Pricing and Hedging Defaultable Participating Contracts with Regime Switching and Jump Risk

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Outline

Participating Life Insurance Contract

Participating contracts is a popular class of insurance contracts. The premiums of these contracts are invested in a reference portfolio. The policyholder not only receives the guaranteed minimum return, but also participates in yields of the reference investment exceeding the minimum guarantee.

Credit Risk

In the 1990s, many insurance companies default in Europe, Japan and the United States, such as the First Executive Corporation, Garantie Mutuelle des Functionnaires and Nissan Mutual Life, etc.

Briys and De Varenne (1994 GPRIT, 1997 JRI), Grosen and Jørgensen (2002, JRI), Bernard, Le Courtois, and Quittard-Pinon (2005, IME), Bernard, Le Courtois, and Quittard-Pinon (2006, NAAJ), Le Courtois and Quittard-Pinon (2008, GRIR), Siu, Lau, and Yang (2008, IJSA), Fard and Siu (2013, IME), Le Courtois and Nakagawa (2013, MF), Cheng and Li (2018, IME)

Jump and Regime Switching

The evolution of asset prices includes important features such as jumps and regime switching.

- In the short term, the evolution of asset prices exhibits fairly extreme movements.
- In the long term, structural changes in the macroeconomic conditions or in the business cycles cause modifications in the evolution pattern of asset prices.

References

- Lévy models: Ballotta (IME, 2005), Riesner (IME, 2006), Kassberger, Kiesel, and Liebmann (IME, 2008), Le Courtois and Quittard-Pinon (GRIR, 2008) and Bauer, Bergmann, and Kiesel (AB, 2010).
- Regime switching Brownian motion models: Hardy (NAAJ, 2001), Siu (IME, 2005) and Lin, Tan, and Yang (NAAJ, 2009).
- Regime switching jump diffusion model: Fard and Siu (IME, 2013).

Our Main Contributions

- We develop a transform-based approach for the pricing of participating life insurance contracts with a constant guaranteed rate and with a floating guaranteed rate, in which we incorporate credit, market (jump), economic (regime switching) risks.
- We show that the contract with a floating guaranteed rate is a riskier but more worthy product when comparing to the contract with a constant guaranteed rate.
- We introduce dynamic and semi-static hedging strategies to hedge jump and regime switching risks in the participating contracts.

Participating Life Insurance Contracts

The life insurance company is supposed to invest A_0 in a reference portfolio and the initial capital of investment funds A_0 is financed by the premium payment of policyholders $L_0 = \alpha A_0$. Therefore, the policyholders can enjoy benefit of excess investment return from a fraction α of the funds.

The contract promises the policyholders that the premium payment L_0 will accumulate by a constant minimum guaranteed rate $\tilde{r}_g$ during the life of the contract. Then, a guaranteed maturity payment is $L_T^g = L_0 e^{\tilde{r}_g T}$. Once the funds run enough well, the policyholders obtain the bonus payment $\delta(\alpha A_T - L_T^g)_+$, where

- T is the maturity of the contract.
- A_T is the maturity value of investment funds.
- δ is the participation coefficient.

The Payoff Structure

The payoff of the contract without early default is as follows:

$$\Theta_L(T) = \begin{cases} A_T & \text{if } A_T < L_T \\ L_T & \text{if } L_T \leqslant A_T \leqslant \dfrac{L_T}{\alpha} \\ L_T + \delta(\alpha A_T - L_T) & \text{if } A_T > \dfrac{L_T}{\alpha} \end{cases}$$

We assume that the insurance company is continuously monitored and the default happens when the funds value A_t falls below a default boundary $B_t = \lambda L_t^g$. Then, the bankruptcy time is

$$\tau = \inf\{t \geqslant 0 : A_t \leqslant \lambda L_t^g\},$$

where

- $0 < \lambda < 1$ is a boundary level parameter.
- $L_t^g = L_0 e^{\tilde{r}_g t}$ is a nominal promise payment at time $t \in [0, T]$.

The Payoff Structure

Then, the pricing formula under the risk-neutral measure $\mathbb{Q}$ is as follows:

$$V = E_{\mathbb{Q}}\left(e^{-\int_0^T r_s \mathrm{d}s} (L_T^g - (L_T^g - A_T)_+ + \delta(\alpha A_T - L_T^g)_+)\mathbb{1}_{\tau \geqslant T} + e^{-\int_0^\tau r_s \mathrm{d}s} A_\tau \mathbb{1}_{\tau < T} \right),$$

where

- r_s is the market interest rate at time s.

We decompose the above pricing formula into four terms as follows:

$$\begin{cases} GF = E_{\mathbb{Q}}\left(e^{-\int_0^T r_s \mathrm{d}s} L_T^g \mathbb{1}_{\tau \geqslant T} \right) \\ PO = E_{\mathbb{Q}}\left(-e^{-\int_0^T r_s \mathrm{d}s} (L_T^g - A_T)_+ \mathbb{1}_{\tau \geqslant T} \right) \\ BO = E_{\mathbb{Q}}\left(e^{-\int_0^T r_s \mathrm{d}s} \delta(\alpha A_T - L_T^g)_+ \mathbb{1}_{\tau \geqslant T} \right) \\ LR = E_{\mathbb{Q}}\left(e^{-\int_0^\tau r_s \mathrm{d}s} A_\tau \mathbb{1}_{\tau < T} \right) \end{cases}$$

The Regime Switching Jump Diffusion Model

The dynamics of the funds value is supposed to follow changes of an exponential regime switching jump diffusion process under the risk-neutral measure $\mathbb{Q}$:

$$A_t = A_0 e^{X_t},$$

where A_0 is the initial funds value and X is a regime switching jump diffusion process:

$$X_t \quad = \int_0^t \langle \hat{\mu}, J_s \rangle \mathrm{d}s + \int_0^t \langle \hat{\sigma}, J_s \rangle \mathrm{d}W_s + \int_0^t \mathrm{d}\langle \hat{N}, J_s \rangle$$

where

- J is a continuous time Markov chain process.
- $\hat{\mu}$ and $\hat{\sigma}$ are constant vectors.
- For each state i, $\hat{N}_i$ is a compound Poisson process with rate $\hat{\lambda}_i$ and the jump size is modeled with an asymmetric double exponential distribution.
- W is an independent standard Brownian motion.

The riskless rate also changes with the state of the economy. Let the riskless rate be $r_t = \langle \hat{r}, J_t \rangle$, where $\hat{r} = (\hat{r}_1, \hat{r}_2, ..., \hat{r}_n)$ and $\hat{r}_i > 0$ is the riskless rate at state e_i.

Let $Z_t = X_t - \tilde{r}_g t$ and Z_t is still a regime switching Kou process. We rewrite the default time τ in a constant barrier form as

$$\tau = \inf\{t \geqslant 0 : Z_t \leqslant \ln \frac{\lambda L_0}{A_0}\},$$

and the default characterization becomes related to a first passage time problem of the regime switching Kou process Z.

The First Passage Time Results

From Le Courtois and Su (2018), we introduce the following first passage time result of X, based on which we deduce closed-form formulas for the price of the contracts.

Proposition 1 Denote the first passage time of X below a constant level b as τ, so that

$$\tau = \inf\{t \geqslant 0 : X_t \leqslant b\}.$$

Let $a_t = \langle a, J_t \rangle$ and the contingent payoff be $h_\tau = \langle J_\tau, \hat{h} \rangle$ where $\hat{h} = (\hat{h}_1, \ldots, \hat{h}_n)$. For any $w > -\tilde{\theta}$, we have

$$E\left(e^{-\int_0^\tau a_s \mathrm{d}s + wX_\tau} h_\tau \right) = Y_0 W^{(a,-)} e^{Q^{(a,-)}(x-b)+wbI_{2n}} \tilde{h},$$

where x is the initial point of X, Y_0 is the initial state vector of Y,

$$\tilde{h} = \left(\left(\hat{h}_1, \ldots, \hat{h}_n \right), \left(\frac{\hat{\theta}_1}{w + \hat{\theta}_1} \hat{h}_1, \ldots, \frac{\hat{\theta}_n}{w + \hat{\theta}_n} \hat{h}_n \right) \right)'.$$

- $$W^{(a,-)} = \begin{pmatrix} \zeta^{(a,-)} \\ I_{2n} \end{pmatrix}$$
- $\zeta^{(a,-)}$ and $Q^{(a,-)}$ are the Wiener-Hopf factors.
- Y_0 is the initial state of Y, which is a continuous time Markov chain process with a finite state space

$$E = \underbrace{E^+}_{\text{Positive jump}} \cup \underbrace{E^0}_{\text{Pure diffusion}} \cup \underbrace{E^-}_{\text{Negative jump}}.$$

The First Passage Time Results

Let M be the fluid embedding of X, where M is a continuous stochastic process whose sample paths are constructed from the sample paths of X by replacing positive jumps with linear segments of slope $+1$ and negative jumps with linear segments of slope -1.

The up-crossing and down-crossing ladder processes $\tilde{Y}^{+}$, $\tilde{Y}^{-}$ of (M, Y) are defined as time changes of Y that are constructed such that Y is observed only when new maxima and minima of M occur respectively. They are Markov processes. $Q^{(a,+)}$ and $Q^{(a,-)}$ are the generator matrices of $\tilde{Y}^{+}$ and $\tilde{Y}^{-}$ and $\zeta^{(a,+)}$ and $\zeta^{(a,-)}$ are the corresponding initial distributions.

The quadruple $(\zeta^{(a,+)}, Q^{(a,+)}, \zeta^{(a,-)}, Q^{(a,-)})$ is a unique Wiener-Hopf factorization of (M, Y) when $a > 0$.

The First Passage Time Results

Definition 1 Let $\left(G^{(a,+)}, G^{(a,-)}\right)$ be a pair of irreducible $2n \times 2n$ matrices, that is, matrices with non-negative off-diagonal elements and non-positive row sums, and $\left(\Pi^{(a,+)}, \Pi^{(a,-)}\right)$ be a pair of $n \times 2n$ matrices with rows made of sub-probability vectors. The quadruple

$$\left(\Pi^{(a,+)}, G^{(a,+)}, \Pi^{(a,-)}, G^{(a,-)}\right)$$

is the Wiener-Hopf factorization of (M, Y) that is associated with $a > 0$ if

$$\Xi(-G^{(a,+)}, W^{(a,+)}) = \Xi(G^{(a,-)}, W^{(a,-)}) = O_{3n \times 2n},$$

where

$$\Xi(S, W) = \frac{1}{2}\Sigma^2 W S^2 + VWS + Q_a W,$$

with the $3n \times 3n$ diagonal matrices:

$$\Sigma = \begin{pmatrix} O_{n\times n} & & \\ & \begin{pmatrix} \hat{\sigma}_1 & & \\ & \ddots & \\ & & \hat{\sigma}_n \end{pmatrix} & \\ & & O_{n\times n} \end{pmatrix}, \quad V = \begin{pmatrix} I_n & & \\ & \begin{pmatrix} \hat{\mu}_1 & & \\ & \ddots & \\ & & \hat{\mu}_n \end{pmatrix} & \\ & & -I_n \end{pmatrix},$$

and the $3n \times 2n$ matrices:

$$W^{(a,+)} = \begin{pmatrix} I_{2n} \\ \Pi^{(a,+)} \end{pmatrix}, \quad W^{(a,-)} = \begin{pmatrix} \Pi^{(a,-)} \\ I_{2n} \end{pmatrix},$$

where I_n and I_{2n} are identity matrices of size $n \times n$ and $2n \times 2n$, respectively, and $O_{3n\times 2n}$ denotes a zero matrix of size $3n \times 2n$.

Numerical Method

From Le Courtois and Su (2018), we introduce a numerical method to compute matrix Wiener-Hopf factorization. Once we have $(\zeta^{(a,+)}, Q^{(a,+)}, \zeta^{(a,-)}, Q^{(a,-)})$, we can compute the value of the contracts, whose closed-form expression will be built on these matrix Wiener-Hopf factors.

Numerical Algorithm for the Computation of $(\zeta^{(a,+)}, Q^{(a,+)}, \zeta^{(a,-)}, Q^{(a,-)})$

- Step 1: Compute $4n$ roots

  $$\Re(\nu_1) \leqslant \Re(\nu_2) \leqslant \ldots \leqslant \Re(\nu_{2n}) \leqslant 0 \leqslant \Re(\nu_{2n+1}) \leqslant \Re(\nu_{2n+2}) \leqslant \ldots \leqslant \Re(\nu_{4n})$$

  from the equation $f(\nu) = 0$ where $f(\nu) = \det(K(\nu)) = 0$ and $K(\nu) = \frac{1}{2}\Sigma^2\nu^2 - V\nu + Q_a$. Let

  $$\tilde{\beta}_i = \nu_i, \qquad i = 1, \ldots, 2n,$$
  $$\bar{\beta}_j = -\nu_{2n+j}, \qquad j = 1, \ldots, 2n.$$

- Step 2: For $i = 1, \ldots, 4n$, compute the $3n \times 1$ vector γ_i by solving a system of linear equations $K(\nu_i)\gamma_i = 0$.

Numerical Method

- Step 3: Let

$$\tilde{\vartheta}_i = (\gamma_{i,1}, ..., \gamma_{i,2n})', \qquad i = 1, ..., 2n,$$

$$\bar{\vartheta}_j = (\gamma_{2n+j,n+1}, ..., \gamma_{2n+j,3n})', \qquad j = 1, ..., 2n,$$

and

$$\tilde{Z} = [\tilde{\vartheta}_1, \tilde{\vartheta}_2, ..., \tilde{\vartheta}_{2n}], \qquad \bar{Z} = [\bar{\vartheta}_1, \bar{\vartheta}_2, ..., \bar{\vartheta}_{2n}].$$

Then, we obtain

$$Q^{(a,+)} = \tilde{Z}\,\mathrm{diag}\{\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_{2n}\}\,\tilde{Z}^{-1},$$

$$Q^{(a,-)} = \bar{Z}\,\mathrm{diag}\{\bar{\beta}_1, \bar{\beta}_2, ..., \bar{\beta}_{2n}\}\,\bar{Z}^{-1}.$$

The matrix exponential is computed as:

$$e^{Q^{(a,+)}x} = \tilde{Z}\,\mathrm{diag}\{e^{\tilde{\beta}_1 x}, e^{\tilde{\beta}_2 x}, ..., e^{\tilde{\beta}_{2n} x}\}\,\tilde{Z}^{-1},$$

$$e^{Q^{(a,-)}x} = \bar{Z}\,\mathrm{diag}\{e^{\bar{\beta}_1 x}, e^{\bar{\beta}_2 x}, ..., e^{\bar{\beta}_{2n} x}\}\,\bar{Z}^{-1}.$$

- Step 4: For $k = 1, ..., n$, compute $2n \times 1$ vector $\tilde{\xi}_k$ by solving a system of linear equations $\tilde{Z}'\tilde{\xi}_k = (\gamma_{1,2n+k}, ..., \gamma_{2n,2n+k})'$ and compute $2n \times 1$ vector $\bar{\xi}_k$ by solving a system of linear equations $\bar{Z}'\bar{\xi}_k = (\gamma_{2n+1,k}, ..., \gamma_{4n,k})'$. Then,

$$\zeta^{(a,+)} = [\tilde{\xi}_1, \tilde{\xi}_2, ..., \tilde{\xi}_n]'$$

and

$$\zeta^{(a,-)} = [\bar{\xi}_1, \bar{\xi}_2, ..., \bar{\xi}_n]'.$$

Contract Valuation

Let $\widehat{GF}$ and $\widehat{LR}$ be the Laplace transform of GF and LR, respectively, and $\widetilde{\widehat{PO}}$ and $\widetilde{\widehat{BO}}$ be the Laplace-Fourier transform of PO and BO, respectively. Then, we obtain

$$\begin{cases}
\widehat{GF}(u) = L_0 \left(Y_0 W_{\tilde{Z}}^{(\hat{r}+u-\tilde{r}_g,-)} e^{Q_{\tilde{Z}}^{(\hat{r}+u-\tilde{r}_g,-)}\left(x-\ln\frac{\lambda L_0}{A_0}\right)} H(0) - J_0 \right) (Q - \mathrm{diag}\{\hat{r}+u-\tilde{r}_g\})^{-1}\mathbb{1}_n. \\
\widehat{LR}(u) = \frac{A_0}{u} Y_0 W_{\tilde{Z}}^{(\hat{r}+u-\tilde{r}_g,-)} e^{Q_{\tilde{Z}}^{(\hat{r}+u-\tilde{r}_g,-)}\left(x-\ln\frac{\lambda L_0}{A_0}\right)+\ln\frac{\lambda L_0}{A_0} I_{2n}} H(1)\mathbb{1}_n. \\
\widetilde{\widehat{PO}}(u,v) = -\frac{A_0}{(\alpha_1 - iv)(\alpha_1 - iv - 1)} \left(Y_0 W_{\tilde{Z}}^{(\hat{r}+u-\tilde{r}_g,-)} e^{Q_{\tilde{Z}}^{(\hat{r}+u-\tilde{r}_g,-)}\left(x-\ln\frac{\lambda L_0}{A_0}\right)-(\alpha_1-iv-1)\ln\frac{\lambda L_0}{A_0} I_{2n}} \right. \\
\qquad \left. H(iv+1-\alpha_1) - J_0\right) (Q - \mathrm{diag}\{\hat{r}+u-\tilde{r}_g-\phi_k^Z(iv+1-\alpha_1)\})^{-1}\mathbb{1}_n. \\
\widetilde{\widehat{BO}}(u,v) = \frac{\delta\alpha A_0}{(\alpha_2 - iv)(\alpha_2 - iv + 1)} \left(Y_0 W_{\tilde{Z}}^{(\hat{r}+u-\tilde{r}_g,-)} e^{Q_{\tilde{Z}}^{(\hat{r}+u-\tilde{r}_g,-)}\left(x-\ln\frac{\lambda L_0}{A_0}\right)+(\alpha_2-iv+1)\ln\frac{\lambda L_0}{A_0} I_{2n}} \right. \\
\qquad \left. H(\alpha_2-iv+1) - J_0\right) (Q - \mathrm{diag}\{\hat{r}+u-\tilde{r}_g-\phi_k^Z(\alpha_2-iv+1)\})^{-1}\mathbb{1}_n.
\end{cases}$$

Contract Valuation

where

- Q is the generator matrix of J.
- $\phi_k^Z(z) = \hat{\mu}_k^Z z + \frac{1}{2}\hat{\sigma}_k^2 z^2 + \hat{\lambda}_k \left(\frac{\hat{p}_k \hat{\eta}_k}{\hat{\eta}_k - z} + \frac{\hat{q}_k \hat{\theta}_k}{\hat{\theta}_k + z} - 1 \right)$.
- $$H(w) = \begin{pmatrix} I_n \\ \begin{pmatrix} \frac{\hat{\theta}_1}{w+\hat{\theta}_1} & & \\ & \ddots & \\ & & \frac{\hat{\theta}_n}{w+\hat{\theta}_n} \end{pmatrix} \end{pmatrix}.$$

Contract Valuation

The two states e_1 and e_2 represent a "good" and a "bad" macroeconomic environment, respectively, where the "good" one bears higher interest rates and makes the funds value dynamics exhibit favorable features, such as less fluctuations, smaller average size of negative jumps and larger average size of positive jumps, etc.

Table 1 **Contract Parameters**

α	$\tilde{r}_g$	δ	T	λ
0.85	0.02	0.9	10	0.8

Table 2 **Funds Dynamics Parameters**

A_0	State	$\hat{r}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{p}$	$\hat{\eta}$	$\hat{\theta}$
100	e_1	0.03	0.2	1.5	0.5	35	45
	e_2	0.02	0.4	3	0.5	45	30

Contract Valuation

Table 3 **Contract and Subcontract Values**

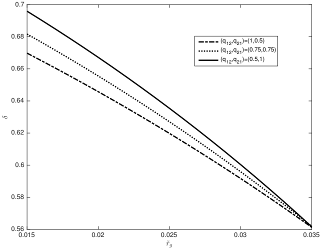
	(q_{12}, q_{21})	GF	PO	BO	LR	Contract	Time
	(1, 0.5)	13.6173	-0.0345	21.7168	55.8348	91.1343	0.2555 min
Laplace-Fourier	(0.75, 0.75)	16.0942	-0.0457	21.4250	53.3456	90.8190	0.3301 min
	(0.5, 1)	19.3834	-0.0628	21.0055	50.1079	90.4340	0.2810 min
	(1, 0.5)	13.8766	-0.0379	21.9741	55.2439	91.0567 (90.9310, 91.1823)	29.8889 min
Monte-Carlo	(0.75, 0.75)	16.3868	-0.0497	21.6857	52.7593	90.7821 (90.6696, 90.8946)	29.8177 min
	(0.5, 1)	19.7011	-0.0674	21.1984	49.5445	90.3765 (90.2834, 90.4696)	30.5151 min

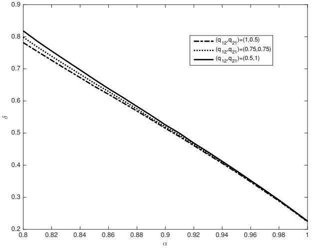
Contract Valuation

The fair contract follows the equilibrium condition: the policyholders' premium investment L_0 is equal to the initial market value of the contract V. This condition enables us to compute the fair participating rate in terms of the other parameters:

$$\delta = \frac{L_0 - GF - PO - LR}{BO^*},$$

where $BO^* = E_{\mathbb{Q}}\left((\alpha A_0 e^{Z_T} - L_0 e^{r^f T})_+ \mathbb{1}_{\tau \geqslant T}\right)$ is the stochastic component of BO, which can be solved by using the same method as in computation of BO.





Constant Guaranteed Rates V.S. Floating Guaranteed Rates

We compare the current contracts (constant contracts) with the contracts having a floating guaranteed rate (floating contracts), whose minimum guaranteed rate is linked to market interest rates. Let the guaranteed interest rates of the floating contracts be $r_s^g = r^f + r_s$ at time s where r_s is the market interest rate and r^f is a constant to control the difference between the guaranteed rates and market interest rates.

The floating guaranteed rates reduce risk exposure to the fluctuations of interest rate, which circumvents an issue of a dramatic narrowing in the safety margin when low interest rates persist for long.

Constant Guaranteed Rates V.S. Floating Guaranteed Rates

The four subcontract terms of the floating contracts are as follows:

$$\begin{cases} GF = L_0 e^{r^f T}\mathbb{Q}(\tau \geqslant T) \\ PO = E_{\mathbb{Q}}\left(-e^{r^f T}(L_0 - A_0 e^{Z_T})_+ \mathbb{1}_{\tau \geqslant T}\right) \\ BO = E_{\mathbb{Q}}\left(\delta e^{r^f T}(\alpha A_0 e^{Z_T} - L_0)_+ \mathbb{1}_{\tau \geqslant T}\right) \\ LR = E_{\mathbb{Q}}\left(A_0 e^{r^f \tau} e^{Z_\tau} \mathbb{1}_{\tau < T}\right) \end{cases}$$

where $Z_t = X_t - \int_0^t r_s^g \mathrm{d}s$.

Constant Guaranteed Rates V.S. Floating Guaranteed Rates

The Laplace or Laplace-Fourier transform results of the four subcontracts terms of the floating contracts are

$$
\begin{cases}
\widehat{GF}(u) = \dfrac{L_0}{u-r^f}\left(1 - Y_0 W_Z^{(\tilde{u}-r^f,-)} e^{Q_Z^{(\tilde{u}-r^f,-)}\left(x-\ln\frac{\lambda L_0}{A_0}\right)} H(0)\mathbb{1}_n\right) \\
\widehat{LR}(u) = \dfrac{A_0}{u} Y_0 W_Z^{(\tilde{u}-r^f,-)} e^{Q_Z^{(\tilde{u}-r^f,-)}\left(x-\ln\frac{\lambda L_0}{A_0}\right)+\ln\frac{\lambda L_0}{A_0} I_{2n}} H(1)\mathbb{1}_n \\
\widetilde{\widehat{PO}}(u,v) = -\dfrac{A_0}{(\alpha-iv)(\alpha-iv-1)}\left(Y_0 W_Z^{(\tilde{u}-r^f,-)} e^{Q_Z^{(\tilde{u}-r^f,-)}\left(x-\ln\frac{\lambda L_0}{A_0}\right)-(\alpha-iv-1)\ln\frac{\lambda L_0}{A_0} I_{2n}}\right. \\
\qquad \left. H(iv+1-\alpha) - J_0\right)\left(Q - \operatorname{diag}\{u - r^f - \phi_k^Z(iv+1-\alpha)\}\right)^{-1}\mathbb{1}_n \\
\widetilde{\widehat{BO}}(u,v) = \dfrac{\delta\alpha A_0}{(\alpha-iv)(\alpha-iv+1)}\left(Y_0 W_Z^{(\tilde{u}-r^f,-)} e^{Q_Z^{(\tilde{u}-r^f,-)}\left(x-\ln\frac{\lambda L_0}{A_0}\right)+(\alpha-iv+1)\ln\frac{\lambda L_0}{A_0} I_{2n}}\right. \\
\qquad \left. H(\alpha-iv+1) - J_0\right)\left(Q - \operatorname{diag}\{u - r^f - \phi_k^Z(\alpha-iv+1)\}\right)^{-1}\mathbb{1}_n
\end{cases}
\tag{1}
$$

Constant Guaranteed Rates V.S. Floating Guaranteed Rates

Table 3 **Contract and Subcontract Values**

	(q_{12}, q_{21})	GF	PO	BO	LR	Contract	Time
Laplace-Fourier	(1, 0.5)	13.6173	-0.0345	21.7168	55.8348	91.1343	0.2555 min
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	(0.5, 1)	19.7011	-0.0674	21.1984	49.5445	90.3765 (90.2834, 90.4696)	30.5151 min

Table 4 **Contract and Subcontract Values**

	(q_{12}, q_{21})	GF	PO	BO	LR	Contract	Time
Laplace-Fourier	(1, 0.5)	13.5018	-0.0344	21.5684	56.1638	91.1997	0.3707 min
	(0.75, 0.75)	15.9877	-0.0458	21.3005	53.6339	90.8763	0.3458 min
	(0.5, 1)	19.2957	-0.0632	20.9117	50.3350	90.4793	0.3503 min
Monte-Carlo	(1, 0.5)	13.7843	-0.0375	21.8485	55.5522	91.1475 (91.0225, 91.2726)	29.4393 min
	(0.75, 0.75)	16.3144	-0.0500	21.5504	53.0190	90.8338 (90.7216, 90.9460)	29.8192 min
	(0.5, 1)	19.6178	-0.0678	21.1404	49.7671	90.4574 (90.3633, 90.5516)	30.5365 min

Constant Guaranteed Rates V.S. Floating Guaranteed Rates

Let y_0 be a discount rate that makes the discounted promised maturity payment equal to the premium, i.e.,

$$L_0 e^{y_0 T} = L_0 E_Q \left(e^{\int_0^T r_s^g \mathrm{d}s} \right).$$

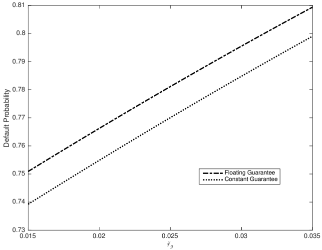
We obtain the explicit form of y_0 as

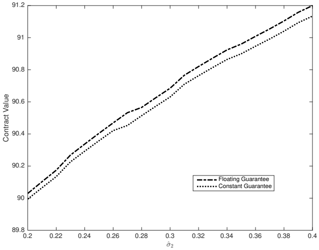
$$y_0 = \frac{\log(J_0 e^{(Q+\mathrm{diag}\{r^f+\hat{r}\})T} \mathbb{1}_n)}{T}.$$

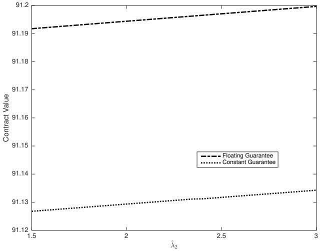
When we compare the constant contracts and floating contracts, we make them bear the same promised maturity payment, i.e., $y_0 = \tilde{r}_g$, and keep the same in other settings.

Constant Guaranteed Rates V.S. Floating Guaranteed Rates

The floating contracts are riskier products accompanied with higher returns.







Hedging Strategy

We develop a dynamic hedging strategy where we choose the optimal delta to minimize the quadratic risk of the change differences between the value of the hedging portfolio and participating contracts at each rebalance time. We also introduce a semi-static hedging strategy developed in He, Kennedy, Coleman, Forsyth, Li, and Vetzal (2006), where we make some adjustments to fit our regime switching case.

Dynamic Hedging Strategy

We compute the delta by solving the following optimization problem:

$$\arg\min_{\delta_k} E((\Delta C_k - \delta_k \Delta S_k)^2 | \mathscr{F}_k).$$

Then, at t_0, the bank account $B_0 = C_0 - \delta_0 S_0$. At each rebalance time t_k, the bank account after rebalancing changes into

$$B_k = e^{\int_{t_{k-1}}^{t_k} r_s \mathrm{d}s} B_{k-1} - (\delta_k - \delta_{k-1}) S_k.$$

Then, at time t_k after rebalancing the overall hedged position has value

$$\Pi(t_k) = -C_k + \delta_k S_k + B_k,$$

where

- B_i is the amount of the bank account at each rebalance time t_i.
- $\Delta C_i, \Delta S_i$ are the value change from t_i to t_{i+1} in the participating contract and the underlying asset, respectively.

Semi-static Hedging Strategy

The semi-static hedging strategy in He, Kennedy, Coleman, Forsyth, Li, and Vetzal (2006) computes the optimal holding $\{e_k, \hat{w}_k\}$ at each rebalance time t_k by solving the following optimization problem

$$\arg\min_{e_k,\hat{w}_k} E\left(\left(\Delta C_k - e_k\Delta S_k - \sum_{j=1}^{\tilde{n}} \hat{w}_{k,j}\Delta\hat{I}_{k,j}\right)^2 |\mathscr{F}_k\right),$$

where

$$\begin{cases} \Delta C_k = C_{k+1} - e^{\int_{k\Delta}^{(k+1)\Delta} r_s \mathrm{d}s} C_k \\ \Delta S_k = S_{k+1} - e^{\int_{k\Delta}^{(k+1)\Delta} r_s \mathrm{d}s} S_k \\ \Delta\hat{I}_{k,j} = \hat{I}_{k+1,j} - e^{\int_{k\Delta}^{(k+1)\Delta} r_s \mathrm{d}s} \hat{I}_{k,j} \end{cases},$$

which takes time value into consideration because of the large rebalance time interval that is induced by infrequent rebalances in the semi-static hedge, where

- Δ is the time step size of the m rebalance times.
- $\hat{I}_i = (\hat{I}_{i,1}, ..., \hat{I}_{i,\tilde{n}})$ are the value of $\tilde{n}$ options at rebalance time t_i.

Semi-static Hedging Strategy

Then, at t_0, the bank account $B_0 = C_0 - e_0 S_0 - \sum_{j=1}^{\tilde{n}} \hat{w}_{0,j} \hat{I}_{0,j}$. The self-financing constraint makes the bank account at time t_k after rebalancing become

$$B_k = e^{\int_{t_{k-1}}^{t_k} r_s \mathrm{d}s} B_{k-1} - (e_k - e_{k-1}) S_k - \sum_{j=1}^{\tilde{n}} (\hat{w}_{k,j} - \hat{w}_{k-1,j}) \hat{I}_{k,j}.$$

Then, at time t_k after rebalancing the whole hedged position has value

$$\Pi(t_k) = -C_k + e_k S_k + \sum_{j=1}^{\tilde{n}} \hat{w}_{k,j} \hat{I}_{k,j} + B_k.$$

Hedging Effectiveness

In the semi-static hedge, at each rebalance time t_k we employ five call options with strike prices $0.8S_k, 0.9S_k, S_k, 1.1S_k, 1.2S_k$ and maturity t_{k+1}. We make $m = 10$ and other parameters are set as before except making $\tilde{r}_g = 0.015$. We generate 100000 sample paths and for each path, we calcuate the discounted profit and loss $e^{-\int_0^{t^*} r_s \mathrm{d}s}\Pi(t^*)$ (APL) and its relative value $e^{-\int_0^{t^*} r_s \mathrm{d}s}\frac{\Pi(t^*)}{V_0}$ (RPL) at default time or maturity t^* of the participating contract where V_0 is the contract value.

Table 5 **Comparison of hedging effectiveness**

Measure	No hedge	Dynamic hdege	Semi-static hedge
Mean(APL)	89.3036	0.9949	14.7473
Std(APL)	108.0681	24.2676	29.3182
$\mathrm{VaR}_{95\%}(\mathrm{APL})$	246.8931	23.1145	49.4263
$\mathrm{CVaR}_{95\%}(\mathrm{APL})$	434.2527	26.5566	71.9829
Mean(RPL)	1.0000	0.1157	0.2133
Std(RPL)	1.2101	0.2580	0.3919

Hedging Effectiveness

Figure 6 Comparison of probability densities of APL