Design of risk sharing for variable annuities

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INNOVATING ACTUARIAL RESEARCH ON FINANCIAL RISK AND ERM

Outline

Introduction

▶ **Motivation :** In the recent years, the pension reform and longevity risk have weakened the annuity market. The revival of annuity market will come from the design of new risk-sharing annuities.

There exists several annuities for which the whole risk is borne by the insurer. Researchers are paying attention on the risk-sharing annuities. Some new designed annuities are pooled annuities where the whole risk is borne by the pool (i.e by the annuitants); e.g the group self-annuitization (GSA).

GSA schemes operate like conventional annuities for which uncertain future mortality risks are shared in the pool.

Introduction

- **Literature :**
  - Piggott *et al.* (2005 [2]) proposed a formal analysis of benefit adjustment from a longevity risk-pooling fund : the GSA
  - Qiao & Sherris (2013 [1]) have extended [2] by showing the extent to which the pooling mechanism can be made effective

Our goal : Propose some (partial or whole) risk-sharing GSA (i.e risk sharing between the pool and the insurer)

Framework

- Filtered probability space $(\Omega, \mathcal{F}_t, \mathbb{F}, \mathbb{P})$ made of a risky S_t and a risk-free B_t assets
- Strategy: Constant proportion $x \in [0, 1]$ is invested on S_t and proportion $(1 - x)$ on B_t; with a dynamic rebalancing. Investment on $[t, T]$ is

$$A(t,T) = A(0,t)\ e^{(\mu x + (1-x)r + \sigma^2 x^2/2)(T-t) + x\sigma W_{T-t}},$$

with $A_t = A(0,t)$; $r,\ \sigma,\ \mu \in \mathbb{R}_+^*$

- Mortality model: force of mortality follows the Hull-White (HW) model

$$d\mu_t^{x_0} = (\theta(t) - a\mu_t^{x_0})dt + \sigma^\mu dW_t^\mu, \quad \text{for all } t \geq 0$$

where $\theta(t) = Ae^{Bt}$ (Gompertz model).

Assumptions

- W_t independent of W_t^μ;
- Consider an open (homogeneous or heterogeneous) pool of policyholders;
- Only longevity and equity risks are considered;

Features of GSA :

▷ Longevity and equity risks fully borne by the pool;

▷ Annual benefits defined iteratively.

Recall on GSA – Notation

- B_t : total benefit of the pool at time t;
- ${}_{x}^{k}B_{i,t}$: benefit at time t of the i^{th} annuitant entering the pool at age x, k period of time ago;
- F_t : total fund of the pool at time t;
- ${}_{x}^{k}\hat{F}_{i,t}$: fund value at t of the i^{th} policyholder including the inheritance from those who died during the period $[t-1, t]$;
- ${}_{x}^{k}F_{i,t}$: fund value at time t of the i^{th} policyholder who entered the pool at age x, k period ago;
- ${}_{s}p_{x,t}$: the survival probability at time t of an annuitant aged x of living s more years;
- $\ddot{a}_x(t)$: annuity factor at time t of a policyholder aged x.

Recall on GSA

Proposed in 2005 by Piggott *et al.* [2]

- Capture the mortality changes using a mortality experience adjustment factor :

$$\text{MEA}_t = \frac{F_t}{\sum_{k\geq 1}\sum_{x} p^{-1}_{x+k-1,t-1} \sum_{A_t} {}^{k}_{x}F_{i,t}};$$

- Capture the updates on the mortality information available at each time by a change expectation adjustment factor :

$$\text{CEA}_t = \frac{\ddot{a}_{x+k}(t-1)}{\ddot{a}_{x+k}(t)};$$

- Capture the investment changes using an interest rate adjustment factor :

$$\text{IRA}_t = \frac{1+R_t}{1+R}$$

Recall on GSA

- Individual benefit at entrance :

$$ {}^{0}_{x}B_{i,t} = \frac{{}^{0}_{x}F_{i,t}}{\ddot{a}_x(t)}. \quad (1) $$

- Individual benefit afterwards i.e $k > 0$:

$$ {}^{k}_{x}B_{i,t} = {}^{k-1}_{x}B_{i,t-1} \ \mathrm{MEA}_t \ \mathrm{IRA}_t \ \mathrm{CEA}_t; $$

Recall on revised GSA

Proposed in 2013 by Qiao & Sherris ([1])

- Increase the effectiveness of the pooling mechanism
- Capture the mortality changes using a single mortality adjustment factor :

$$\text{TEA}_t = \frac{F_t}{\sum_{k\geq 1}\sum_{x} p^{-1}_{x+k-1,t-1} \, \frac{\ddot{a}_{x+k}(t)}{\ddot{a}_{x+k}(t-1)} \, \sum_{A_t} {}^{k}_{x}F_{i,t}};$$

- TEA_t improve the dependence of mortality changes across the pools
- Individual benefit for $k > 0$:

$${}^{k}_{x}B_{i,t} \; = \; {}^{k-1}_{x}B_{i,t-1} \; \text{TEA}_t \; \text{IRA}_t;$$

On the risk-sharing GSA

- Shifting a constant or variable proportion of the whole and / or partial risk to the insurer;
- Then reduction of the risk borne by the pool and increase of annual benefits;
- $\overline{{}^{k}_{x}R_{i,t}}$ denotes the individual annual benefit of the risk-sharing (revised) GSA;
- *Lower bound threshold sharing* : individual benefit at least equal to the threshold $\overline{{}_{x}R_{i}}$;
- *Direct or static sharing* : the risk is statically shared between the insurer and the pool.

Lower bound threshold sharing

- $\gamma_t \in [0, 1]$ is the proportion of risk shifted to the insurer;
- Annual lower bound : $\overline{{}_{x}R_i}(\gamma_t) = \gamma_t \; {}_{x}^{0}R_{i,t}$
- Defined the individual annual benefit :

$$\overline{{}_{x}^{k}R_{i,t}}(\gamma_t) \;=\; \max\left({}_{x}^{k}R_{i,t},\; \overline{{}_{x}R_i}(\gamma_t)\right);$$

where ${}_{x}^{k}R_{i,t}$ is defined by a GSA or equals to

$${}_{x}^{k}R_{i,t} = \overline{{}_{x}^{k-1}R_{i,t-1}}(\gamma_{t-1})\ \text{Adjust}_t.$$

Static sharing

- $\beta'_t,\ \beta_t \in [0,1]$ are respectively the proportion of mortality and equity risks shifted to the pool;

- Defined the individual annual benefit of the risk-sharing GSA :

$$\begin{aligned} \overline{{}^{k}_{x}B_{i,t}}(\beta_t,\beta'_t) &= \overline{{}^{k-1}_{x}B_{i,t-1}}(\beta_t,\beta'_t)\left[\beta_t \ \mathrm{MEA}_t \ \mathrm{CEA}_t + (1-\beta_t)\right] \\ &\quad \times \left[\beta'_t \ \mathrm{IRA}_t + (1-\beta'_t)\right]; \end{aligned}$$

- Individual annual benefit of the risk-sharing revised GSA

$$\begin{aligned} \overline{{}^{k}_{x}B_{i,t}}(\beta_t,\beta'_t) &= \overline{{}^{k-1}_{x}B_{i,t-1}}(\beta_t,\beta'_t)\left[\beta_t \ \mathrm{TEA}_t + (1-\beta_t)\right] \\ &\quad \times \left[\beta'_t \ \mathrm{IRA}_t + (1-\beta'_t)\right]; \end{aligned}$$

Numerical parameters

- Consider a $n = 9$ years term annuity with $x = 15\%$, $r = R = 1\%$ and a closed heterogeneous pool :

Group i	Individual premium $F_{i,0}$	Initial age x_i	Number of annuitants $N_{i,0}$
G1	1000€	65	100
G2	700€	70	700

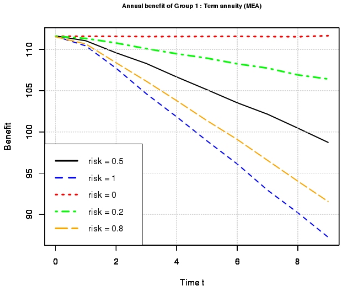
- $\mu = 5.8702\%$; $\sigma = 20.4172\%$: calibrated from the S&P500 indexes using MLE

- Consider a unisex mortality table of individuals initially aged 65 and 70 (source: IA|BE mortality tables 2015)

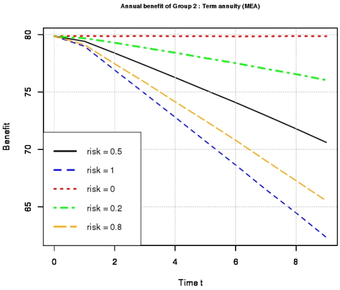
- Calibration of HW model : MSE

- Simulation approach : MC

Results : Static sharing

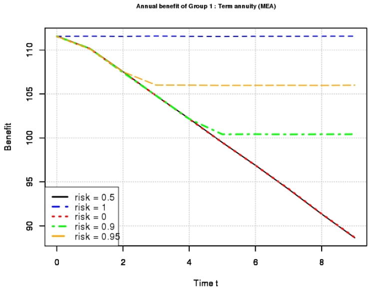


Group1

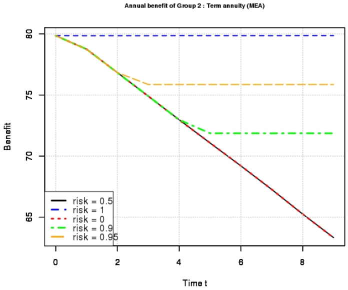


Group2

Results : Lower bound threshold sharing

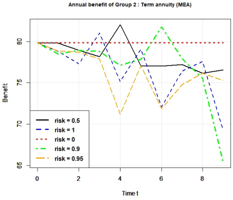


Group1

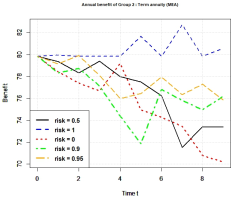


Group2

Results



Static



Threshold

	Group1		Group2	
	Static(β)	Threshold (γ)	Static(β)	Threshold (γ)
$\beta = 5\%$	110.6	105.9	79.1	75.8
$\beta = 50\%$	102.5	94.4	73.3	67.5
$\beta = 95\%$	94.9	94.4	67.8	67.5

Table 1: Annuitants' annual benefit at $t = 7$ for a $n = 9$ years term annuity; $\gamma = 1 - \beta$ and $\beta = \beta'$.

Conclusion and perspectives

A range of annuities can be built from the risk-sharing GSA : we have

(*i*) Conventional annuity if no risk borne by the pool

(*ii*) GSA or revised GSA if no risk borne by the insurer

(*iii*) Intermediate cases if the risk of the insurer is in $(0, 1)$

SO...?? Find $R(\beta'_t, x)$ and ${}_s p_{x_k}(\beta_t)$ such that $\ddot{a}_{x_k}(t, \beta_t, \beta'_t)$ describes (i), (ii) and (iii) for different values of β_t and β'_t ?

$$\ddot{a}_{x_k}(t, \beta_t, \beta'_t) = \sum_{s=0}^{\infty} \left(\frac{1}{(1 + R(\beta'_t, x))} \right)^s {}_s p_{x_k}(\beta_t)$$

Conclusion and perspectives

Question : Up to which extend could this be reasonable enough for an insurer?

Answers (Future work!):

⇒ Find the risk proportions that minimise the solvency capital (SC) of the insurer.

⇒ Compute the insurer's risk premium for a given risk proportion

What next...

- Design and price successive annuities using risk-sharing annuity.

References

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THANK YOU FOR YOUR KIND ATTENTION !