Modelling Dynamic Prepayment and Default with Survival Analysis and Machine Learning in Credit Protection Insurance

Alessia Eletti, Marco Aleandri

Sapienza Università di Roma

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Agenda

- Background
- Market dynamics and actuarial model
- Survival analysis with machine learning
- Dynamic lapse rate and TVOG
- Study case: CPI on US loans

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Background

- Richardson et al., Lapse rate.

- Outreville, Whole-life insurance lapse rates and the emergency fund hypothesis.

- Loisel et al., From deterministic to stochastic surrender risk models: impact of correlation crises on economic capital.

- Barsotti et al., Lapse risk in life insurance: correlation and contagion effects among policyholder behaviors.

- Nolte et al., Don’t lapse into temptation: a behavioral explanation for policyholder surrender.

Background

Emergency Fund Hypothesis

Lapse risk is mainly driven by a natural – and irrational to some extent – response to the need of money due to personal conditions in time of distress

Interest Rate Hypothesis

Lapse risk is mainly driven by rational reasonings (e.g., interest rate arbitrage and preference for different products) due to the policyholder‘s risk appetite

Policyholder-related factors

- Age
- Family
- Salary
- …

Product-related factors

- Duration
- Premium
- Guaranteed rate
- …

Macroeconomic factors

- Market yields
- Unemployment rate
- Gross domestic product
- …

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Market dynamics and actuarial model

$$dr_t = a(b - r_t)dt + \sigma dZ_t$$

$$v(t,T) = -\frac{1}{T-t} lnA(t,T) + \frac{1}{T-t}B(t,T)r(t)$$

$$g_t = \alpha + \beta r_t$$

- Vasicek model is chosen so as to allow for negative interest rates
- Parameters are considered as additional surrender drivers

$$I_t = (r_{t-1} + s)\frac{(1+i)^t}{(1+i)^n - 1}C_0$$

$$P = C_0 U_{\overline{x:n}|i} = C_0 \frac{IA_{\overline{x:n}|i}}{\ddot{a}_{\overline{x:n}|i}}$$

$$V_t = C_t U_{\overline{x+t:n-t}|i}$$

Note: the mathematical reserve is deterministic as the stochastic interest rate doesn’t affect the capital amortization

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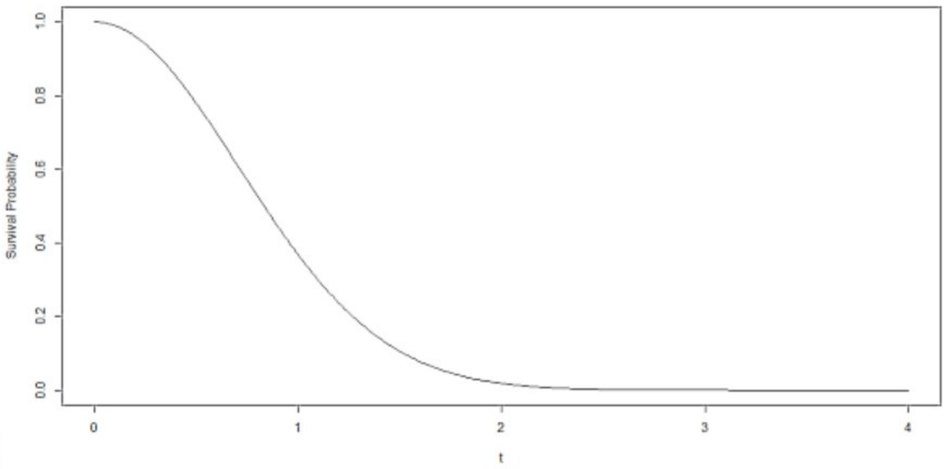
Survival analysis with Machine Learning

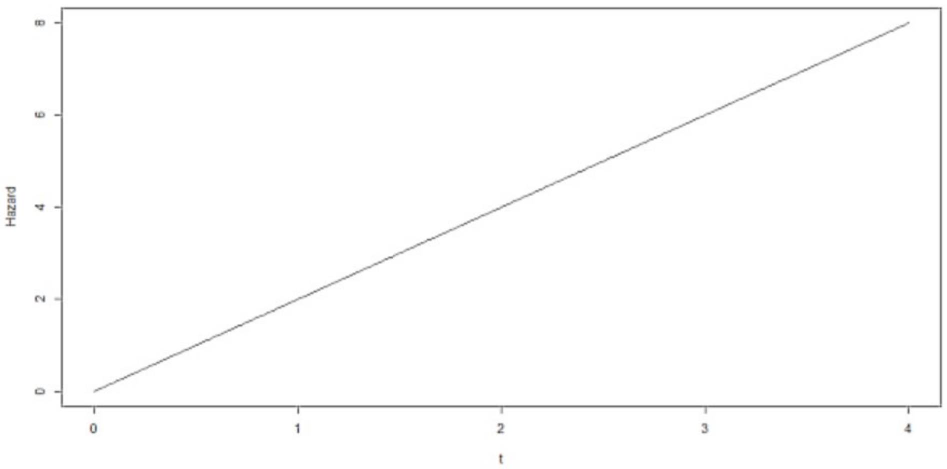
Accelerated Failure Time model

$$\log(T) = \beta'\boldsymbol{x} + \sigma\epsilon$$

$$S(t \mid \boldsymbol{x}) = S_0\big(t \cdot \exp(-\beta'\boldsymbol{x})\big)$$



$$h(t \mid \boldsymbol{x}) = h_0\big(t \cdot \exp(-\beta'\boldsymbol{x})\big)\exp(-\beta'\boldsymbol{x})$$



Survival analysis with Machine Learning

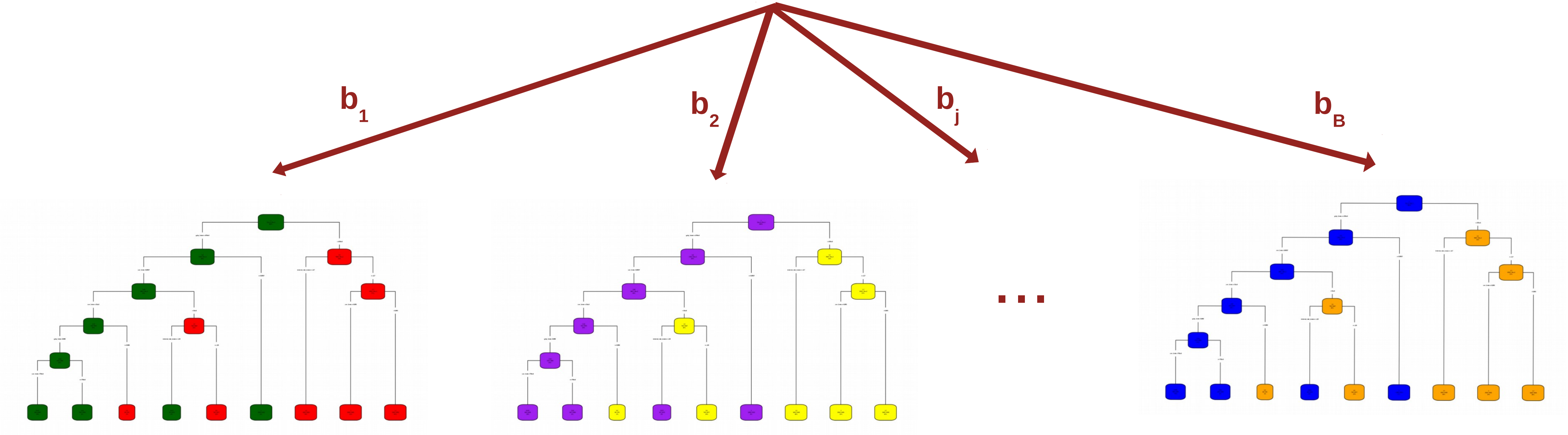
Accelerated Failure Time model

	Survival function	Hazard function
Weibull	$S(t)=\exp(-\lambda t^{p})$	$h(t)=\lambda p t^{p-1}$
Log-logistic	$S(t)=\frac{1}{1+\exp(\theta)t^{\kappa}}$	$h(t)=\frac{\exp(\theta)\kappa t^{\kappa-1}}{1+\exp(\theta)t^{\kappa}}$
Log-normal	$S(t)=1-\Phi\left(\frac{(ln x)-\mu}{\sigma}\right)$	$h(t)=\frac{\frac{1}{x\sigma}\Phi\left(\frac{(ln x)-\mu}{\sigma}\right)}{\Phi\left(-\frac{(ln x)-\mu}{\sigma}\right)}$
Gamma	$S(t)=1-\frac{\gamma(\alpha,\beta x)}{\Gamma(\alpha)}$	$h(t)=\frac{x^{\alpha-1}e^{-\beta x}}{\Gamma(\alpha)-\gamma(\alpha,\beta x)}$

Survival analysis with Machine Learning

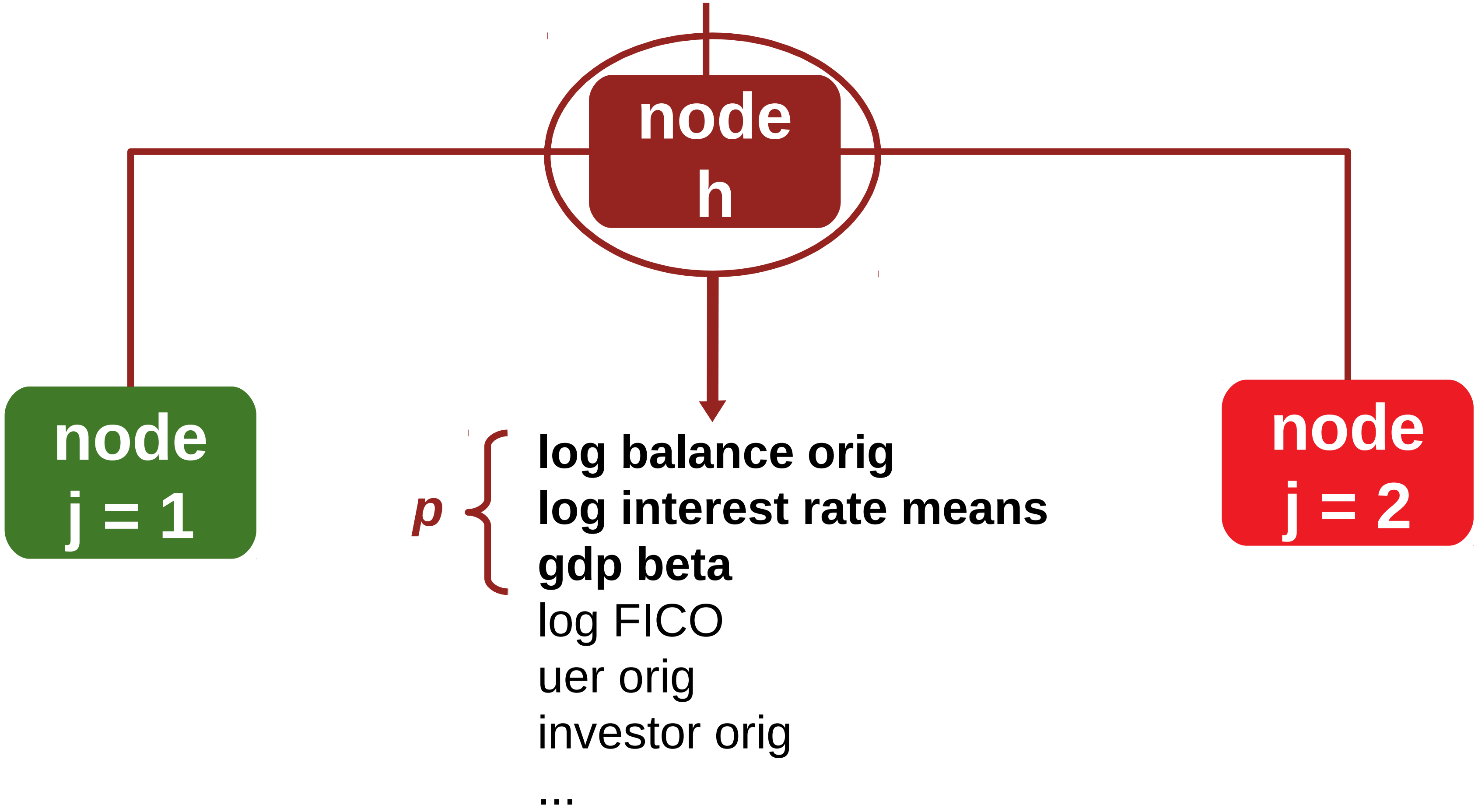
Survival Random Forest

B bootstrap samples are drawn from the original data



Survival analysis with Machine Learning

Survival Random Forest



node h

node j = 1

node j = 2

p

log balance orig

log interest rate means

gdp beta

log FICO

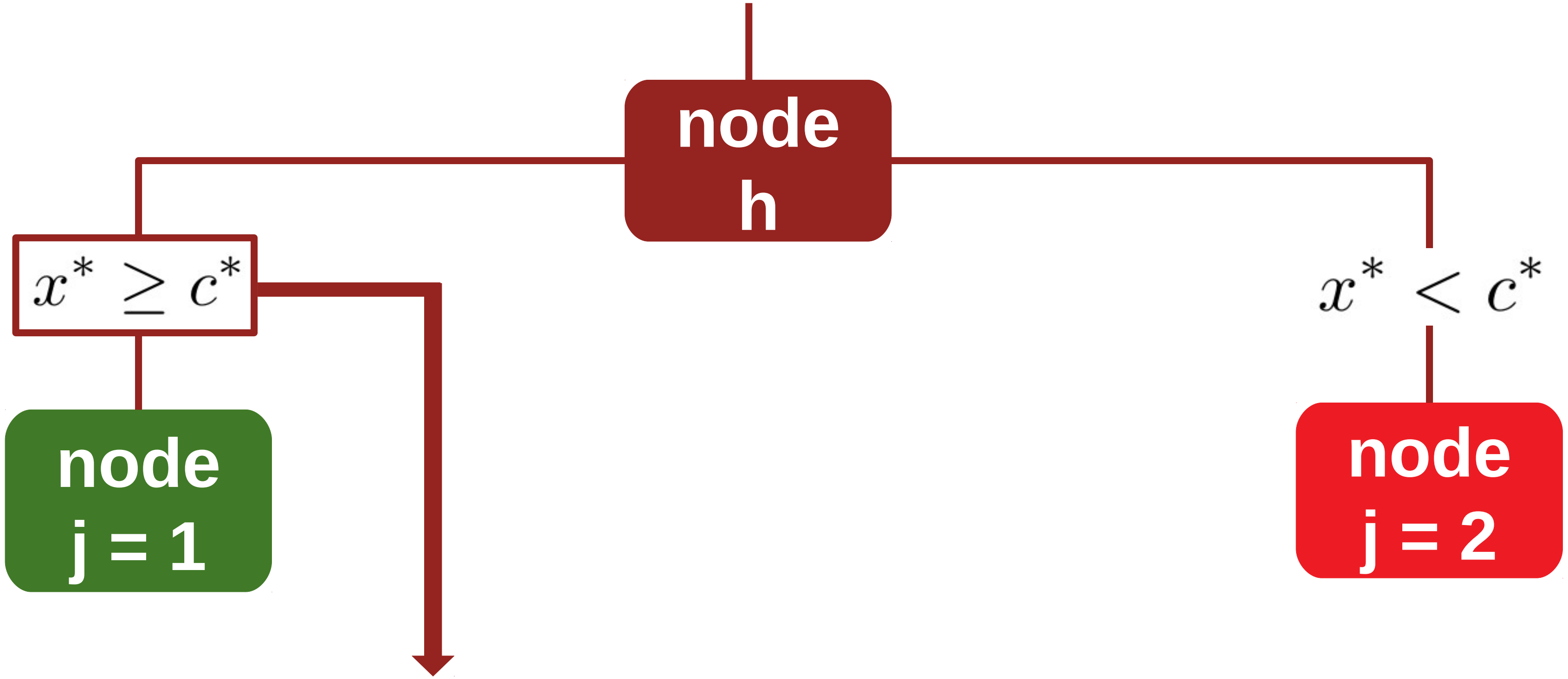
uer orig

investor orig

...

Survival analysis with Machine Learning

Survival Random Forest



log-rank test

$$|L(x^*, c^*)| \geq |L(x, c)|$$

Survival analysis with Machine Learning

Survival Random Forest

log-rank statistic

$$L(x,c) = \frac{\sum_{i=1}^{N}\left(d_{i,1} - Y_{i,1}\frac{d_i}{Y_i}\right)}{\sqrt{\sum_{i=1}^{N}\frac{Y_{i,1}}{Y_i}\left(1-\frac{Y_{i,1}}{Y_i}\right)\left(\frac{Y_i-d_i}{Y_i-1}\right)d_i}}$$

ordered unique death times

$$t_1 < t_2 < \ldots < t_N$$

number of deaths at (j, t_i)

$$\mathrm{d}_{i,j}$$

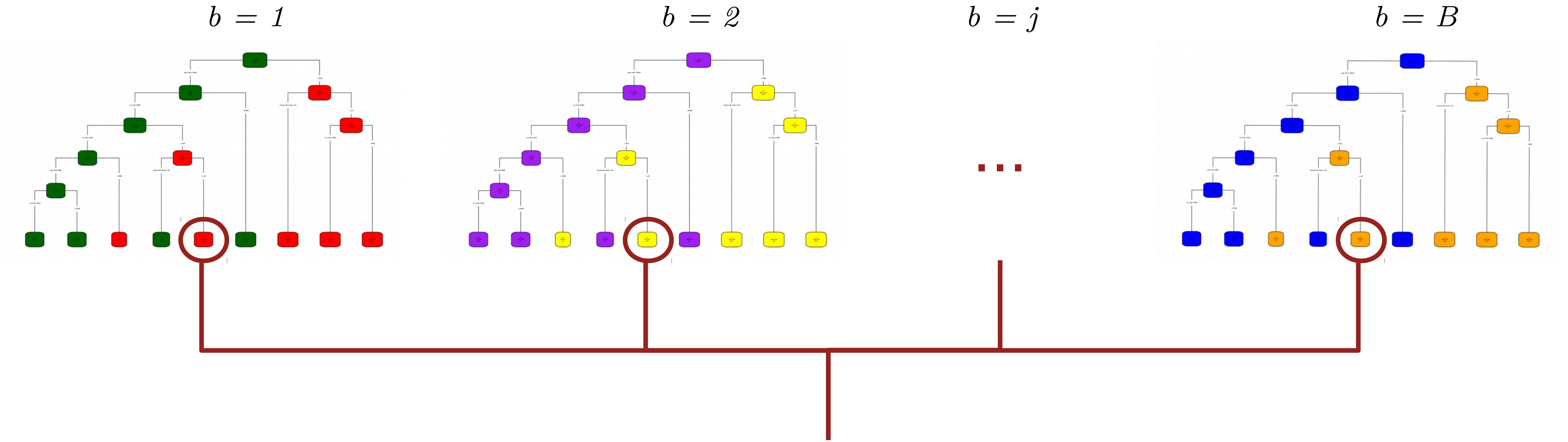
number of individuals at (j, t_i)

$$\mathrm{Y}_{i,j}$$

$$\hat{\Lambda}_j(t) = \sum_{t_{i,j}<t}\frac{d_{i,j}}{Y_{i,j}}$$

Survival analysis with Machine Learning

Survival Random Forest



$$\widehat{\Lambda}_j^{(E)}(t) = \frac{1}{B}\sum_{b=1}^{B}\widehat{\Lambda}_j^{(b)}(t)$$

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Dynamic lapse rate and TVOG

Surrender probability

$$\widehat{Q} \equiv \frac{\widehat{S}(t) - \widehat{S}(t+1)}{\widehat{S}(t)}$$

Lapse rate

$$\widehat{l}_r := 1 - \left[1 - \widehat{Q}_P(r)\left(1 - \frac{\widehat{Q}_D(r)}{2}\right)\right]\left[1 - \widehat{Q}_D(r)\left(1 - \frac{\widehat{Q}_P(r)}{2}\right)\right]$$

Certainty equivalent profit

$$D_{ce} = f(i, q_x; r_{ce}, C_0, s, \beta)$$

Stochastic profit

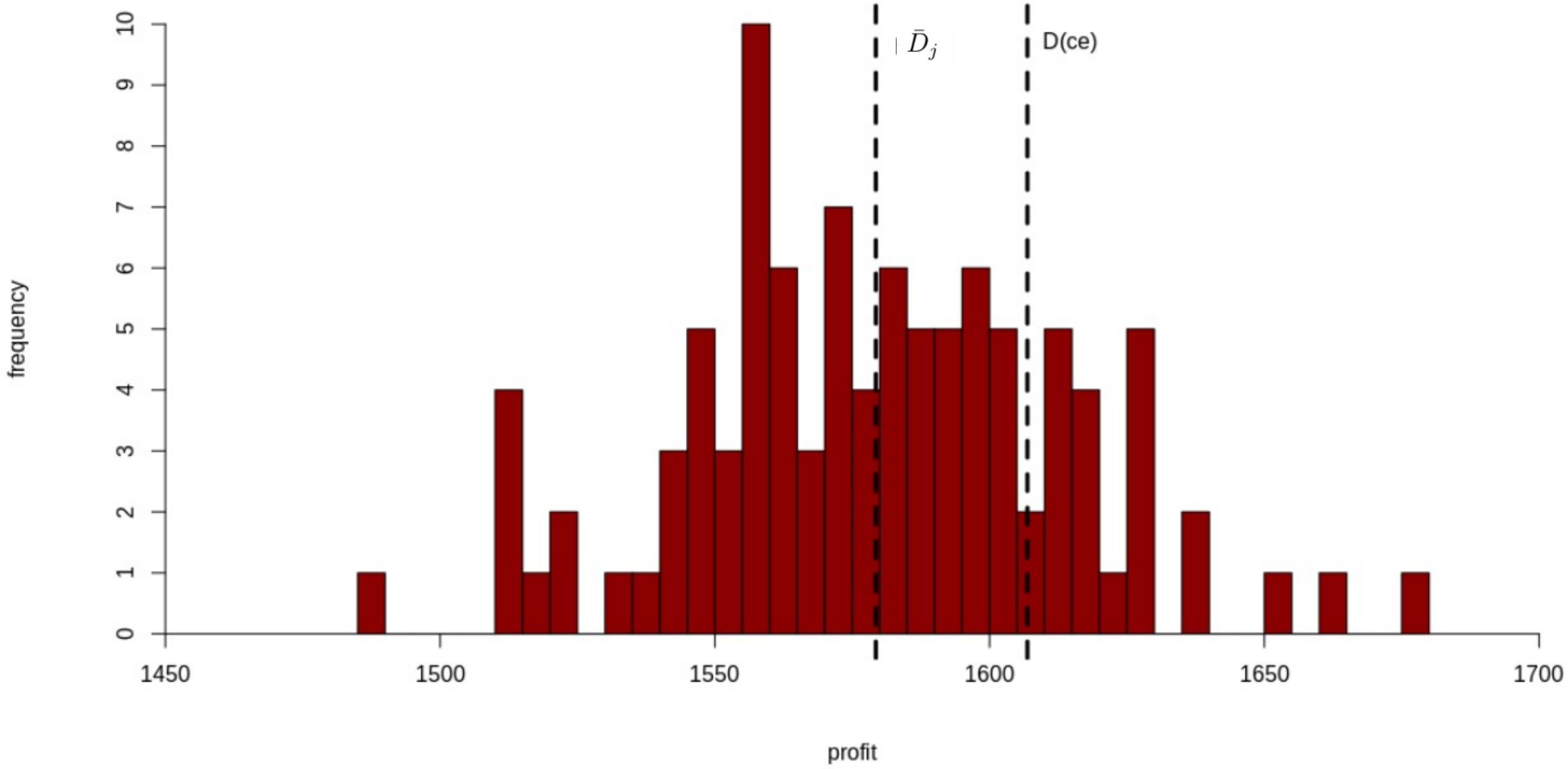
$$D_j = f(i, q_x; r_j, C_0, s, \beta)$$

Average stochastic profit

$$\overline{D}_j = \frac{1}{N}\sum_{j=1}^{N} D_j$$

Dynamic lapse rate and TVOG

$$D_{ce} > \bar{D}_j$$



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Study case: CPI on US loans

Dataset and data pre-processing

- Containes origination and performance information on 50.000 borrowers synthesised in cross-section form
- Left and right censored data with maximum loan observation time of 60 trimesters, i.e. 15 years
- Features
  - origination balance (logarithm)
  - interest rate origination value, mean and variance (logarithm)
  - unemplyment rate and gdp origination value
  - unemplyment rate and gdp period-wise indicator
  - logarithm of FICO credit score
  - real estate and investor type

Study case: CPI on US loans

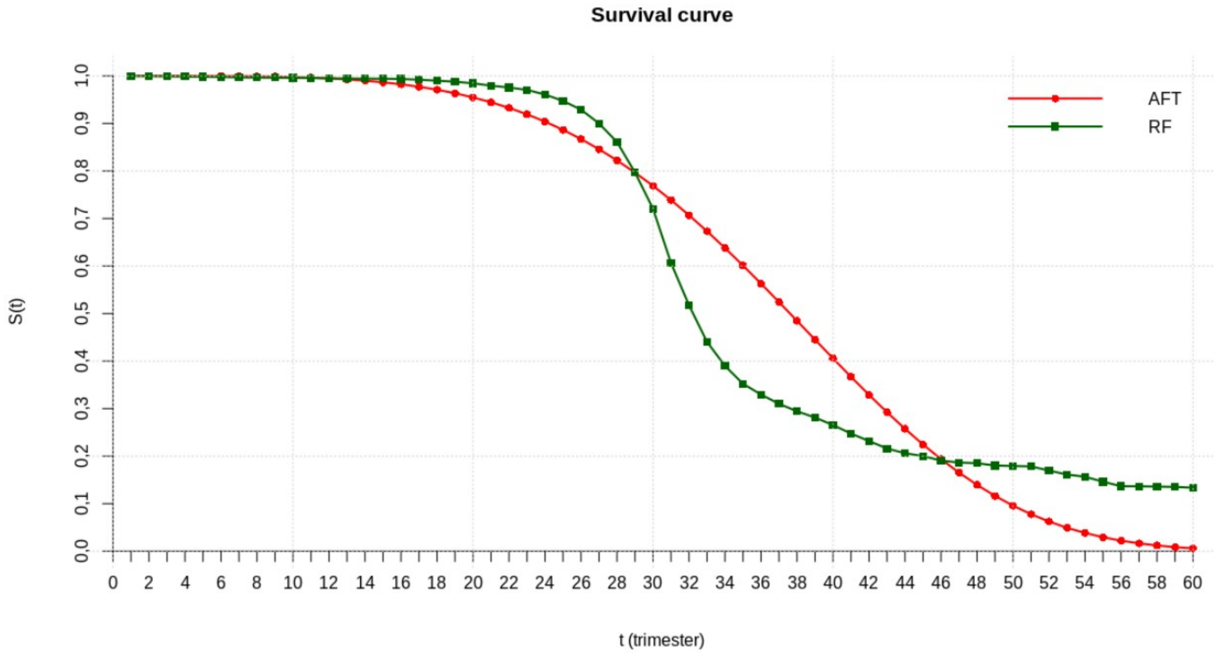
Parametric vs non-parametric model

- Model 1: Accelerated Failure Time model with Weibull distribution.

- Model 2: Random Survival Forest.

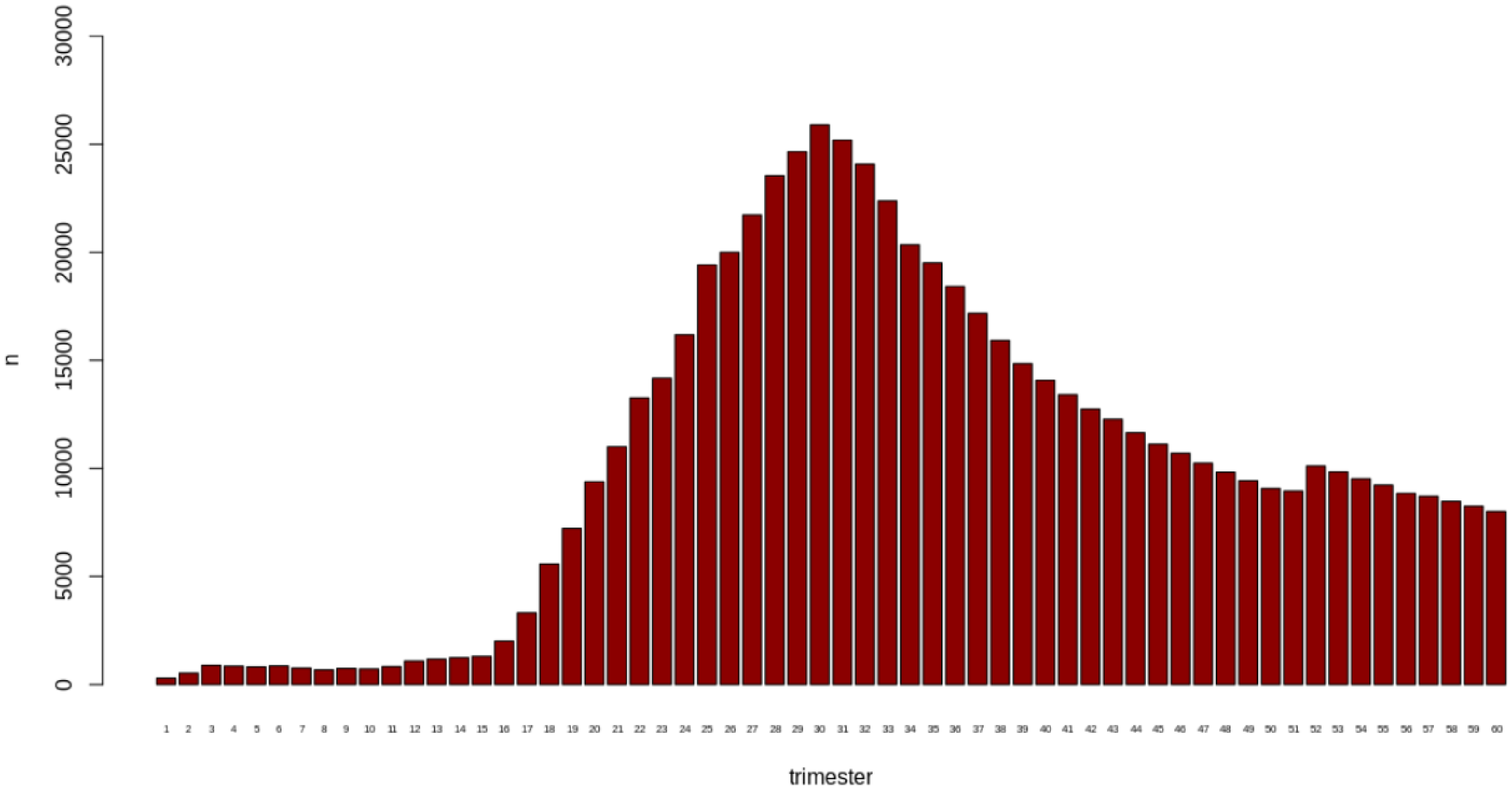
- Logistic regression.

- When evaluating the two models, censored data is dealt with by computing an AUC for each time *t.*



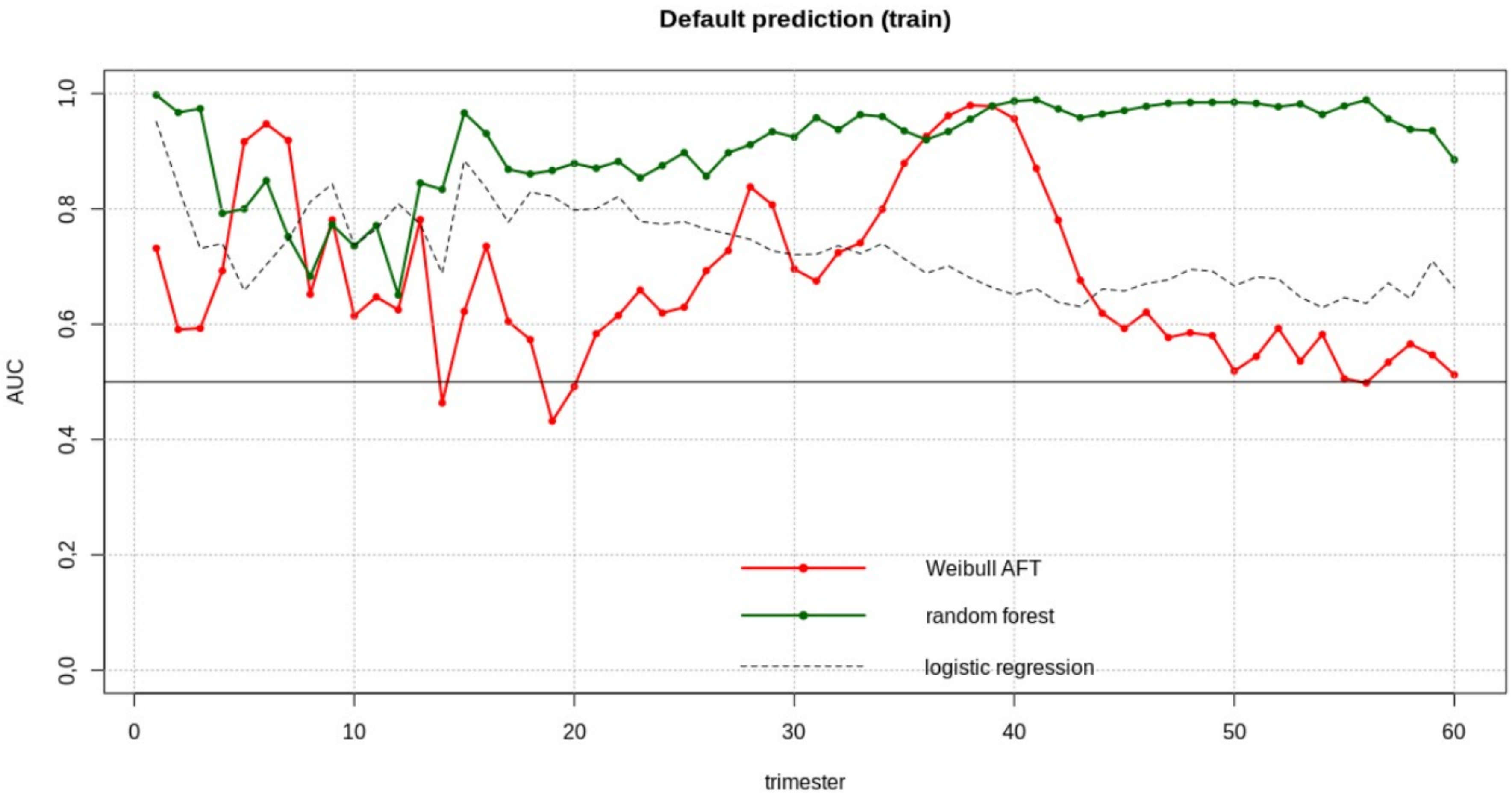
Study case: CPI on US loans

Parametric vs non-parametric model



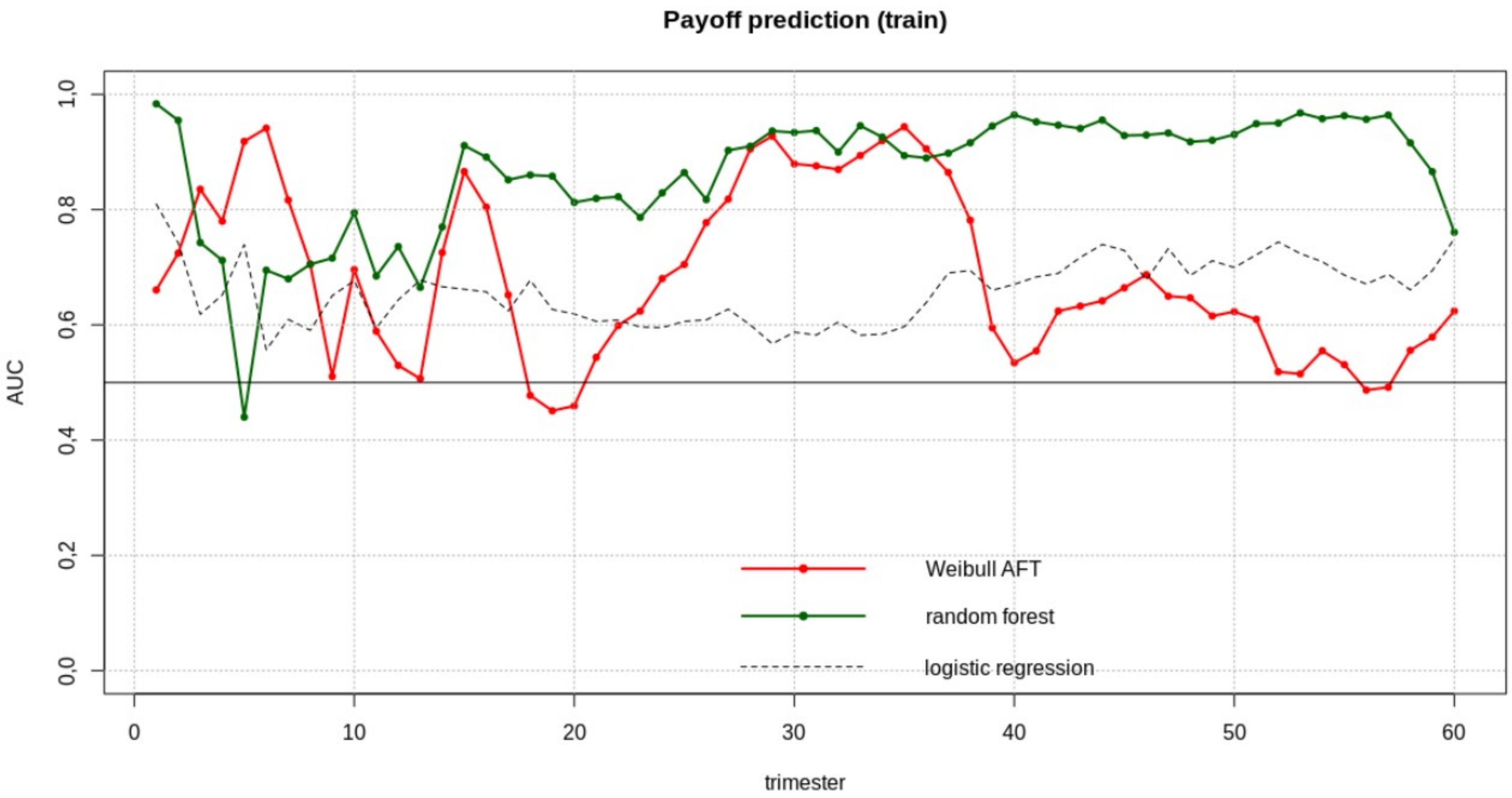
Study case: CPI on US loans

Parametric vs non-parametric model



Study case: CPI on US loans

Parametric vs non-parametric model



Study case: CPI on US loans

Parametric vs non-parametric model

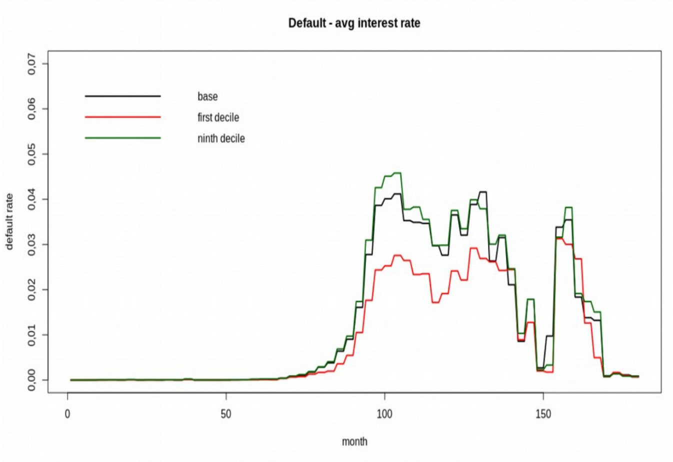
$$AUC = \frac{\sum_{t=1}^{N} AUC_t n_t}{\sum_{t=1}^{N} n_t}$$

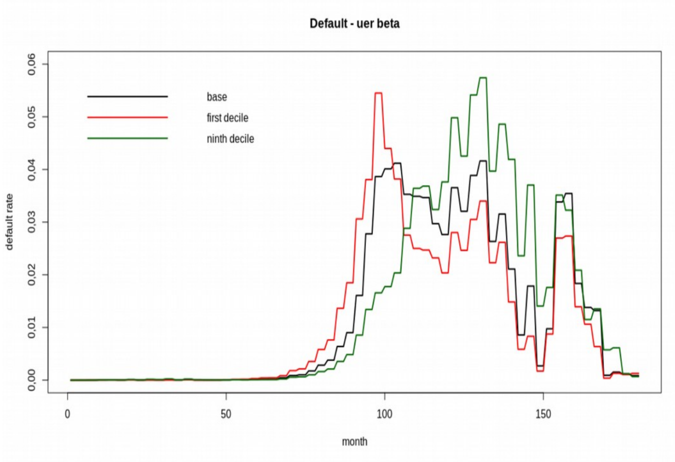
Default	Train	Validation
AFT model	71%	72%
Random Forest	**94%**	**91%**
Logistic regression	71%	71%

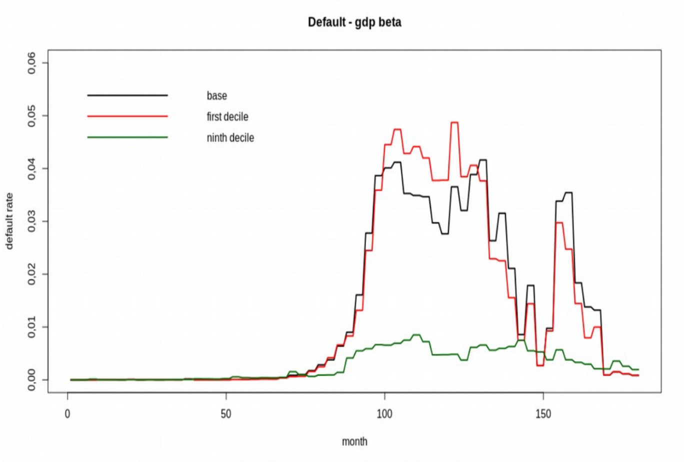
Prepayment	Train	Validation
AFT model	73%	73%
Random Forest	**91%**	**89%**
Logistic regression	64%	64%

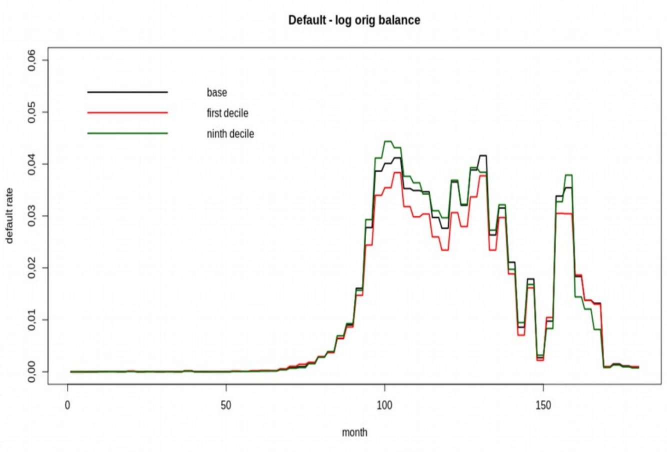
Study case: CPI on US loans

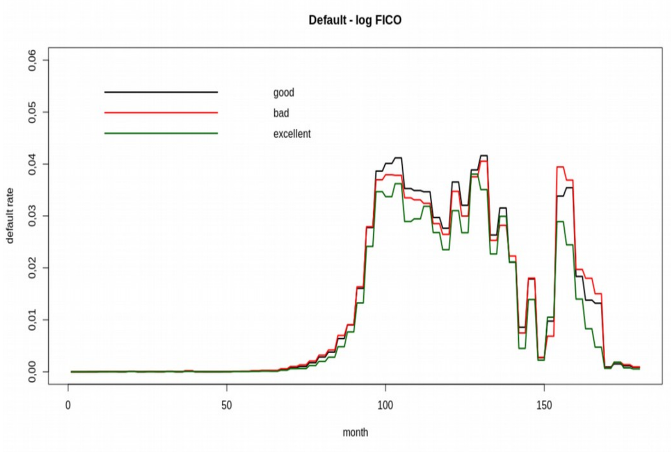
Default and prepayment estimation

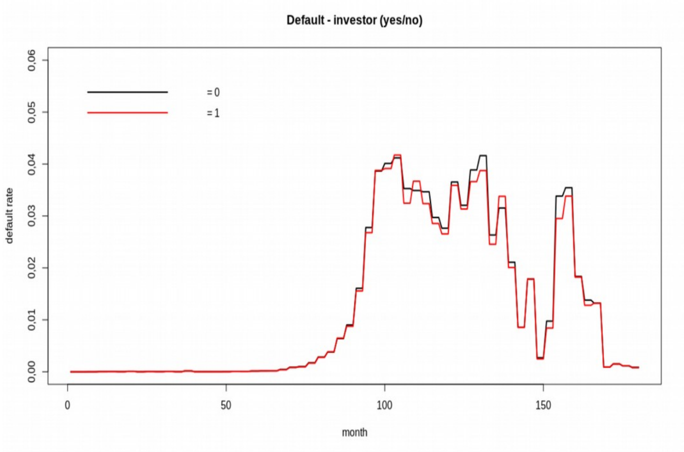

Default - avg interest rate


Default - uer beta


Default - gdp beta

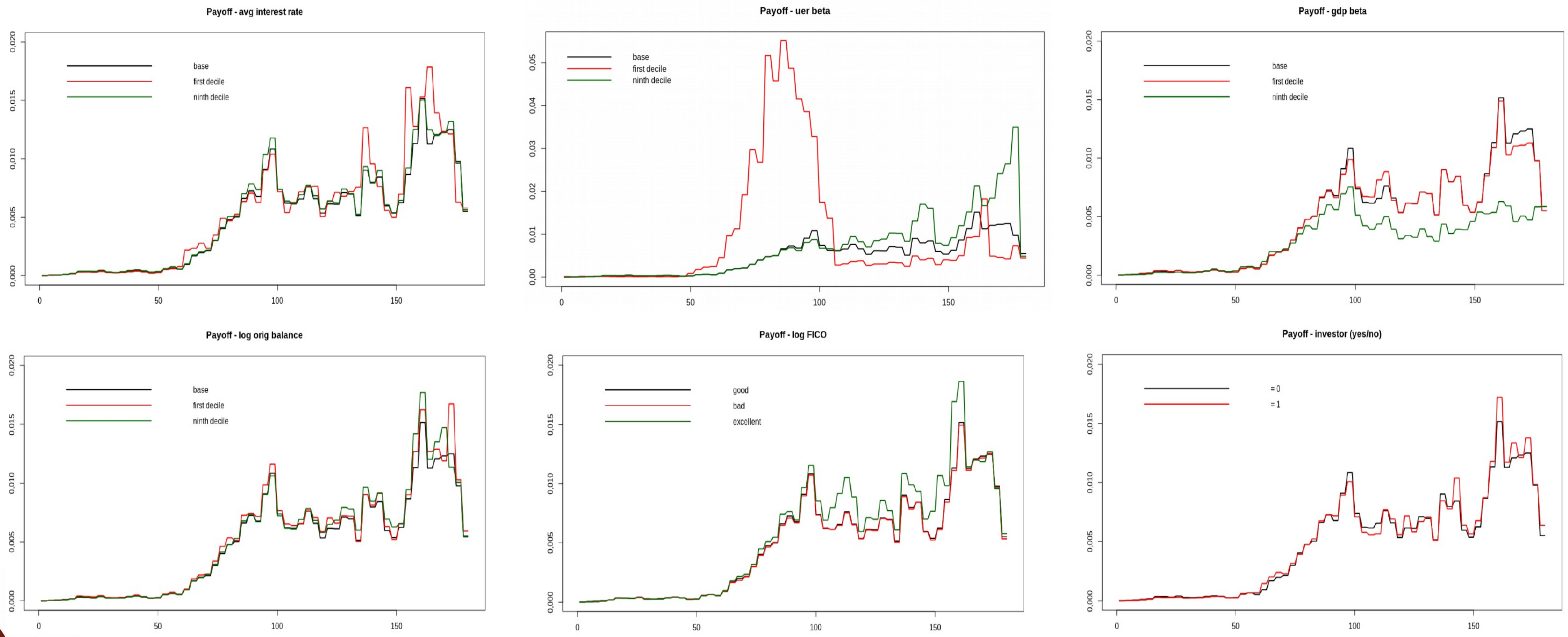

Default - log orig balance


Default - log FICO


Default - investor (yes/no)

Study case: CPI on US loans

Default and prepayment estimation



Payoff - avg interest rate

Payoff - uer beta

Payoff - gdp beta

Payoff - log orig balance

Payoff - log FICO

Payoff - investor (yes/no)

Study case: CPI on US loans

TVOG analysis

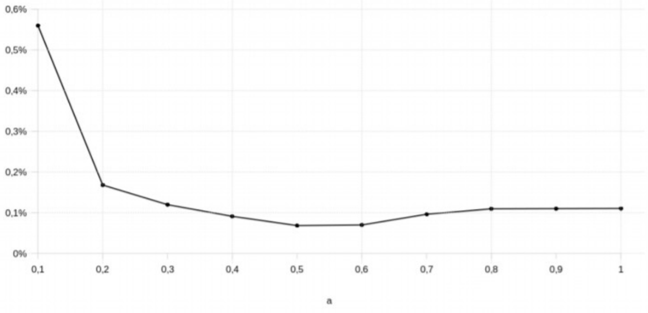


Fig. 32: TVOG shape by a.

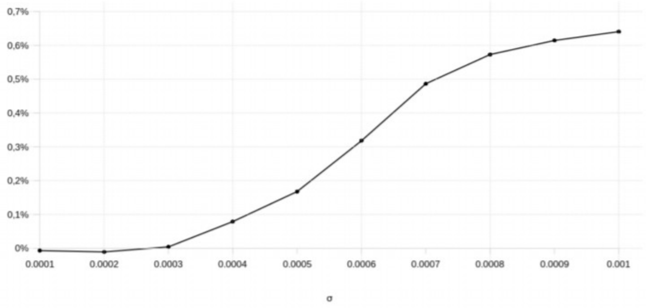


Fig. 33: TVOG shape by σ.

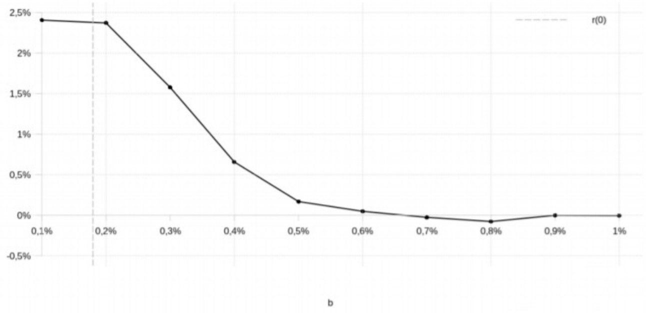


Fig. 34: TVOG shape by b.

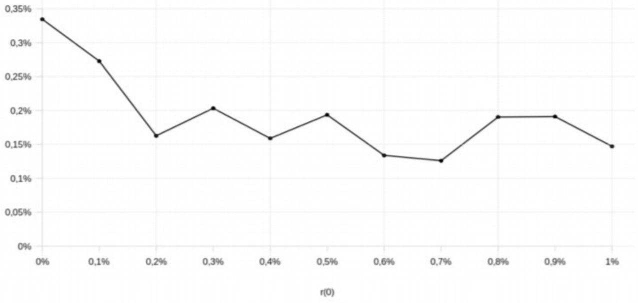


Fig. 35: TVOG shape by $r(0)$.

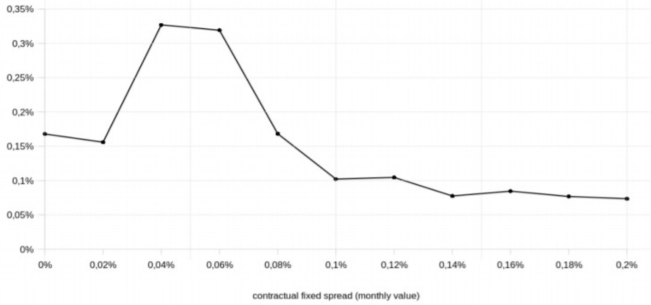


Fig. 36: TVOG shape by s.

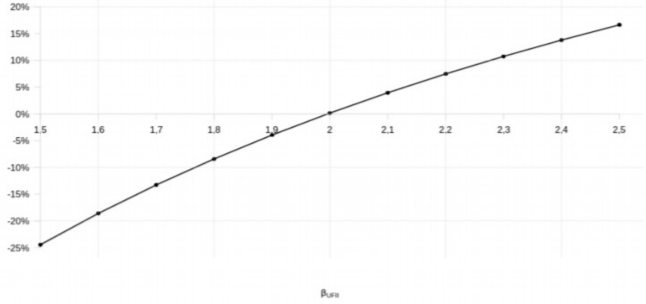


Fig. 37: TVOG shape by β.

Study case: CPI on US loans

Limitations, extensions and conclusion

- Random forest **outperforms** the AFT model (*through-the-cycle model*). This is due to the fact that RF relies on a higher number of degrees of freedom.
- Random forest also **outperforms** the logistic regression (*point-in-time model*). Indeed, in the latter stepwise selection of features at each time leads to further loss of information.
- The **significantly better performance** of the random forest comes at the cost of **very slight overfitting**. This is typical as ensemble learners tend to fit both linear and more complex non linear relations while keeping low generalization error.
- TVOG tends to vary between 0% and 6% for all parametrizations seen, except for values of β higher than the baseline value. Negative interest rates and the dual nature of default, in fact, can lead to negative TVOG.

Study case: CPI on US loans

Limitations, extensions and conclusion

- **Cox model**: the main goal of this study was to confront traditional parametric model and semi-parametric random forest but we could have chosen Cox proportional hazard model (which is also a common survival model) as the counterpart in the confrontation with RF.

- **Competing risks model**: our model doesn’t take into consideration the correlation between default and prepayment. This is partly justified by the fact that these options tend to be the consequence of opposite situations for the policyholder. CRM account for their interaction and thus are a possible extension of our model.

- **Mixture cure modelling**: in credit-risk modelling a large part of the population never defaults/prepays, hence the survival probability will never actually be zero but will level at some value.

Q&A

Any questions?