

# A Double-Sigmoid approach for dynamic policyholder behavior

Fabio Baione, Davide Biancalana, Paolo De Angelis & Ivan Granito

*23 May 2019*



# Key points of the work

- The policyholders' behavior is a determining factor for the valuation of technical provisions for a life insurance company under the Solvency 2 framework.
- In particular, the paper considers the problem of estimating the lapse rates in a portfolio of with profit policies when conditions in the financial market change over time.

## *Some preliminary considerations*

### THE AIM OF THE WORK

1. To choose a specific regression model to describe the **Dynamic Policyholder Behavior (DPHB)**.
2. To analyze two different actuarial approaches for dynamic lapses rate compared with a non-dynamic lapses rate, based on the experience of an Italian life insurance company.
3. To investigate the impact on **Technical Provisions** and to value the cost of the surrender option (**SoC**) due to the different approaches.

### THE STEPS OF THE WORK

1. An **investigation** of the academic and professional literature.
2. An **implementation of the two models** investigated for the analysis in order to understand their economic and actuarial sense.
3. An **assessment of the BEL** of an homogeneous portfolio of participating life policies with guarantees by considering the effect of a **Dynamic Policyholder Behavior** model.

# Scientific and Professional literature [Bauer et al. 2017]

## Theoretical framework

- ✓ Insurance **market completeness and frictionless** as drivers of a **“full” rational policyholder behavior** seeking to **maximize the value of the contract**. [Bacinello et al. 2011]
- ✓ **Market frictions and the asymmetric positions** of the insurer and the insured produce a dissonance with which two agents value policy cash flows. [Moenig and Zhu 2016]
- ✓ **Incompleteness of the insurance market** and the impact of preferences and idiosyncratic risks. [Fei et al. 2015]
- ✓ **Asymmetric Information, adverse selection and moral hazard** as the basis for pricing formulas in the context of a life-cycle model. [Zhu and Bauer 2011, 2013]

## Empirical evidence

Three main hypothesis are used to explain the policyholder behavior. [Eling and Kochanski (2013)].

- ✓ **Interest rate hypothesis**: policyholder lapses in response to change in interest rates.
- ✓ **Policy replacement hypothesis**: policyholder as an arbitrageur lapses existing contract to purchase a new more profitable contract.
- ✓ **Emergency fund hypothesis**: policyholder lapses to meet unexpected funding needs.

# Regulatory Framework

## Regulation (Solvency II Delegated Acts)

### **Article 26 – Policyholder behavior**

When determining the likelihood that policyholders will exercise contractual options, including lapses and surrenders, insurance and reinsurance undertakings shall conduct an analysis of past policyholder behaviour and a prospective assessment of expected policyholder behaviour.

That analysis shall take into account all of the following

- a) how beneficial the exercise of the options was and will be to the policyholders under circumstances at the time of exercising the option;
- b) the influence of past and future economic conditions;
- c) the impact of past and future management actions;
- d) any other circumstances that are likely to influence decisions by policyholders on whether to exercise the option.

The likelihood shall only be considered to be independent of the elements referred to in points (a) to (d) where there is empirical evidence to support such an assumption

**Article 31** – allow for "dependency between two or more causes of uncertainty"

**Article 32** – take into account "all factors" which may affect the likelihood that policyholders will exercise contractual options or realize the value of financial guarantees.'

## Ivass Letter 05/06/2018

Assumptions (Surrenders) underlying the calculation of technical reserves:

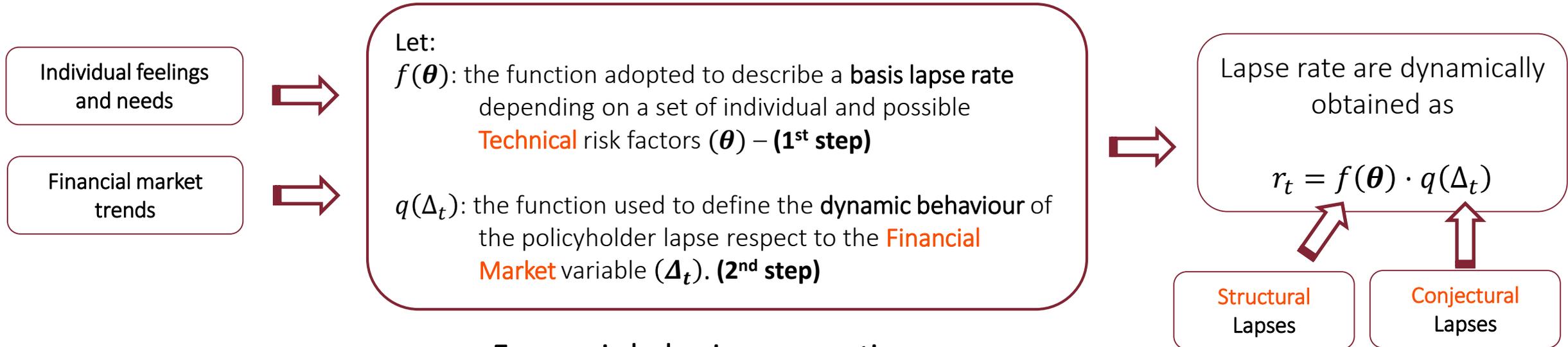
- Identify of a number of appropriately balanced clusters in order to generate assumptions consistent with the characteristics of each homogeneous risk class of contracts and at the same time allow reliable statistical analyzes.
- Calibrations on time series of robust and historically adequate data containing information on the surrender capitals or the number of surrenders.
- Ex-post verification through appropriate consistency analysis (e.g. backtesting)

Concerning the (Dynamic) Policyholder Behavior:

- evaluate the degree of awareness of the policyholders with respect to the value of the contractual options and the possible relationships with the variables that describe the performance of the financial markets.
- Use correlation analysis between the policyholders behavior and the spread between the credited returns to the policy and the returns obtained from alternative investments.
- Cluster the portfolio in a way that is consistent with the different "detachable" policyholders' behavior, also due to the different types of contracts.

# A Two-Stage Dynamic Policyholder Behavior model: Basic assumptions

“The common structure for these models is that the surrender rate is divided into two main parts consisting of a base rate reflecting irrational behavior and a rate that depends on some economic factors reflecting rational behavior” (QIS3)



## Economic behavior assumptions:

- $\Delta_t = MR_t - CR_t \Rightarrow$  **Market Spread as the difference between the yield of a financial market benchmark and the policy crediting rate**
- $\Delta_t \cong 0 \Rightarrow$  the lapse rate depends only on subjective variables (e.g. age, sex, income, wealth,..., other individual risk factors);
- $\Delta_t \rightarrow -\infty \Rightarrow$  the **negative** difference between the market and the policy crediting rate is high enough to stop an ever-increasing number of policyholders from withdraw. The **lapse rate decreases** and tends to a lower asymptote.
- $\Delta_t \rightarrow +\infty \Rightarrow$  market performs better than the insurance policy to convince an increasing number of policyholders to withdraw even in the presence of other subjective decision-making factors. The **lapse rate increases** and tends to an upper asymptote.

# Surrender models depending on economic variables: a survey

## Technical Specification for the Preparatory Phase -TP.6.44 and QIS 5 - TP.7.44

- Arctangent:  $q(\Delta_t) = a + b \cdot \arctan(m \cdot \Delta_t - n)$
- Parabolic:  $q(\Delta_t) = a + b \cdot \text{sign}(\Delta_t) \cdot \Delta_t^2$
- Modified Parabolic:  $q(\Delta_t, CR_t) = a + b \cdot |\Delta_t| \cdot k + c^{(CR_{t-1} - CR_t)^j}$
- NY State Law 126:  $q(\Delta_t, FV_t, CSV_t) = a + b \cdot \text{sign}(\Delta_t) \cdot \Delta_t \cdot k - c \left( \frac{FV_t - CSV_t}{FV_t} \right)$
- Exponential:  $q(CR_t, MR_t) = a + b \cdot e^{(m \cdot CR_t / MR_t)}$
- Lemay's:  $q(FV_t, GV_t) = a + \alpha + b \cdot \frac{FV_t}{GV_t}$

where

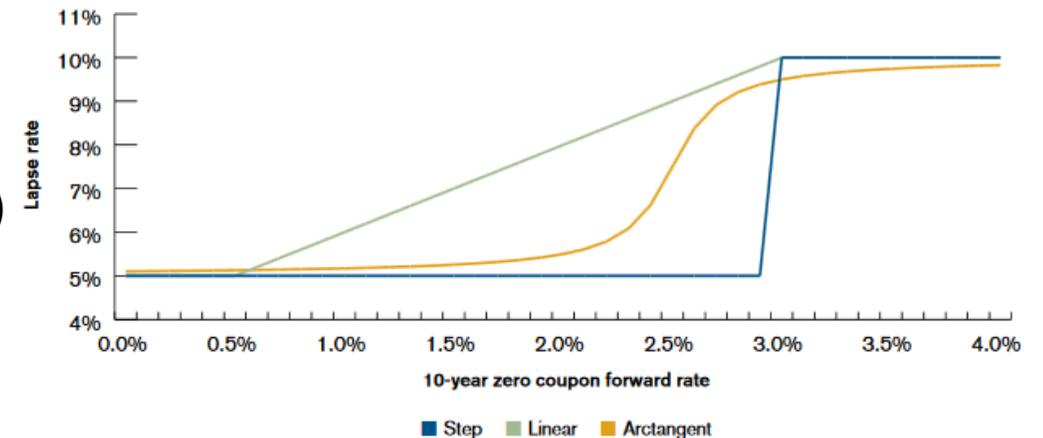
- $a, b, c, m, n, j, k$  are coefficients
- $\alpha$  denotes underlying (possible time dependent) base lapse rate,
- FV denotes the fund (account) value of the policy,
- GV denotes the guaranteed value of the policy,
- $\Delta$  equals reference market rate less crediting rate less surrender charges,
- CR denotes the crediting rate,
- MR denotes the reference market rate,
- CSV denotes the cash surrender value and
- $\text{sign}()$  equals 1 if ( ) is positive and -1 if ( ) is negative.

- **Logistic function** (C. Kim 2005)

$$q(\Delta_t) = \frac{e^{\beta_0 + \beta_1 \cdot \Delta_t}}{1 + e^{\beta_0 + \beta_1 \cdot \Delta_t}}$$

- Step Rate Increase or (Bounded) **Linear Increase** (Milliman 2013)

$$q(\Delta_t) = \min \left( r_{max}; \max \left( \frac{\Delta_t - \beta_2}{\beta_1 - \beta_2} \cdot r_{min}; r_{min} \right) \right)$$



# Dynamic policyholder behavior model: S-Shaped curve

- One of the most popular function used to describe the dynamic policyholder behavior is based on **S-Shaped curve** (e.g. arctangent, logit etc.).

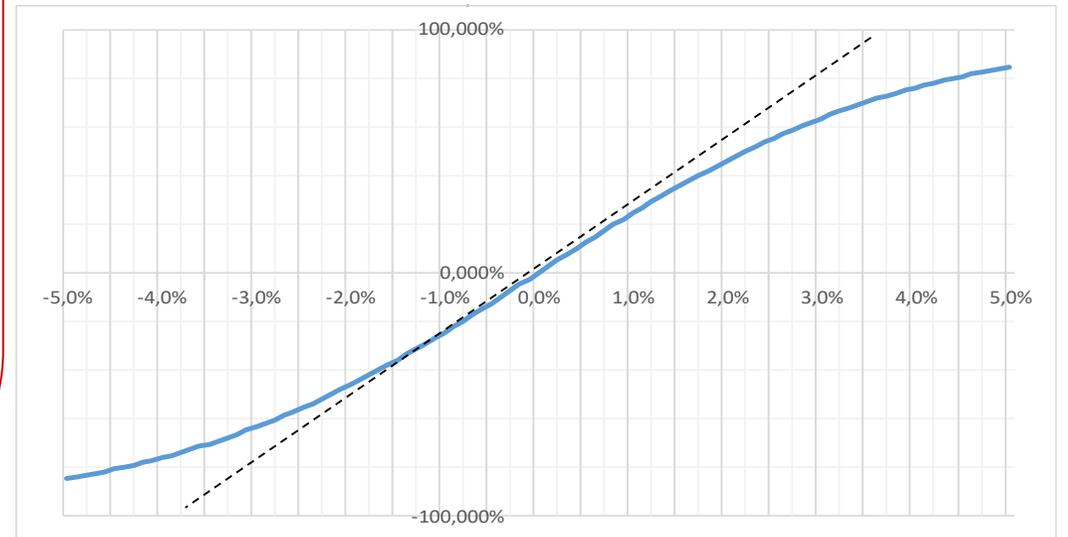
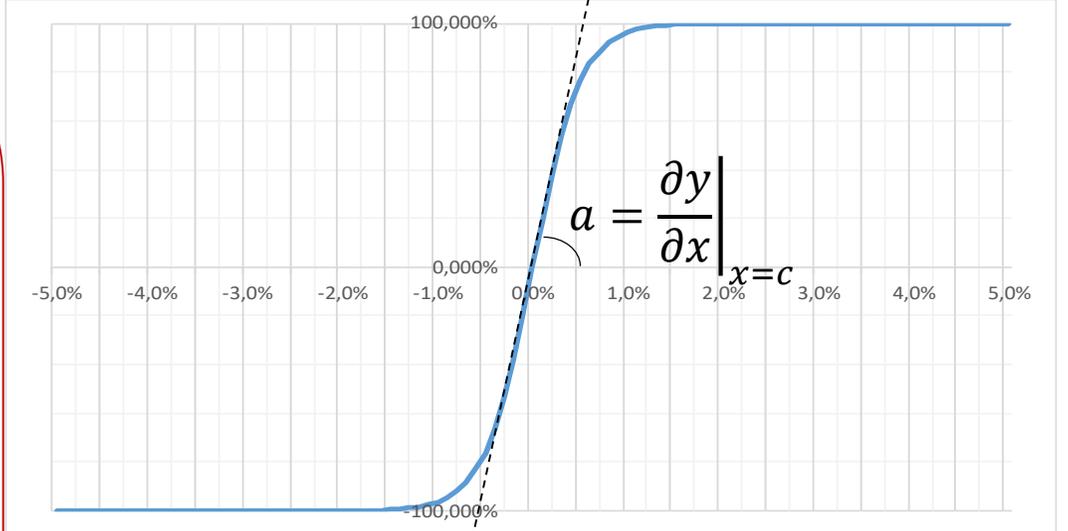
$$q(\Delta_t) = q_{min} + \frac{q_{max} - q_{min}}{1 + \exp(-a \cdot (x - c))}$$

Where

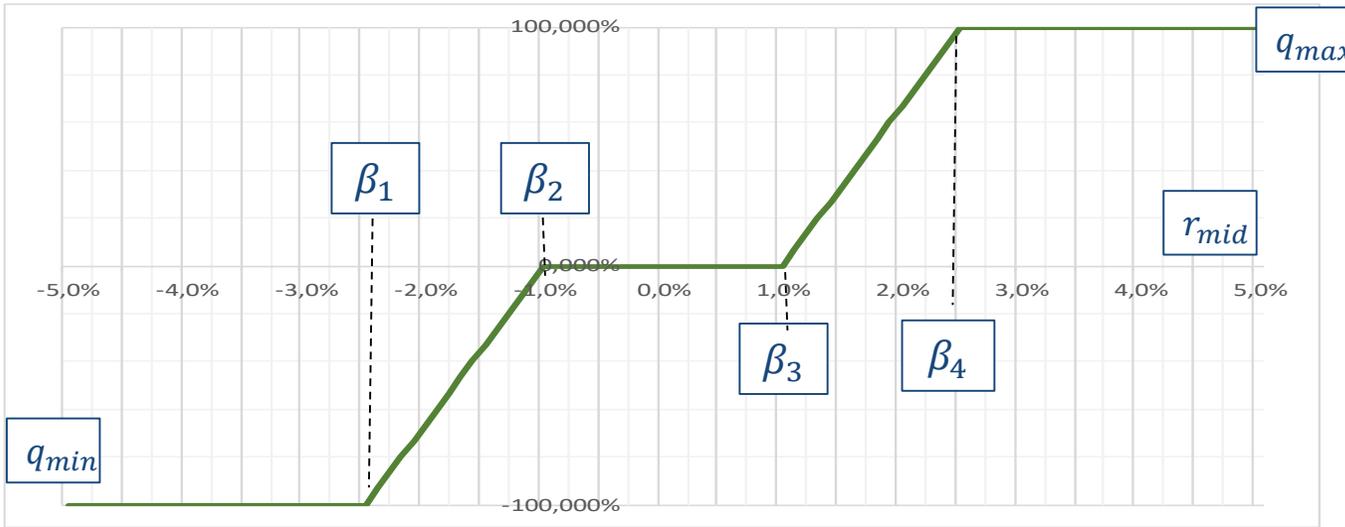
- $a > 0$  is the steepness of the curve
- $c$  is the middle point i.e.  $q(c) = \frac{q_{max} + q_{min}}{2} = q_{mid}$

However a S-Shaped curve - depending on the **steepness** - could be too sensitive to “market spread” values around zero that is the surrender rate too rapidly increases/decreases respect to small values of the economic variable.

As an alternative it is possible to use a function that for **small differences** between market rate and policy crediting rate defines **lapse rates close to the base level**. Moreover, as this “market spread” value increases (decreases) the surrender rate should increase (decrease) more or less rapidly.



# Dynamic policyholder behavior model: Double Linear curve



Common market approach for with-profit business  
 Autorité de Contrôle Prudentiel (2013) - Institute and Faculty of Actuaries  
 Seminar 2016

$$q(\Delta_t) = \begin{cases} \max\left(\frac{\Delta_t - \beta_2}{\beta_1 - \beta_2} \cdot q_{min}; q_{min}\right) & \Delta_t < \beta_2 \\ q_{mid} & \beta_2 \leq \Delta_t < \beta_3 \\ \min\left(\frac{\Delta_t - \beta_3}{\beta_4 - \beta_3} \cdot q_{max}; q_{max}\right) & \Delta_t \geq \beta_3 \end{cases}$$

## Double Linear (or Double Step)

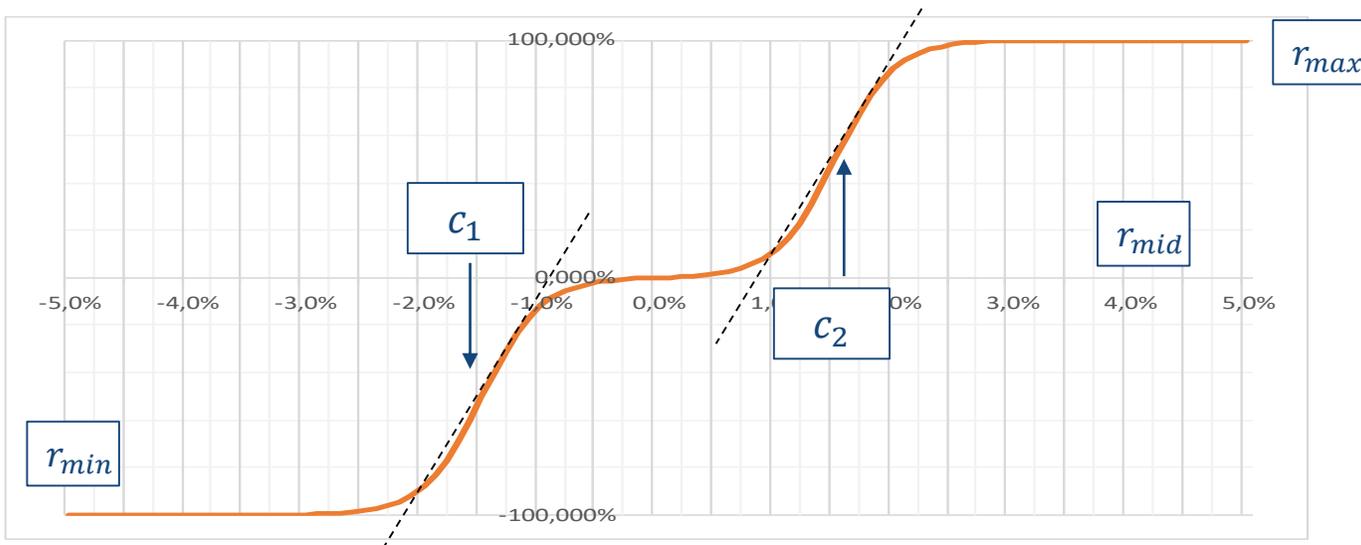
### Pros ✓

- Continuous
- Easy to implement
- Parameters are easily understandable
- It can be easily reduced to a Linear shape

### Cons ⚠

- Not differentiable
- Not smoothed
- Parameter estimates are based on MSE or MLE for non differentiable functions.
- Parameter estimates need Expert Judgement.

# Dynamic policyholder behavior model: Double Sigmoid curve



## Double Sigmoid (or Double Logit)

### Pros ✓

- Continuous
- Differentiable
- Smoothed
- Easy to implement
- It can be easily reduced to a S-Shaped
- High flexible

### Cons ⚠

- Parameter are not immediately understandable
- Parameter estimates are based on MSE or MLE for non differentiable functions.
- Parameter estimates needs Expert Judgement.

### Additive model

$$q(\Delta_t) = q_{min} + \frac{q_{mid} - q_{min}}{1 + \exp(-a_1 \cdot (\Delta_t - c_1))} + \frac{q_{max} - q_{mid}}{1 + \exp(-a_2 \cdot (\Delta_t - c_2))}$$

### Multiplicative model

$$q(\Delta_t) = \frac{1}{q_{mid}} \left( q_{min} + \frac{q_{mid} - q_{min}}{1 + \exp(-a_1 \cdot (\Delta_t - c_1))} \right) \cdot \left( q_{mid} + \frac{q_{max} - q_{mid}}{1 + \exp(-a_2 \cdot (\Delta_t - c_2))} \right)$$

# A 2-step dynamic policyholder behavior model

Remark:  $f(x) = \frac{1}{1+\exp(-a \cdot (x-c))} = \frac{1}{2} \cdot \left[ 1 + \tanh\left(\frac{x-c}{w}\right) \right]$ , with  $w = 2/a$

A multiplicative model based on a Logit GLM (1<sup>st</sup> step) for **basis lapse rate** estimate and a double sigmoid function (2<sup>nd</sup> step) for **DPHB** we can formalize the lapse rate at time  $t$  as:

$$r_t = f(\theta) \cdot \left[ \alpha + \beta \cdot \left[ \tanh\left(\frac{\Delta_t - c_1}{w_1}\right) \cdot \delta + \tanh\left(\frac{\Delta_t - c_2}{w_2}\right) \cdot \gamma \right] \right], \quad w_1, w_2 > 0 \quad (2)$$

Assuming a  $r_{max} > r_{min}$  in  $(0,1)$ ,  $\alpha$  and  $\beta$  are calculated by solving the following linear system:

$$\begin{cases} \lim_{\Delta_t \rightarrow +\infty} \left[ \alpha + \beta \cdot \left[ \tanh\left(\frac{\Delta_t - c_1}{w_1}\right) \cdot \delta + \tanh\left(\frac{\Delta_t - c_2}{w_2}\right) \cdot \gamma \right] \right] = q_{max} \\ \lim_{\Delta_t \rightarrow -\infty} \left[ \alpha + \beta \cdot \left[ \tanh\left(\frac{\Delta_t - c_1}{w_1}\right) \cdot \delta + \tanh\left(\frac{\Delta_t - c_2}{w_2}\right) \cdot \gamma \right] \right] = q_{min} \end{cases}$$

Where  $q_{max} = \frac{r_{max}}{f(\theta)}$  and  $q_{min} = \frac{r_{min}}{f(\theta)}$

$$\alpha = \frac{q_{max} + q_{min}}{2}$$

$$\beta = \frac{q_{max} - q_{min}}{2 \cdot (\delta + \gamma)} = \frac{1}{2}$$

$$\delta = q_{mid} - q_{min}$$

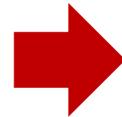
$$\gamma = q_{max} - q_{mid}$$

# Case study: An application based on a Italian Life Insurance data set

- ✓ Contract type: participating (with-profit) life insurance policy.
- ✓ Actuarial form: Single Premium Endowment- 20 years duration- Financial technical rate: 0,00%.
- ✓ Revaluation method: cliquet.

## Data Set

- Time series: monthly from January 2009 to December 2017
- Number of policies and lapses classified by:
  - Premium clusters (A:0-5k ; B:5k-15k ; C:>15k);
  - Guarantees clusters (1:(0-1%); 2;(1%-2%));
- Market Spread
  - monthly observations calculated as the difference between the yield to maturity of a financial market benchmark (10 years maturity Italian Treasury Bullet Bond “BTP”) and the policy crediting rate.



## Policyholder risk classes

- A1 : Premium 0-5k ; Guarantee cluster 0-1%
- A2 : Premium 0-5k ; Guarantee cluster 1%-2%
- B1 : Premium 5-15k ; Guarantee cluster 0-1%
- B2 : Premium 5-15k ; Guarantee cluster 1%-2%
- C1 : Premium >15k ; Guarantee cluster 0-1%
- C2 : Premium >15k ; Guarantee cluster 1%-2%

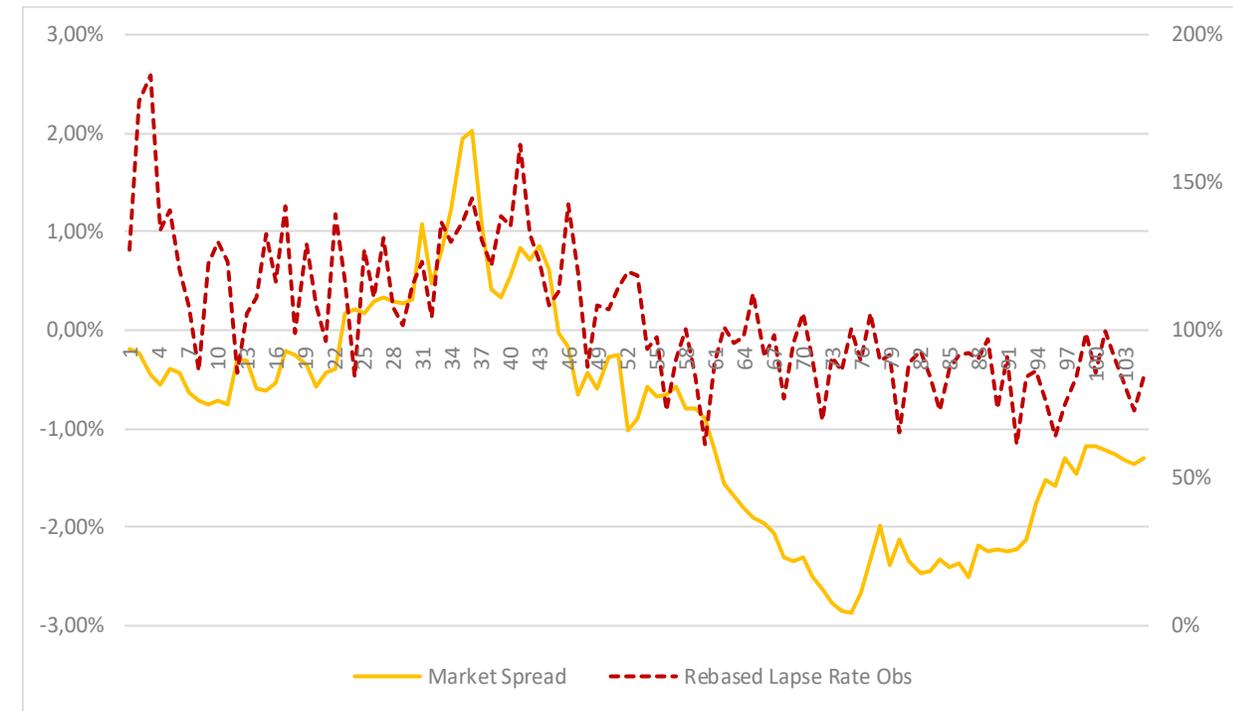
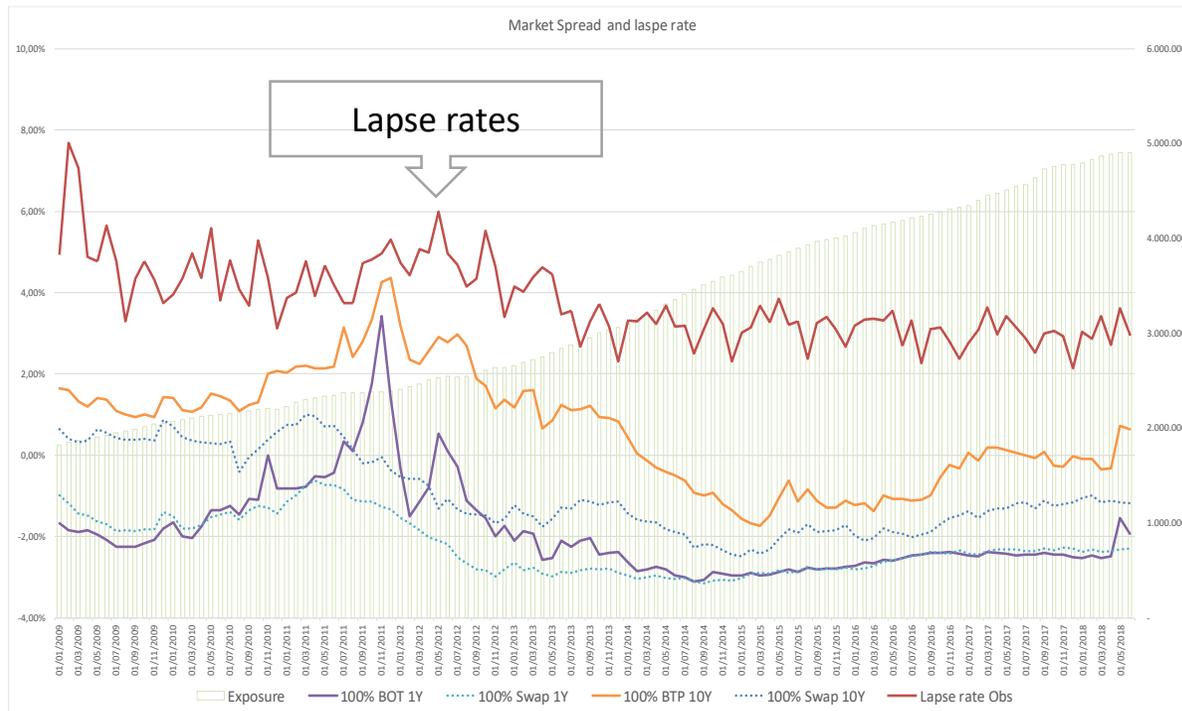
# Case study – Benchmark selection

Correlation between Spread and Yield to return of segregated funds

| Benchmark      | Bot 1Y | Swap 1Y | BTP 10Y      | Swap 10Y |
|----------------|--------|---------|--------------|----------|
| Mean Yield     | -2,80% | -3,13%  | -0,26%       | -1,85%   |
| Pearson's Rho  | 52,9%  | 52,7%   | <b>64,7%</b> | 55,4%    |
| Kendall's Tau  | 43,0%  | 30,1%   | <b>47,8%</b> | 33,0%    |
| Spearman's Rho | 64,9%  | 48,5%   | <b>70,4%</b> | 52,2%    |
| % Bmk          | 100%   | 100%    | 100%         | 100%     |

Correlation between Spread and Credited Policy Rate

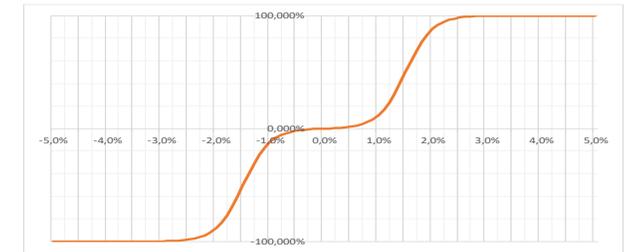
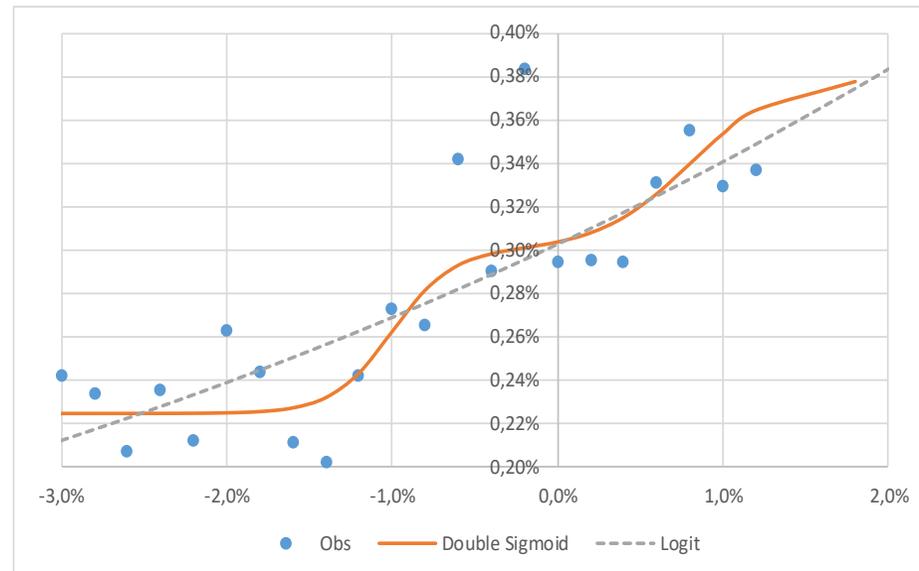
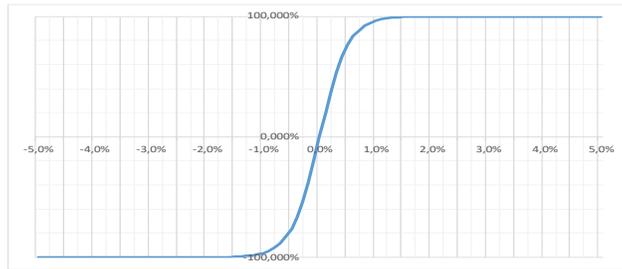
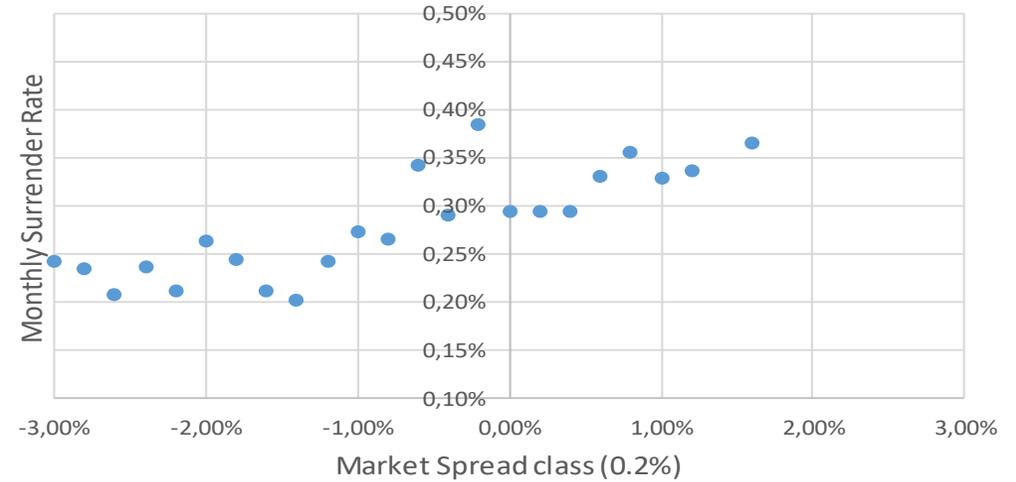
| Benchmark      | Bot 1Y        | Swap 1Y       | BTP 10Y       | Swap 10Y      |
|----------------|---------------|---------------|---------------|---------------|
| Mean Yield     | <b>-0,11%</b> | <b>-0,01%</b> | <b>-0,04%</b> | <b>-0,03%</b> |
| Pearson's Rho  | <b>59,4%</b>  | <b>68,5%</b>  | <b>62,9%</b>  | <b>59,6%</b>  |
| Kendall's Tau  | <b>53,4%</b>  | <b>51,5%</b>  | <b>47,1%</b>  | <b>40,3%</b>  |
| Spearman's Rho | <b>75,1%</b>  | <b>73,5%</b>  | <b>70,2%</b>  | <b>62,1%</b>  |
| % Bmk          | 300%          | 500%          | <b>80%</b>    | 150%          |



# Case Study: from time series analysis to dependency analysis



Market spread values are grouped to empathize the policyholder behavior



# Case Study: A 2-Step model

1<sup>st</sup> step: **basis lapse** for each risk class is estimated using a Logit Regression based on a GLM without considering market spread as covariate:

- $f(\theta_j) = E(\text{lapse} | \theta_j), j = 1, \dots, J$  : expected basis lapse rate for the  $j$ -th risk class.

where  $J = 6$  is the number of risk classes.

2<sup>nd</sup> step: for **DPHB**

For each (month)  $t$  we estimate the expected lapse rate as

$$f_t = \sum_{j=1}^J \frac{f(\theta_j) \cdot N_{t,j}}{N_{t,j}}$$

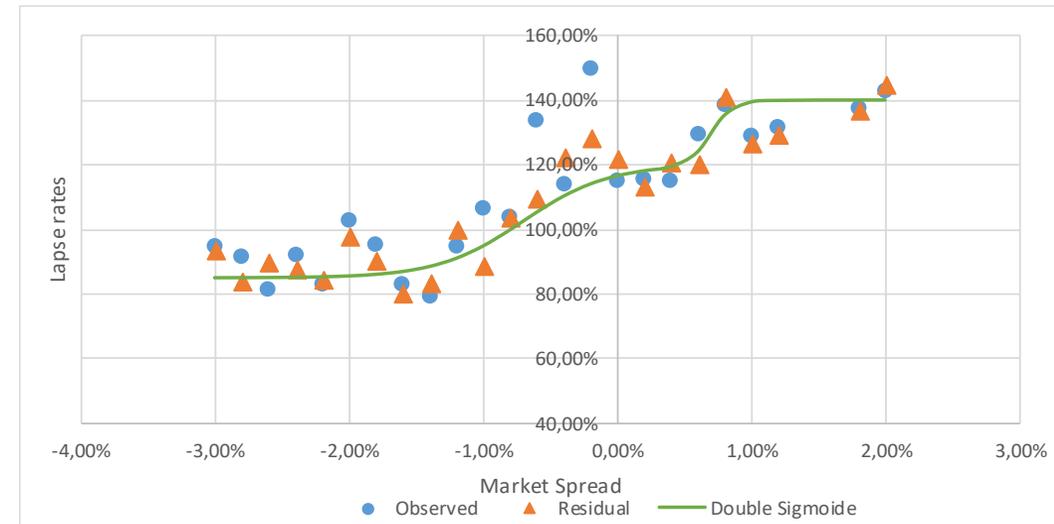
where  $N_{t,j}$  is the exposure (number of contracts or amount of Technical Provisions) at time  $t$  of the risk class  $j$ . Then, we compute:

$$q^{res}(\Delta_t) = \frac{r_t^{obs}}{f_t}$$

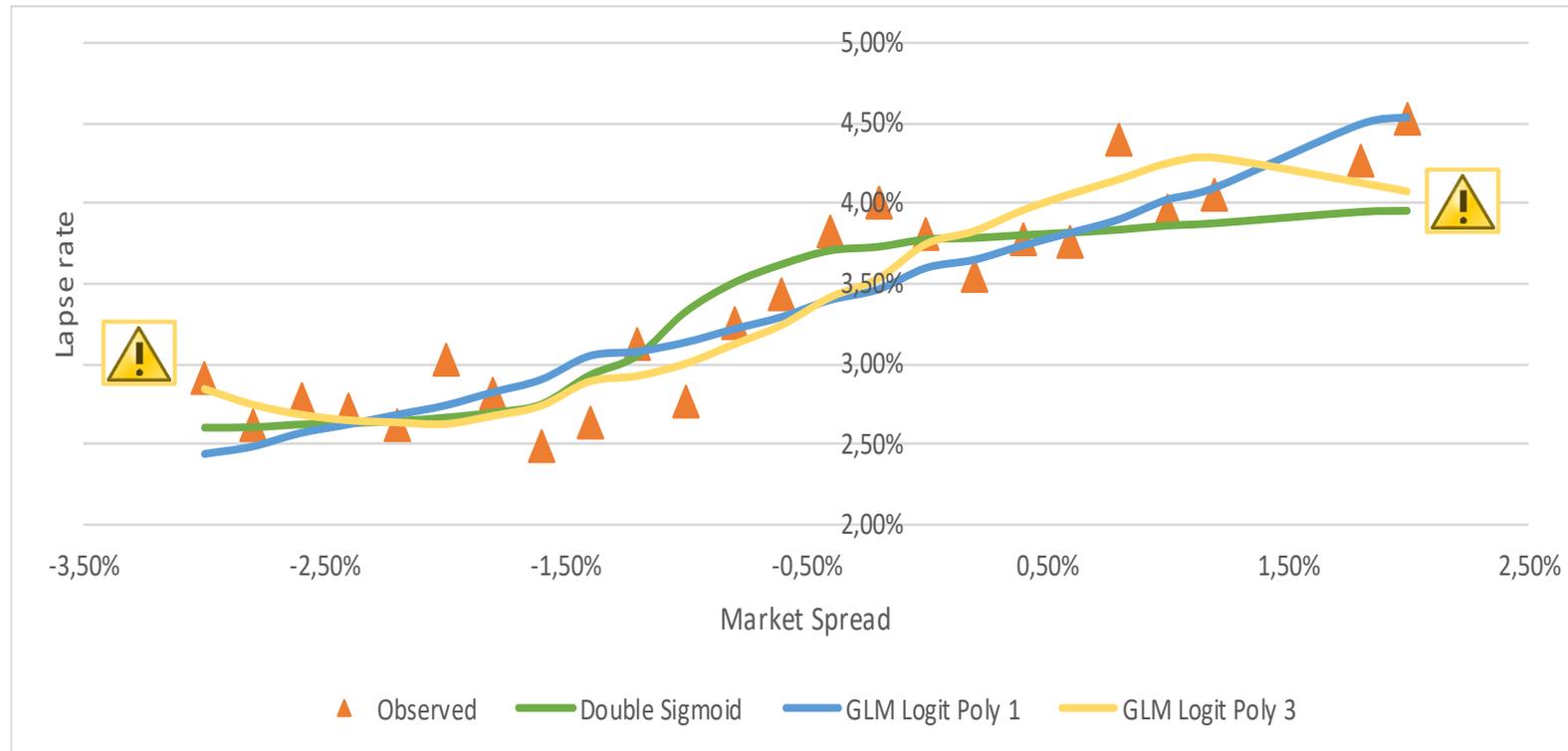
$$q(\Delta_t) = \alpha + \beta \cdot \left[ \tanh\left(\frac{\Delta_t - c_1}{w_1}\right) \cdot \delta + \tanh\left(\frac{\Delta_t - c_2}{w_2}\right) \cdot \gamma \right]$$



| Risk Class | Premium level | Min. Gar | Exposure | Observed | Estimated |
|------------|---------------|----------|----------|----------|-----------|
| A1         | ≤5k€          | ≤1%      | 3%       | 3,89%    | 3,28%     |
| B1         | 5k€-15k€      | ≤1%      | 4%       | 3,00%    | 3,13%     |
| C1         | ≥15k€         | ≤1%      | 3%       | 2,39%    | 2,84%     |
| A2         | ≤5k€          | >1%      | 37%      | 3,18%    | 3,23%     |
| B2         | 5k€-15k€      | >1%      | 32%      | 3,10%    | 3,09%     |
| C2         | ≥15k€         | >1%      | 21%      | 2,87%    | 2,80%     |



# Case study – A comparison between a Logit GLM and 2-step models for lapse rate estimate



The **yellow** and **blu** lines show the fitting of a GLM logit model using the market spread as a covariate.

The GLM model we consider is fitted by using a 1° (**blu**) and 3° grade (**yellow**) polynomial function to describe the relation between the logit of the lapse rate and the market rate.

The 3° grade polynomial form used allows a better fitting. However, it violate the economic assumption of policyholder behavior respect to the market spread.

| Average Lapse rate |         |
|--------------------|---------|
| Observed           | 3,1232% |
| Double Sigmoid     | 3,1329% |
| GLM Logit Poly 1   | 3,0766% |
| GLM Logit Poly 3   | 3,0788% |

# Open issues in DPHB modelling

- To demonstrate the relation between the policyholders behavior and the market spread is an hard task as:
  - The selection of the **time horizon** is strategic in the analysis especially when financial crisis occurs.
  - The **reference market asset** should represent an alternative to the segregated fund. Hence, it can be considered a Coupon Rate to be compared with the policy's credited rate or an Yield to Maturity for bonds with a duration similar to the average maturity or strategic horizon of the insurance contracts.
  - **The clustering of the portfolio** may reduce the size, deepness and completeness of the data adding volatility or a difficulty in represents a "rationale" economic behavior in each cluster.
  - The **correlation analysis** can confirm dependence but do not always indicate a known functional form of the dependency.
- The choice of the model to be used to describe the policyholder behavior depends on:
  - the subjective assumption on the policyholder behavior:
    - **optimal** dynamic lapsation assuming rational and **risk-neutral** (or risk-averse) investors;
    - **suboptimal** dynamic lapsation assuming rational in a **real-world** framework.
  - The ability of the model to represent past lapsation experience and to be back-tested
- The calibration of a suboptimal model that incorporate dynamic and deterministic lapse:
  - Requires a **large, deep and detailed data set**;
  - Requires large use of **expert judgment**;
  - is based on **real-world data**. 

The risk-neutral approach used in the calculation of the BEL/SCR in Solvency 2 implies a distortion in the probability measure thereby it is necessary to introduce a parameter adjustment in order to use the model in S2 framework

# Final Remarks

- Suboptimal dynamic lapse models proposed in the literature used to describe the relationship between a market spread and lapse rates are usually based on an increasing monotonous function eventually bounded by two asymptotes (higher and lower) with just one change of concavity.
- The model we propose is based on a different assumption on the policyholder behavior and modeled through a double sigmoid function that is a monotonous growing function between two asymptotes, but with one or two concavity changes to better fit the data.
- The approach proposed is coherent with a widely adopted model by actuarial software used in the insurance undertakings but it is more robust mathematically as it is based on a continuous and differentiable function.
- As the need to analyze the dependence between the lapse rates and the spread arises from the economic assumption that a correlation between the two variables exists, the 2-steps model (2S) imposes a functional form to the estimated lapse rates coherent with this economic assumption.
- The GLM aims to obtain the best possible fitting on the observed data and, in presence of observed lapse rates not perfectly increasing can generate lapse rates not increasing, violating the economic assumption of positive correlation between the financial spread and lapse rates.

# Main references

1. Anne MacKay, Maciej Augustyniak, Carole Bernard, Mary R. Hardy [2017], Risk Management of Policyholder Behavior in Equity-Linked Life Insurance, *Journal of Risk and Insurance*, 84, 2, 661.
2. Changki Kim [2005], Modeling surrender and lapse rates with economic variables, *North American Actuarial Journal*, 9(4):56–70. (2005).
3. Christian Knoller, Gunther Kraut, Pascal Schoenmaekers [2016], On the Propensity to Surrender a Variable Annuity Contract: An Empirical Analysis of Dynamic Policyholder Behaviour, *Journal of Risk and Insurance*, 83, 4, 979.
4. Daniel Bauer, Jin Gao, Thorsten Moenig, Eric R. Ulm, Nan Zhu [2017], Policyholder Exercise Behavior in Life Insurance: The State of Affairs, *North American Actuarial Journal*, 21:4, 485-501.
5. Kristian Buchardt, Thomas Møller, Life Insurance Cash Flows with Policyholder Behavior, *Risks*, 3, 3, 2 (2015).
6. Lipovetsky, Stan. (2010). Double logistic curve in regression modeling. *Journal of Applied Statistics*. 37. 1785-1793. 10.1080/02664760903093633.
7. Sven Nolte, Judith C. Schneider, Don't lapse into temptation: a behavioral explanation for policy surrender, *Journal of Banking & Finance*, 2017, 79, 1.
8. Milliman (2013), Dynamic lapse risk in an era of quantitative easing, Milliman Research Report <http://www.milliman.com/uploadedFiles/insight/2013/Dynamic-lapse-risk.pdf>
9. Martin Eling ,Michael Kochanski (2013), Research on Lapse in Life Insurance—What Has Been Done and What Needs to Be Done, *The Journal of Risk Finance* 14(4):392-413.

# A Double-Sigmoid approach for dynamic policyholder behavior

Thank you!  
Questions?

**Fabio Baione**, Davide Biancalana, Paolo De Angelis & Ivan Granito

*fabio.baione@unrioma1.it*

