trends are similar to that observed for males (life expectancy is higher than for general population and it increases substantially in line with the latter in relative terms) but with less significant differences due to amount effects, except for workers in arts and sports where, as seen for males, amount-weighted life expectancies are about 5 years higher than those of the general population.

FIG. 2 – LIFE EXPECTANCY AT 65 YEARS IN THE OBSERVATION PERIOD (\*) – FEMALES  
lives (1st graph) and amounts (2nd graph)



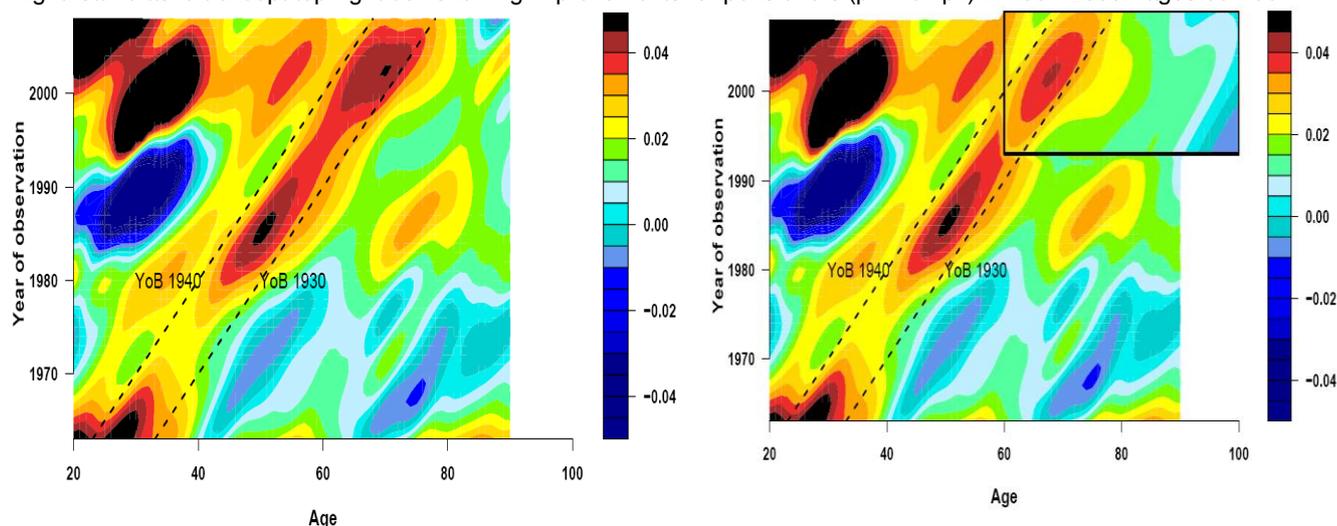
(\*) Missing data has been left-truncated.

As seen for males, female civil servants have a higher life expectancy – by about 2 years – than the general population for the whole period of observation; the same spread is registered for pension funds and life office annuitants. Female self-employed and private employees have lower values of life expectancy, however still higher than the population. Further analyses of historic data have revealed interesting trends. Previous mortality studies [9] on the population had already reported the presence of cohort effects (i.e. significant mortality improvements relevant to certain generations), in particular for males. This is shown in the heat map of mortality improvements in the left-hand side of Fig. 3 – where the mortality improvements have been smoothed using the p-spline method (areas in red and yellow indicate an improvement, a worsening the blue ones).

**FIG. 3 –HEAT MAP OF MORTALITY IMPROVEMENTS IN THE ITALIAN POPULATION AND FOR PENSIONERS– MALES**

Left: mortality improvements - Italian population – males- 1962-2008 - ages 20-90

Right: same as left except top-right box showing improvements for pensioners (priv. empl.) in 1991-2009 - ages 60-100



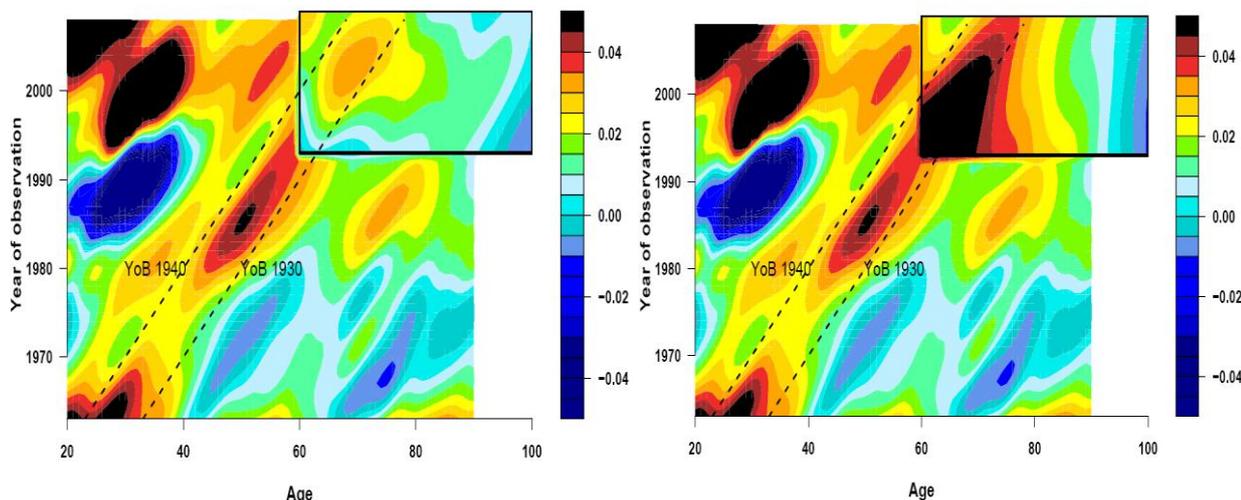
Source: Elaboration on HMD – Human mortality database data.

The data examined in this study showed a similarity between cohort effect for pensioners and the general population (the heat map in the box at the top right of the right graph in Fig. 3 relate to mortality improvements data for private employees pensioners); that is what can be reasonably be expected considering that a significant proportion of the general population at those ages are private pensioners. A further insight comes by stratifying the data: it is evident that cohort effect improvements are more pronounced for those pensioners receiving a higher amount of pension (see Fig. 4 – box at the top right in the right-hand graph) than for those receiving lower incomes (see Fig. 4 – box at the top right in the left-hand graph).

**FIG. 4 – MORTALITY IMPROVEMENTS HEAT MAP FOR PENSIONERS EARNING DIFFERENT PENSION AMOUNT - MALES**

Left: like Fig. 3 except top-right box showing improvements for priv. empl. with pension lower than 1,200 € monthly

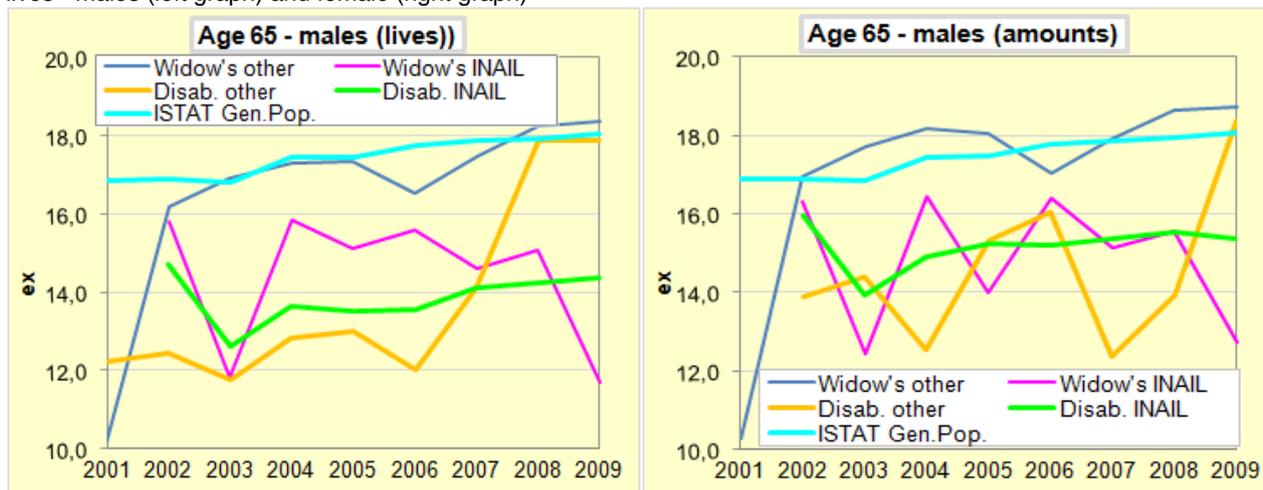
Right: like Fig.3 except top-right box showing improvements for priv. empl. with pension higher than 1,200 € monthly



Source: Elaboration on HMD – Human mortality database data, private employees normal retirement pensioner data.

Such evidences have supported the decision to investigate the impact on mortality for Italian pensioners and life office annuitants considering effects of cohorts and size of pension and the choice of projections models that are able to capture such effects. Data volumes for widows and disability pensions are less numerous (in 2009 almost 1.5 million positions for less than 14.5 euro billions), as data is missing from the most important pension institution and as the information is available for a shorter time. Fig. 5 shows life expectancy at age 65 for these groups (INAIL data are considered separately due the specific nature of workers compensation, while other widows and disability pensioners data sets have been grouped to increase statistical numerousness).

**FIG. 5 – WIDOW’S AND DISABILITY PENSIONERS: 65 YEARS OLD LIFE EXPECTANCY IN THE PERIOD 2001-2009 (\*)**  
lives - males (left graph) and female (right graph)



(\*) Due to not statistically s data, for some institution values are shown only starting from a certain year onward.

The results show some irregularities due to scarcity of data. At any rate, some general trend can be noted. For widows and widowers pensioners, it can be seen that life expectancy for INAIL male pensioners is always lower than the general population, while life expectancy for the other groups is closer to the population; for females, there is the opposite trend: life expectancy for INAIL pensioners is slightly lower than population, for the other groups the expectation is still lower.

For disability pensioners, instead, life expectancy for both male and females INAIL pensioners is always lower than general population, while for the other groups it gradually approaches population values for both genders. The study also includes analyses on mortality differentials (“selection”), showing ratios of mortality rates for different groups. In particular, three main group of pensioners have been set as benchmarks (private employees, self-employed and the group given by sum of the two) and ratios have been calculated in respect with such benchmark groups and population.

**FIG. 6 – COMPARISON OF SELECTED DATA SETS (YEARS 2002-2009) WITH CORRESPONDING “BENCHMARK” COLLECTIVES**  
ratio between mortality rates (amount-weighted) for each group and rates (lives) of benchmark collectives - males, %

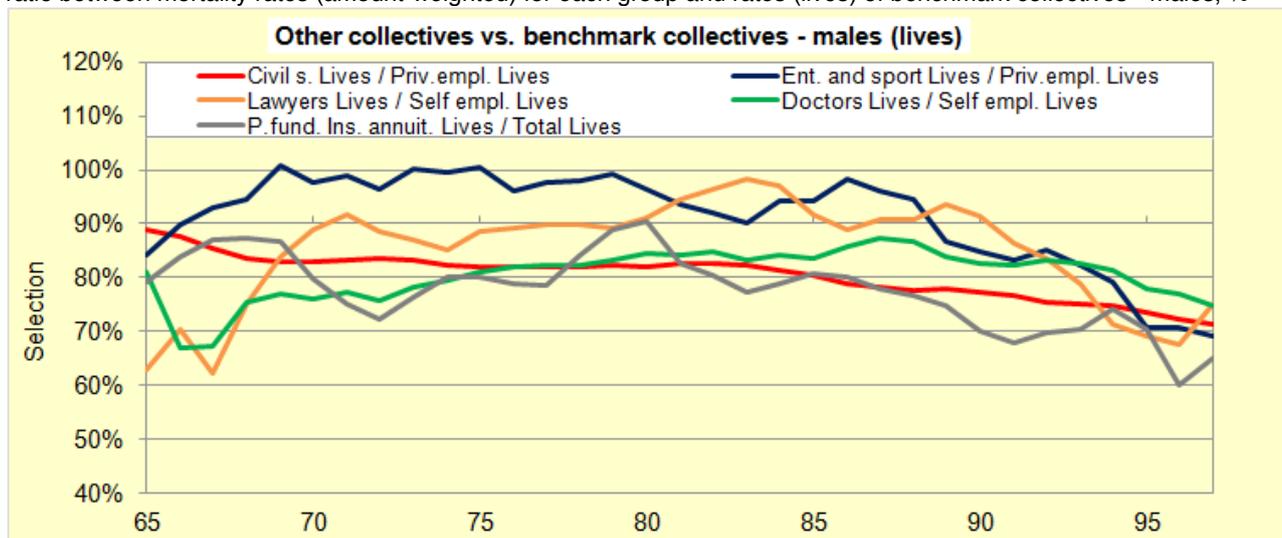
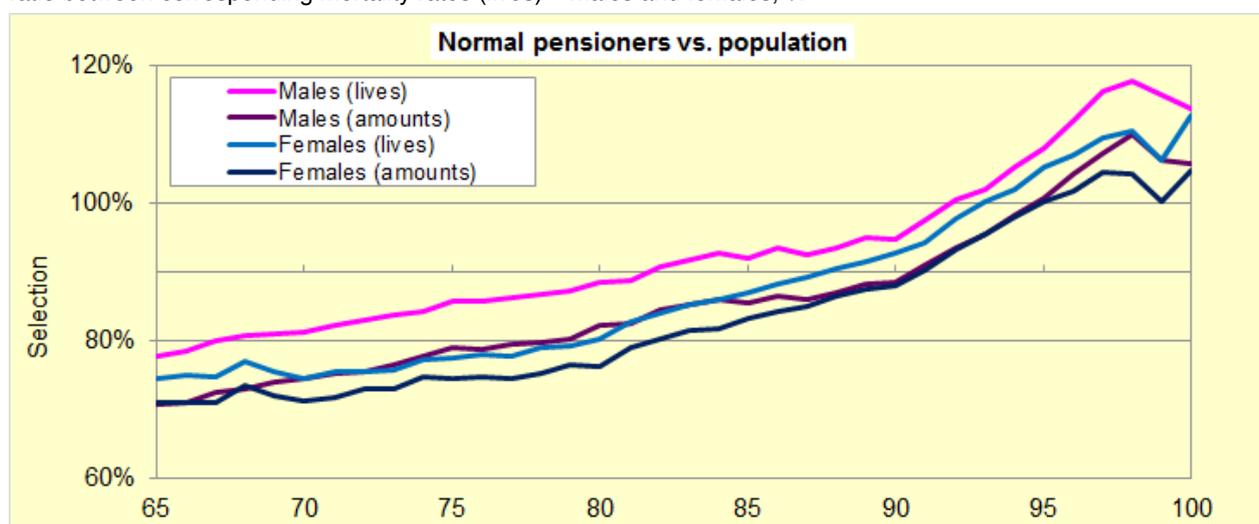


Fig. 6 illustrates the average selection in amount-weighted mortality of pensioners (male) compared to lives in the period 2002-2009. It has to be noted that:

- for private employees and self-employed the impact of the pension amount is low on mortality, with differentials within 10%, mostly at younger ages;
- lawyers, doctors and pension funds / life office pensioners and annuitants mortality data exhibits a greater selection effect compared with that of the benchmark collective (total private employee and self-employed pensioners), with a ratio that, despite fluctuations, is between 80% and 90%;
- for civil servants there is a clear selection effect compared to private employees, with a ratio close to 70%;
- for workers in arts and sports there is a greater differential, with a mortality rate equal to half that of private employees.

Fig. 7 shows the average selection (lives) in the same period 2002-2009 for all pensioners data (males and females) in respect with the general population.

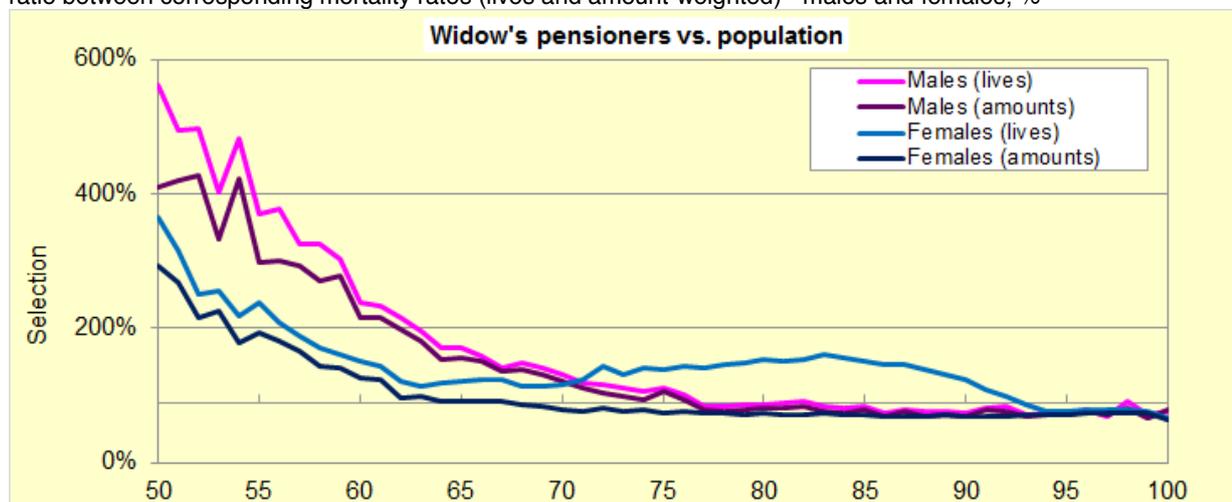
**FIG. 7 – COMPARISON OF NORMAL RETIREMENT PENSIONERS DATA (YEARS 2002-2009) WITH GENERAL POPULATION**  
ratio between corresponding mortality rates (lives) – males and females, %



It can be seen that there is a considerable selection effect at younger ages, for both genders, gradually disappearing at older ages (where it can be seen a higher mortality rate than the general population that may be attributed to the small volumes of data). Selection effect for amount-weighted mortality rates is higher for both genders.

Similar analyses have been carried out on widows and disability pensioners. Fig 8 shows a comparison of widow's pensioners mortality with the general population.

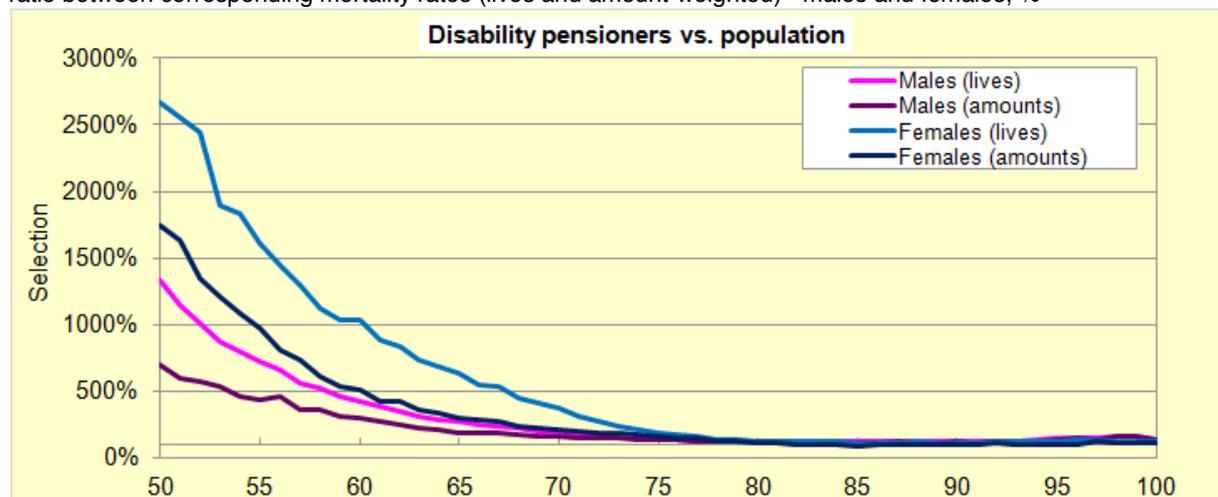
**FIG. 8 – COMPARISON OF WIDOW'S PENSIONERS (YEARS 2002-2009) TO GENERAL POPULATION**  
ratio between corresponding mortality rates (lives and amount-weighted) - males and females, %



The differential is very high at the younger ages, where widow's (and widower's) pensioners mortality is equal to several multiples that of the general population, in particular for males. Amount-weighted mortality experience is generally close to lives. At older ages the differentials decline significantly converging to a ratio close to 100%.

Figure 9 shows the average selection of disability pensioners compared with general population.

**FIG. 9 – COMPARISON OF DISABILITY PENSIONERS MORTALITY (YEAR 2002-2009) TO GENERAL POPULATION**  
ratio between corresponding mortality rates (lives and amount-weighted) - males and females, %



Extra-mortality is very considerable at the younger ages until age 65-70 especially for females. Amount-weighted mortality rates are significantly lighter. At very old ages the differential is substantially nihil.

It should be stressed that the results (see also Fig. 5) for widow's and disability pensioners have been obtained by aggregating data from different institutions so to increase statistical significance. As a consequence, the results after the aggregation can differ from those submitted by each data contributor.

From the results presented it can be seen that pensioners have a higher life expectancy than the general population while disability and widow's pensioners, even if with a differential depending on age, have a lower life expectancy compared to general population. This result seems to be consistent with the life expectancy of the general population being driven by a large proportion of pensioners with higher life expectancy (normal retirement pensioners data) and a small proportion of pensioners with a lower life expectancy (such widow's and disability pensioners); it should be noted that the study does not include all pensioners in Italy.

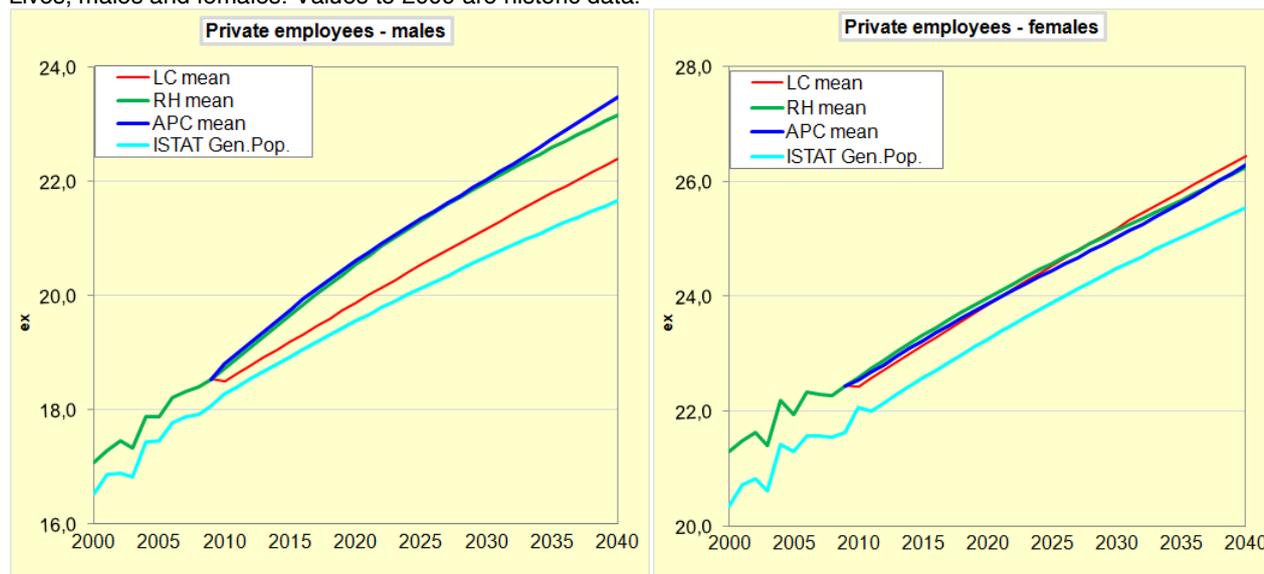
The last part of the study covers all-pensioners mortality projections for the period 2010-2040. The forecast was carried out on different sub-sets (private employees, self-employed and the sum of the two dataset, using lives data), selected because of their numerosity and accuracy of data collected. Projections have been performed through different models, both stochastic and deterministic.

The stochastic models selected were the Lee-Carter Poisson log-bilinear model and the Renshaw-Haberman model with a cohort effect, and multiple scenarios associated with a probability distribution, namely including a central scenario, and high and a low scenarios (respectively the 5<sup>th</sup> and 95<sup>th</sup> percentile) have been developed.

The deterministic model selected in the study was the Age-Period-Cohort (APC) model first developed by the Continuous Mortality Investigation (CMI) Bureau in the United Kingdom where it is widely used by pension funds and insurance companies. As a consequence of analysis on historical data, the APC model has been applied with different long-term mortality improvement rate assumptions: 2% as a "central best-estimate" scenario, 1,25% as a low scenario and 2,75% as a high scenario.

**FIG. 10 – PROJECTION OF LIFE EXPECTANCY AT AGE 65 PERIOD 2010-2040 - PRIVATE EMPLOYEES**

Lives, males and females. Values to 2009 are historic data.



**FIG. 11 – PROJECTION OF LIFE EXPECTANCY AT 65 YEARS FOR 2010-2040 - SELF-EMPLOYED**

Lives, males and females. Values up to 2009 are historic data.

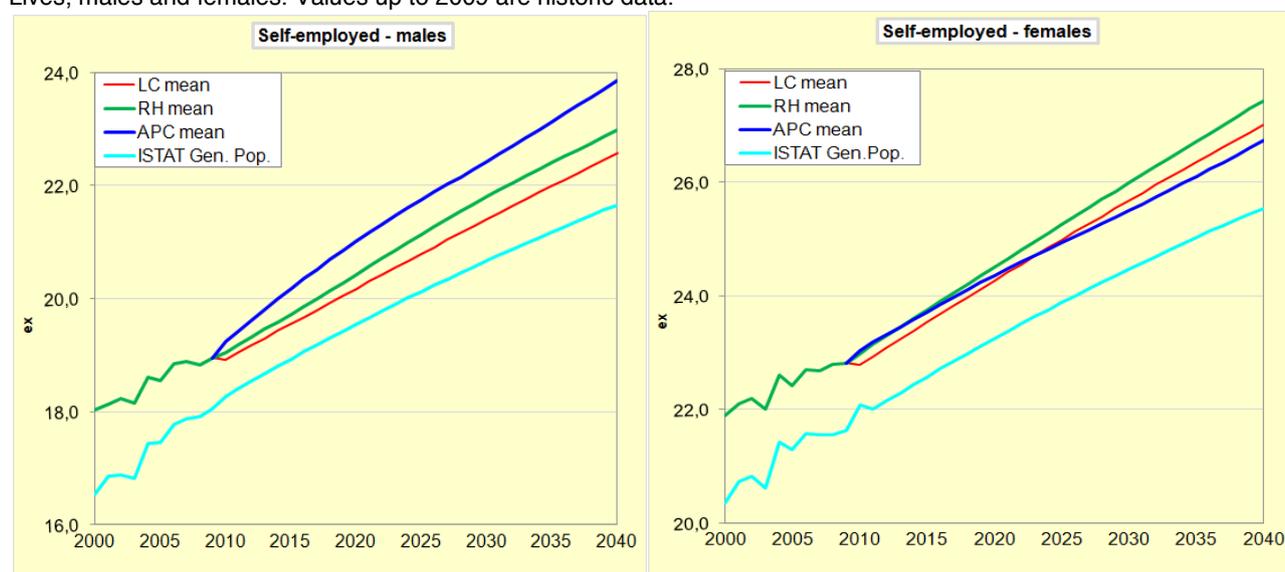


Fig. 10 and 11 outline the results of the projections for a life expectancy of age 65 in central scenarios until 2040, compared with projections for the general population made by ISTAT for 2011-2065. It can be noted that:

- it is confirmed that the historically observed higher life expectancy for these groups persists also in the forecast period, with values remaining higher than those projected for the general population;
- for males, life expectancy calculated by the Lee-Carter model are the lowest of the three models (less than 22.5 years in 2040 for private employees, just over 22.5 years for the self-employed), followed by the Renshaw -Haberman (23 years) and the APC model (about 23.5 for private employees, almost 24 for the self-employed) that give higher values;
- for females, the results and differentials in life expectancy among the different models are closer: between 26 and 26.5 years for private employees, while for the self-employed the APC model had projected life expectancy at 26.7 years, the Lee-Carter at 27 years while the Renshaw-Haberman at almost 27.5 years.

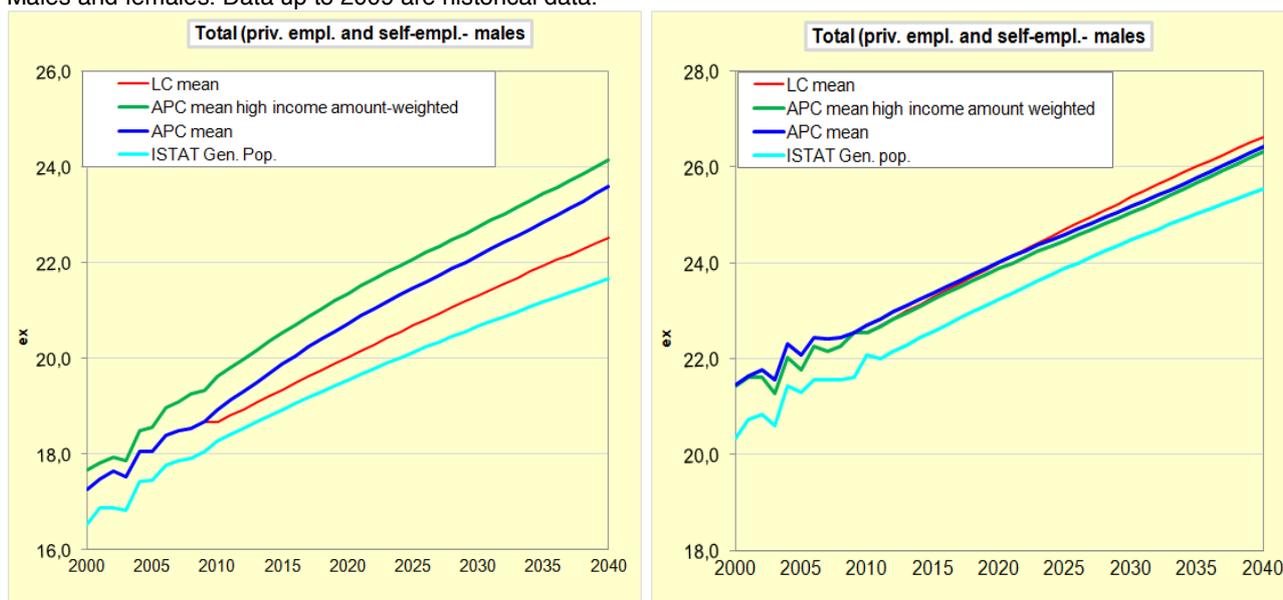
The ISTAT projections on the general population that had been considered as benchmark for this study have published at the end of 2011 ("The demographic future of the country", ISTAT

2011) and they have been calculated through a “cohort component model” that estimates the evolution of the population with an iterative process making assumptions on the balance (births and deaths) and migration of the population and using a specific model appropriate for forecasts on general population.

In addition, projections for the aggregated data (private employees and self-employed) have been carried out, using the Lee-Carter and APC models, and a special projection with APC model that used amount-weighted mortality of pensioners with monthly incomes at least equal to 1,200 euro.

**FIG. 12 – PROJECTION OF LIFE EXPECTANCY AT 65 YEARS FOR 2010-2040 - NORMAL RETIREMENT PENSIONER DATA (PRIV. EMPL. AND SELF-EMPLOYED)**

Males and females. Data up to 2009 are historical data.



In addition to the trends that have been already commented on, it can be observed that:

- for males, the lowest life expectancy at age 65 is calculated by the Lee-Carter model (around 22.5 years in 2040), while highest is the amount-weighted projection through the APC for the pensioners earning at least 1,200 euro per month (more than 24 years compared to 23.5 years for normal retirement pensioners without amount effect);
- for females, values of life expectancy are closer among different models and similar to those of private employees, with substantially no incidence for the amount effect.

Overall, all the forecasts lead to expectations of a further growth in longevity of pensioners, although trends are different depending on the characteristics of the collective or the models that have been adopted. The projected life expectancy of all-pensioner data is consistently higher, for all the groups been considered, than the general population values calculated by ISTAT, in line with historically observed trends. Despite the projections are performed using different models and datasets, it is plausible to assume that the projections reflects a continuation of historic trends, that is, that pensioners have a higher life expectancy compared to general population and that disability and widow's pensioners have a lower expectation.

There are several factors to be taken into account in the projections reflecting the uncertainty that is inevitably embedded in longevity estimates. However, the extrapolation of trends conducted in this study responds to a principle of *best estimate*, i.e. an attempt to estimate with best efforts, through a scientific approach accompanied by analysis and data support, a phenomenon whose future outcome remains uncertain. In this regard, it is believed that the application of the projections just described to other collectives (e.g. civil servants, lawyers, doctors or other categories that are not treated in this study) should be made with care considering whether and to what extent historic trends in mortality for such collectives (see Fig. 6) could recur in the future and taking into account the other indications given in Section 4 par. 4.3 *Information on the use of projections, uncertainties and limitations of the study*.

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