Should Commodity Investors Follow Commodities' Prices?

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Outline

- Motivation:
  Commodity Futures as an Asset Class.
- Model:
  Portfolio Choice in Commodity Indexes.
  Constant relative risk aversion.
- Results:
  Policy and Performance: Index vs. Price Observations.

Commodity Futures Investing

- Commodity Futures:
  "No trade deserves more the full protection of the law, and no trade requires it so much; because no trade is so much exposed to popular odium." (Adam Smith, 1776)
- Commodity Futures as an Asset Class:
  Inflation hedges (Bodie, 1983)
  Similar return as equities (Bodie and Rosansky, 1980)
  Negative correlation with equities (Gorton and Rouwenhorst, 2006)
  Individual commodities uncorrelated (Erb and Harvery, 2006)
  Positive, predictable returns (Levine, Ooi, and Richardson, 2016)
- Commodity Futures in Practice:
  Rising popularity since 2004 (Singleton, 2014)
  Investment through commodity index ETFs (Basak and Pavlova, 2016)

Related Models

- Enlargement of filtrations.
- Logarithmic utility:
  Karatzas and Pikovsky (1996), Grorud and Pontier (1998), Amendinger, Imkeller, Schweizer (1998), Corcuera et al. (2004), Guasoni (2006)
- Power and exponential utilities in complete markets:
  Amendinger, Becherer, Schweizer (2003)
- Filtering theory.
- Portfolio choice with partial information.
  Lakner (1995, 1998), Brennan (1998), Brennan and Xia (2001), Rogers (2001), Brendle (2006), Cvitanic et al (2006).
- Asset Pricing with learning:
  Detemple (1986), Dothan and Feldman (1986), Veronesi (2000).

This Paper

- Portfolio choice for a commodity index
- With or Without observing commodities' prices.
- Power utility and long horizon.
- Commodities: transitory price shocks
- Myopic policies far from optimal. Large intertemporal demand.
- Additional price information large even for risk-averse investors.
- Gains in equivalent safe rate of about 0.5%.

Commodity Futures

- P_t spot price of commodity at time t. Cannot be held like financial asset.
- F_t^T futures price at time t for expiration T. Zero cost.
- At time t, buy contracts expiring at $t + \Delta t$ equal to portfolio amount at t.
- At time $t + \Delta t$, liquidate contract (and buy new contracts expiring at $t + 2\Delta t$ equal to portfolio amount at $t + \Delta t$)
- Return on $[t, t + \Delta t]$, assuming zero safe rate:

$$\frac{F_{t+\Delta t}^{t+\Delta t} - F_t^{t+\Delta t}}{F_t^{t+\Delta t}} = Q_t^{t+\Delta t} \frac{P_{t+\Delta t}}{P_t} - 1 = Q_t^{t+\Delta t} \underbrace{\frac{P_{t+\Delta t} - P_t}{P_t}}_{\text{spot return}} + \underbrace{\left(Q_t^{t+\Delta t} - 1\right)}_{\text{roll return}}$$

  where $Q_t^{t+\Delta t} = P_t / F_t^{t+\Delta t}$ is the spot-futures ratio at time t.
- With roll-return of order dt, dynamics for rolled-over futures portfolio S_t is:

$$\frac{dS_t}{S_t} = \mu_t dt + \frac{dP_t}{P_t}$$

- Key difference: P_t is stationary.
  Empirically and theoretically.

Commodity Index Model

- n commodities. Return on futures portfolio of i-th commodity:

$$\frac{dS_t^i}{S_t^i} = \mu^i dt + \sigma^i dU_t^i \qquad dU_t^i = -\lambda^i U_t^i dt + dW_t^i$$

  W_t^i independent Brownian motions.

- Commodity index with weights w^i:

$$\frac{dS_t}{S_t} = \sum_{i=1}^{n} w^i \frac{dS_t^i}{S_t^i} = \left(\mu - \sum_{i=1}^{n} w^i \sigma^i \lambda^i U_t^i\right) dt + \sigma d\tilde{W}_t$$

  where $\mu = \sum_{i=1}^{n} w^i \mu^i$ and $\sigma \tilde{W}_t = \sum_{i=1}^{n} w^i \sigma^i W_t^i$ (W Brownian motion).

- Spot returns depend on spot prices $P_t^i = P_0^i e^{\sigma^i U_t^i}$...
- ...and so do optimal investment strategies. Notation:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dY_t \quad Y_t = \sum_{i=1}^{n} p_i U_t^i \quad p_i = w^i \sigma^i / \sigma \quad \sum_{i=1}^{n} p_i^2 = 1$$

One Asset Prelude

- One commodity only, $n = 1$. Compare Föllmer and Schachermayer (2008)
- Constant relative risk aversion. Utility $U(x) = x^{1-R}/(1-R)$.
- Wealth X_t satisfies budget equation $\frac{dX_t}{X_t} = \pi_t \frac{dS_t}{S_t}$. π_t portfolio weight.
- Maximize equivalent safe rate $\lim_{T\to\infty} \frac{1}{T} \log E[X_T^{1-R}]^{\frac{1}{1-R}}$
- Optimal policy:
$$\pi_t = \frac{\mu}{\sigma^2} - \frac{\lambda_1}{\sigma\sqrt{R}} U_t^1$$
- No R in the denominator! Interpretation?
- Equivalent safe rate:
$$\delta = \frac{\mu^2}{2\sigma^2} + \frac{\lambda_1}{2(1+\sqrt{R})}$$
- Risk-premium, plus market timing. Risk premium without risk aversion!
- Why?

Intertemporal Balance

- Myopic and intertemporal hedging decomposition:

$$\pi_t = \underbrace{\frac{\mu - \sigma\lambda_1 U_t^1}{R\sigma^2}}_{myopic} + \underbrace{\frac{(R-1)\mu + (1-\sqrt{R})\sigma\lambda_1 U_t^1}{R\sigma^2}}_{intertemporal}$$

$$= \frac{\mu}{\sigma^2} - \frac{\lambda_1}{\sigma\sqrt{R}} U_t^1$$

- Myopic demand offset by terms in intertemporal component.
- Transitory risks are not like permanent risks.
- They are “less risky”, so more risk averse investors take more such risks (than it they were permanent).
- Discontinuity for $\lambda \downarrow 0$, as transitory becomes permanent in the limit.
- Strategic vs. tactical exposures.
  Strategic independent of risk aversion and state: captures risk premium.
  Tactical independent of risk premium, captures imbalance in state.

Dynamics and Information

- Dynamics with observation of **commodities** prices:

$$\frac{dS_t}{S_t} = \left(\mu - \sum_{i=1}^{n} w^i \sigma^i \lambda^i U_t^i\right) dt + \sigma d\tilde{W}_t$$
$$dU_t^i = -\lambda^i U_t^i dt + dW_t^i$$

- Dynamics with observation of **index** price only (Kalman filter):

$$\frac{dS_t}{S_t} = \left(\mu - \sum_{i=1}^{n} w^i \sigma^i \lambda^i \tilde{U}_t^i\right) dt + \sigma d\tilde{W}_t$$
$$d\tilde{U}_t^i = -\lambda^i \tilde{U}_t^i dt + (p' - \gamma_t b') d\tilde{W}_t$$
$$\frac{d\gamma_t}{dt} = -\lambda\gamma_t - \gamma_t\lambda + I - (p' - \gamma_t b')(p' - \gamma_t b')'$$

$p = (p_1, \ldots, p_n)$, λ as diagonal matrix, γ_t $n \times n$ matrix.

- Time-dependent Kalman filter.

Long Term Filter

Proposition

For any initial γ_0, the solution γ_t to the Riccati differential equation converges:

$$\lim_{t \to +\infty} \gamma_t = \gamma,$$

where the matrix γ satisfies the Riccati algebraic equation

$$-\lambda\gamma - \gamma\lambda' + I - [p' - \gamma b'][p' - \gamma b']' = 0.$$

The dynamics of the filters $\tilde{U}_t^i$ becomes

$$d\tilde{U}_t^i = -\lambda_i \tilde{U}_t^i dt + \alpha_i d\tilde{W}_t \qquad \textit{where} \qquad \alpha_i' = p_i - \sum_{k=1}^{n} p_k \lambda_k \gamma_{ik}.$$

- Convergence relies on results on controllable and stabilizable systems.
- Bad news: Riccati matrix equation has no explicit solution in general.

Observing Commodities

Theorem

If an investor trades the index by observing the prices of all commodities:

$$\pi_t^{C*} = \frac{\mu}{\sigma^2} - \frac{p\left(\lambda - \mathbf{A}^C\right) U_t}{R\sigma} \qquad \textit{(Optimal Portfolio)}$$

$$\mathrm{EsR}^C = \frac{\mu^2}{2\sigma^2} + \frac{\mathrm{tr}\left(\mathbf{A}^C\right)}{2(1-R)} \qquad \textit{(Equivalent Safe Rate)}$$

$$\mathbf{A}^C = \lambda - \mathbf{C}^{-\frac{1}{2}}\left(\mathbf{C}^{\frac{1}{2}}\frac{\lambda^2}{2}\mathbf{C}^{\frac{1}{2}}\right)^{\frac{1}{2}}\mathbf{C}^{-\frac{1}{2}} \qquad \textit{and} \qquad \mathbf{C} = \frac{I}{2} + \frac{(1-R)p'p}{2R}$$

- Explicit solution to Riccati equation.
- Strategic vs. Tactical as with one-asset.
- Long-run verification theorem.

Observing Index

Theorem

If an investor trades the index observing only the index:

$$\pi_t^{I*} = \frac{\mu}{R\sigma^2} + \frac{(1-R)\beta^{I\prime}\alpha}{R\sigma} - \frac{\left(p\lambda - \alpha'\mathbf{A}^I\right)\tilde{U}_t}{R\sigma}$$

$$\mathrm{EsR}^I = \frac{\mu^2}{2R\sigma^2} + \frac{(1-R)^2(\beta^I\alpha)^2}{2R} + \frac{(1-R)\mu\beta^I\alpha}{R\sigma} + \frac{\operatorname{tr}(\alpha\alpha'\mathbf{A}^I)}{2(1-R)} + \frac{(1-R)\operatorname{tr}(\alpha\alpha'\beta^{I\prime}\beta^I)}{2}$$

where

$$\beta^I = -\frac{\mu\left(p\lambda - \alpha'\mathbf{A}^I\right)}{R\sigma}\left(\lambda + \frac{(1-R)\alpha p\lambda}{R} - \frac{\alpha\alpha'\mathbf{A}^I}{R}\right)^{-1},$$

and $\mathbf{A}^I$ is the symmetric, definite-positive solution of the matrix Riccati equation

$$\frac{\mathbf{A}^I\lambda + \lambda\mathbf{A}^I}{2} + \frac{1}{2R}\lambda p' p\lambda + \frac{(1-R)}{2R}\mathbf{A}^I\alpha\alpha'\mathbf{A}^I - \frac{(1-R)}{R}\frac{\lambda p'\alpha'\mathbf{A}^I + \mathbf{A}^I\alpha p\lambda}{2} = 0$$

- No explicit solution. Easy to solve numerically.
- Qualitative structure similar. Quantitative differences?

Commodities

- S&P GSCI Index.
- 6 commodities explain about 85% of index return variance.
- Rolled-over commodity futures:
  Each month, invest in two-month contract. Sell month afterwards.
- Understand optimal portfolios and equivalent safe rates.
  With or without observing commodities.

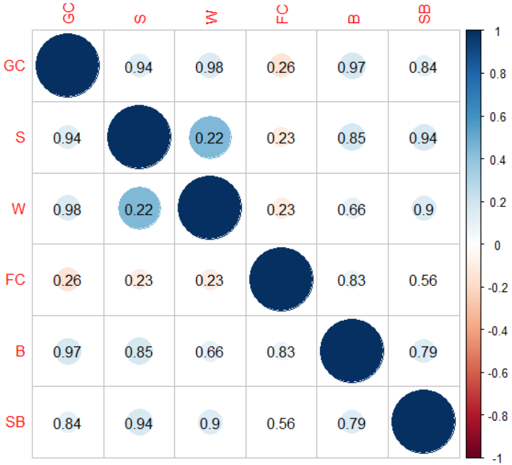
Commodities: Mean-Reverting

Calibrated Parameters

Commodity	Symbol	p_i	λ_i	$\omega_i(\%)$	$\sigma_i(\%)$
Wheat	**W**	0.07	0.11	9.2%	31.6%
Soybeans	**S**	0.08	0.21	6.8%	24.9%
Sugar	**SB**	0.04	0.12	2.5%	33.0%
Feeder Cattle	**FC**	0.05	0.17	7.6%	14.2%
Brent Crude Oil	**B**	0.85	0.12	58.9%	31.1%
Gold	**GC**	0.06	0.12	8.4%	16.0%
Residual	**RE**	0.51	0.01		
GSCI		$\mu(\%)$	$\sigma(\%)$		
		2.4	19.84		

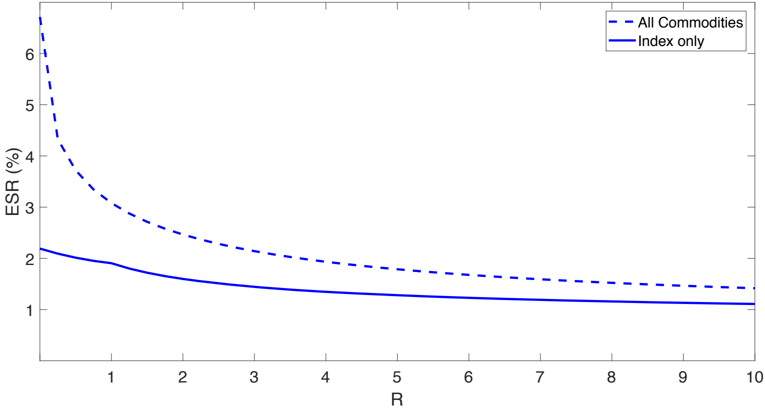
- Monthly Returns 1993:05-2018:02
- Weights estimated from sensitivities. Do not have to add to one.

Commodities: Uncorrelated Returns



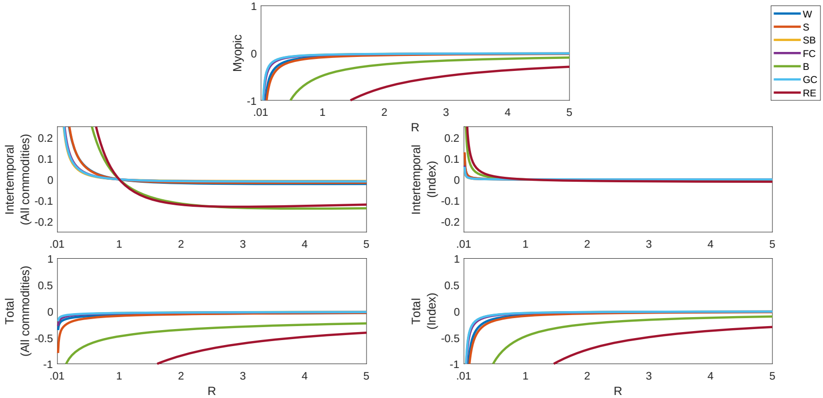
Colors = Correlations. Numbers = p-values.

Commodities: Equivalent Safe Rate



- Information in single commodity prices is economically significant (even highly risk averse investors)
- Gains in equivalent safe rate of about 0.5%.
- Market-timing component sensitive to drift of individual commodities
- Myopic investor values its strategy only on its growth rate, does not benefit from shifts in investment opportunities

Commodities: Optimal Portfolio



- Sensitivities of optimal portfolios with respect to the state variables $(U_t \& \tilde{U}_t)$ associated each commodity.
- Myopic sensitivities decline to zero as R increases and are the same in both full a partial information
- Left and Right panels show effect of partial information
- Sensitivity is driven by...
- ...intertemporal component with full info, myopic component with partial info

Conclusion

- Should Commodity Investors Follow Commodities' Prices?
- Long term investors should, even the more risk averse.
- Mean exposure to commodities insensitive to risk aversion...
- ...and optimal strategies benefit from the extra information.
- Gains similar to earning an extra risk-free 0.5% on wealth.

Thank You!

Questions?