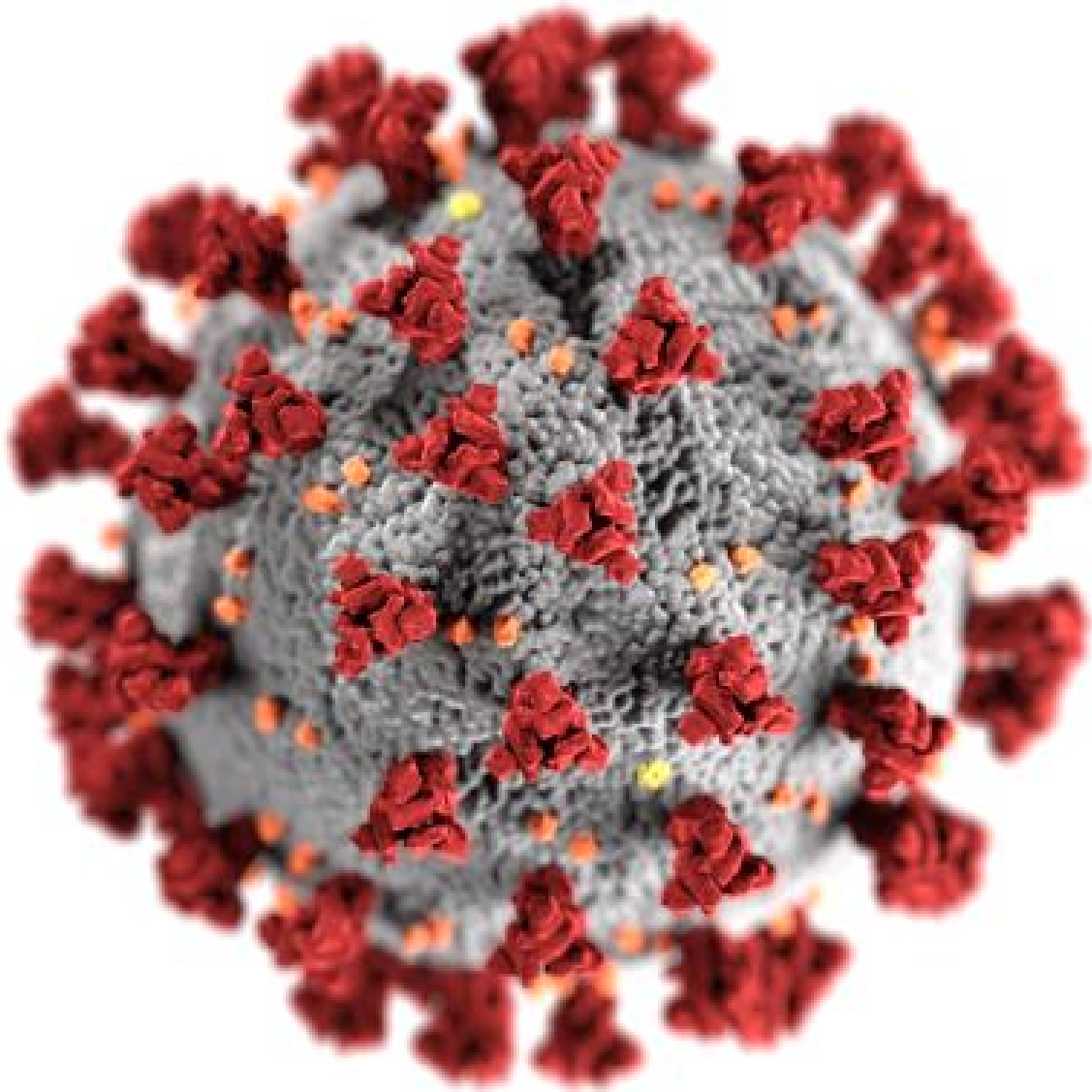
Webinar

Allowing for shocks in portfolio mortality models

Stephen J. Richards

Tuesday 31st May 2022, 16:00 (Italy), 15:00hrs (UK)

1 Motivation



Motivation

- How does COVID-19 affect portfolio mortality?
- How can actuaries avoid bias when deriving bases?

Bias is a business problem

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Consider an annuity portfolio or pension scheme:

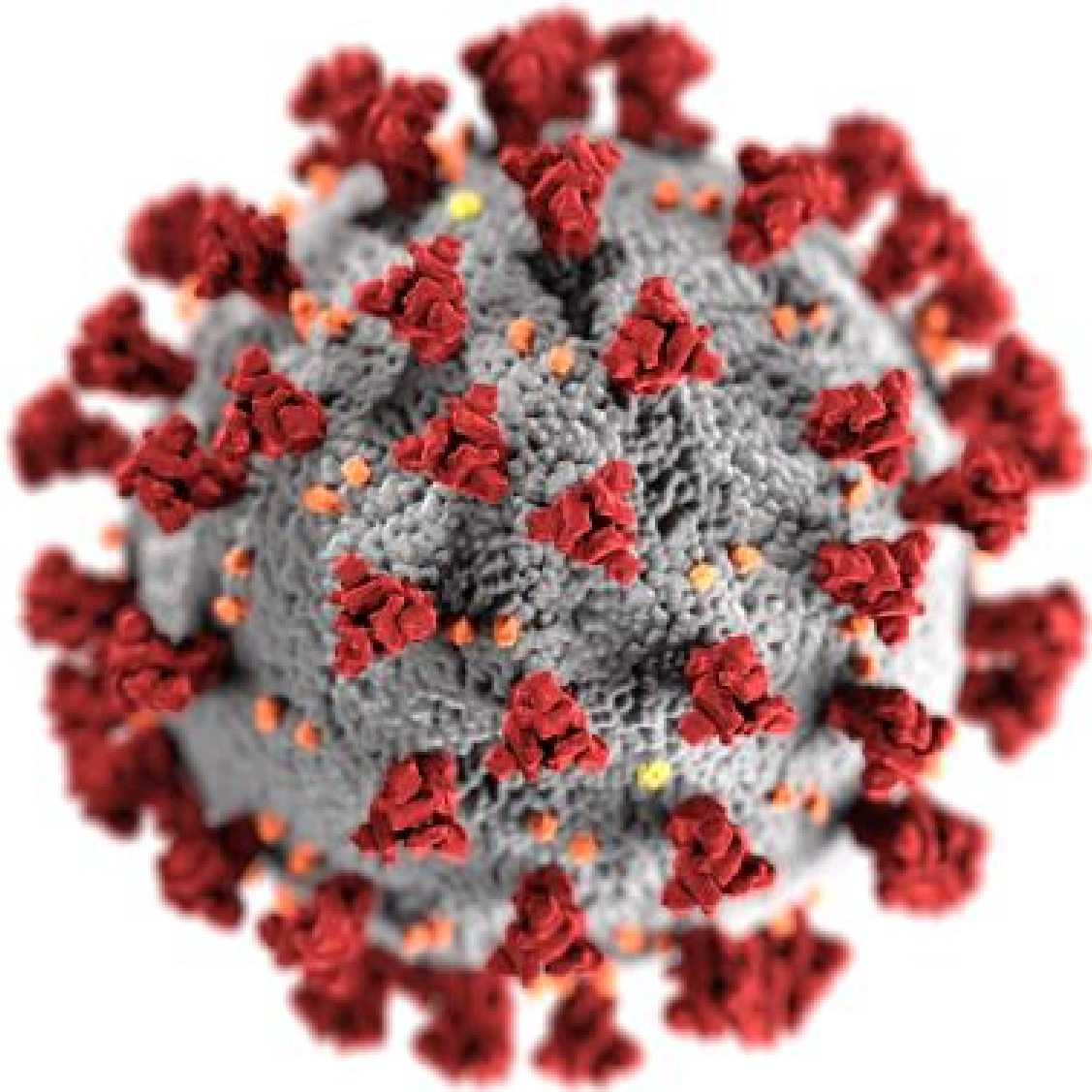
Reserving

✗ Imprudent to include excess shock mortality when deriving a long-term basis.

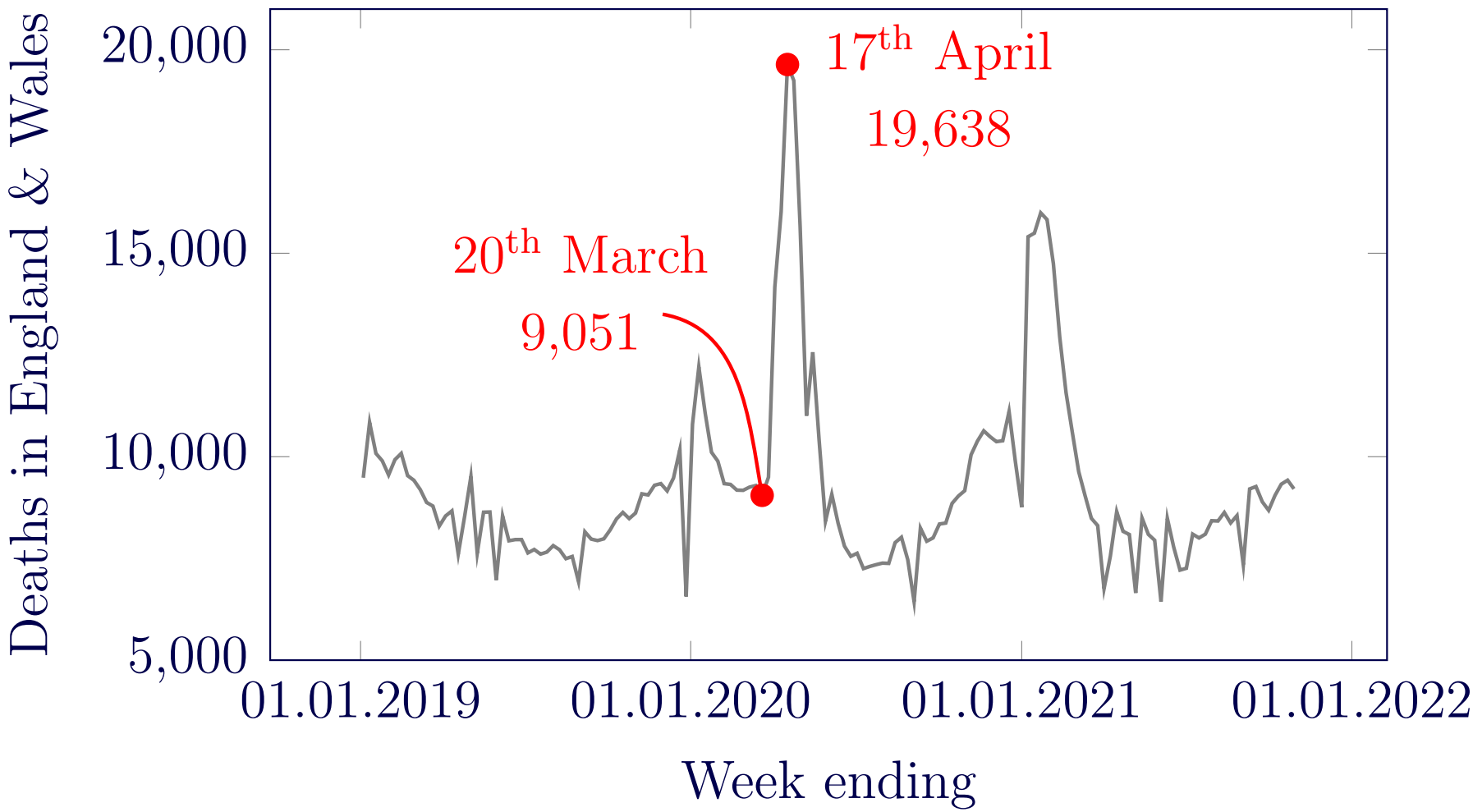
Pricing

✗ Including excess shock mortality underprices risk in bulk annuities and longevity swaps.

2 Mortality shocks

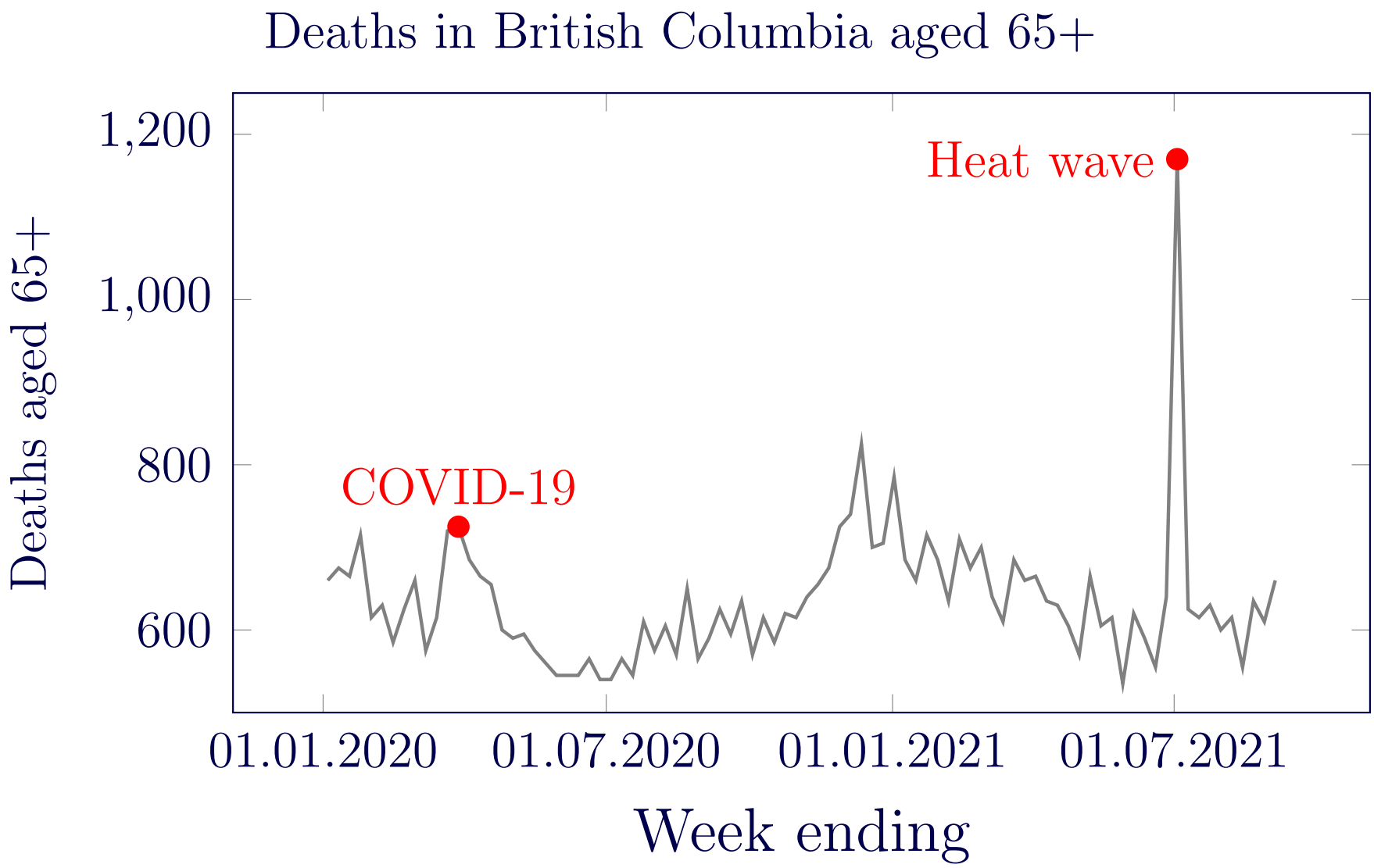


All-cause deaths aged 65+



Source: ONS data for deaths in England & Wales by date of registration.

COVID-19 v. climate stress



Source: Statistics Canada.

Mortality shocks

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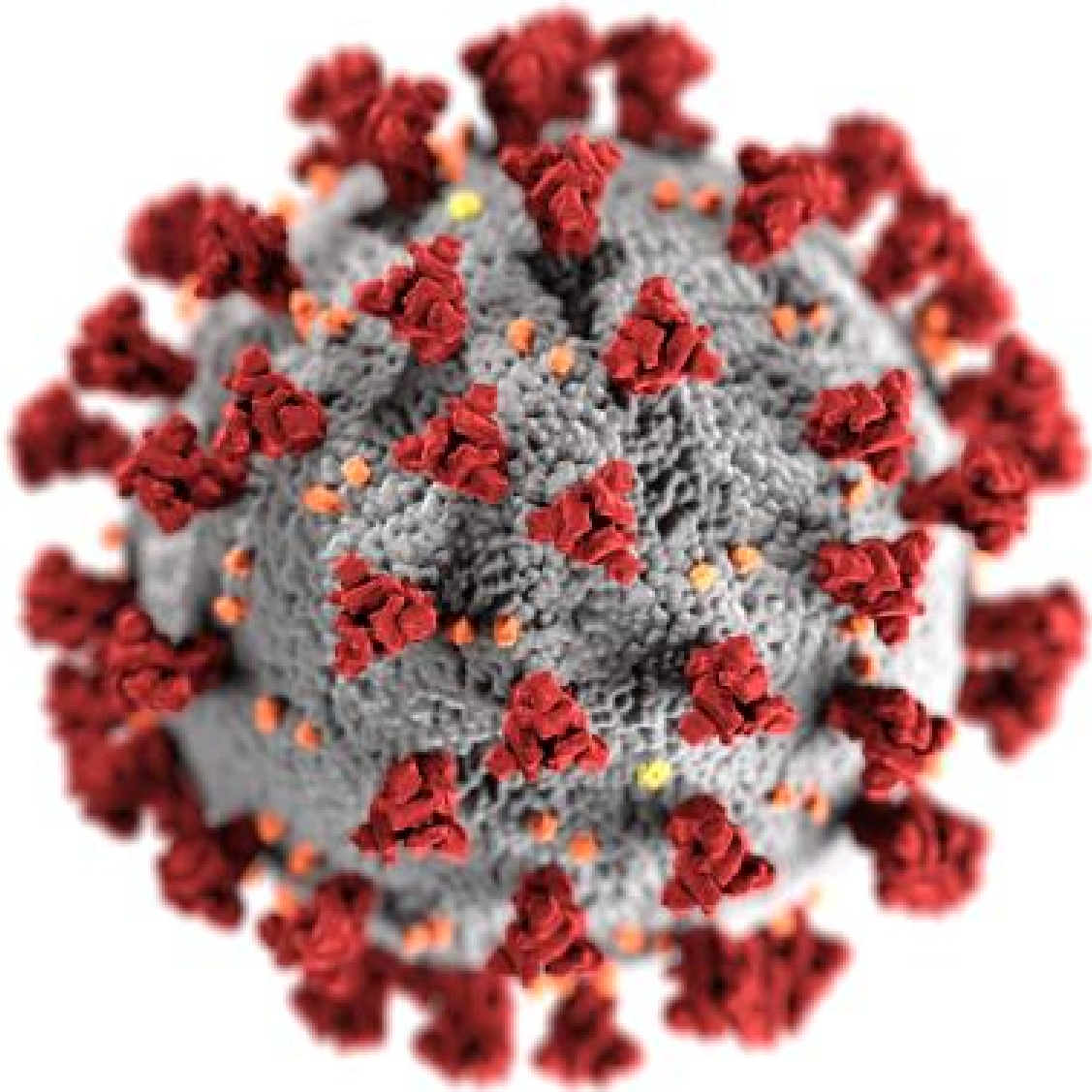
- Shocks occur over a very short space of time.
- Annual rates cannot capture the nature of shocks.
- Continuous-time methods are required.

3 Crude mortality hazard



Daily mortality hazard

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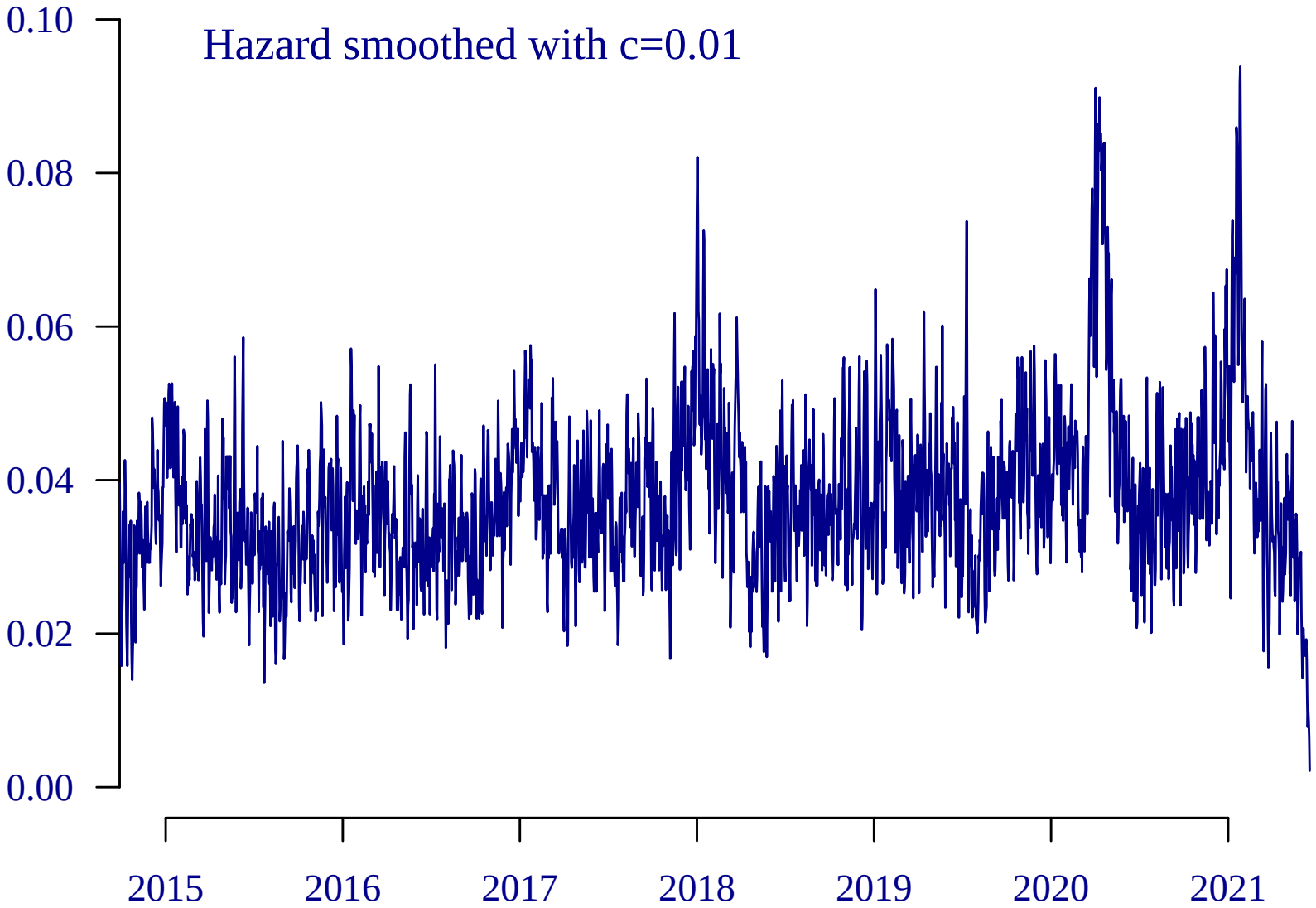
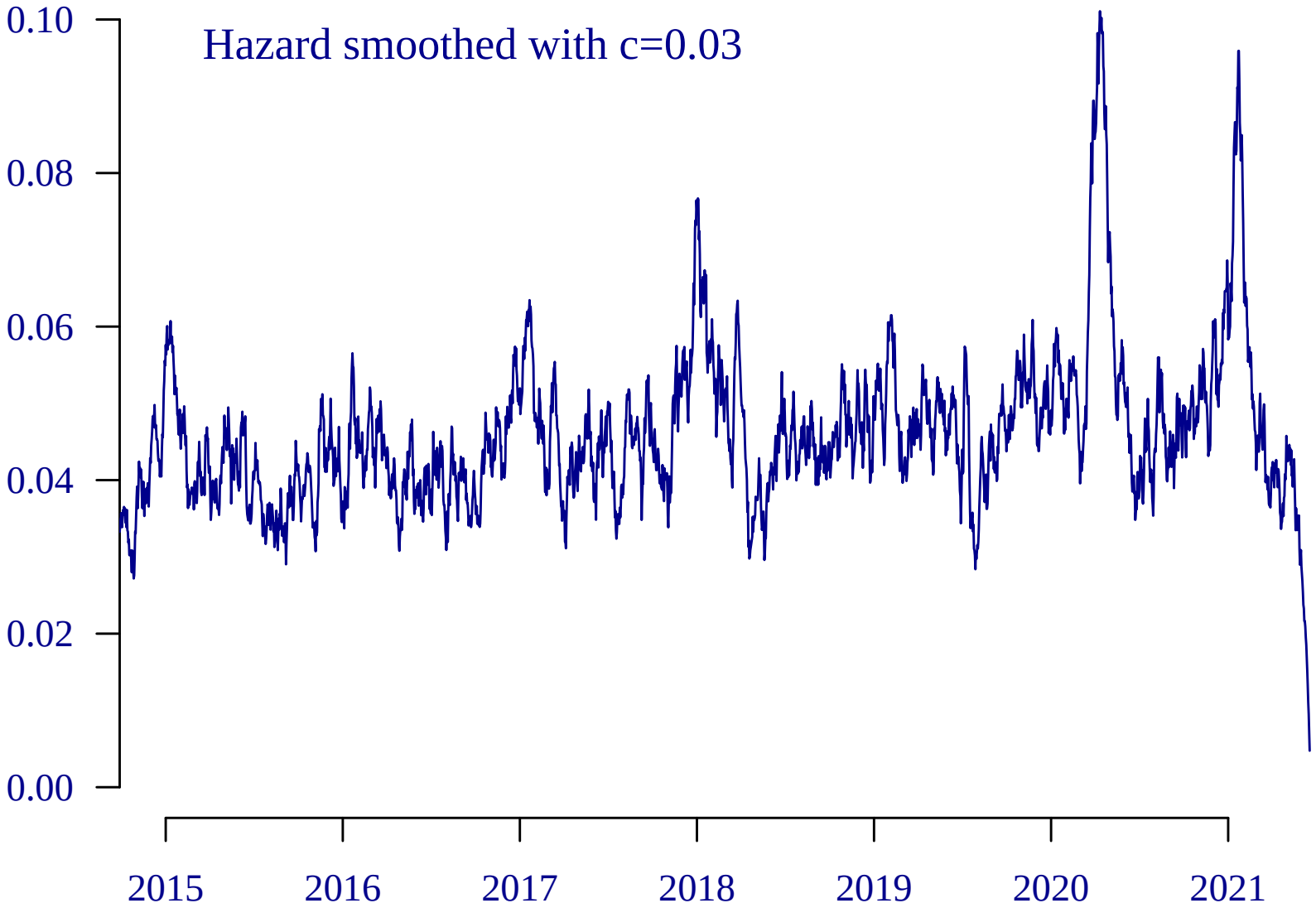
- Richards [2022a] proposed a new estimator for actuarial portfolios.
- Daily mortality hazard calculated in Excel or R.
- Vary smoothing parameter, c, to reveal patterns.

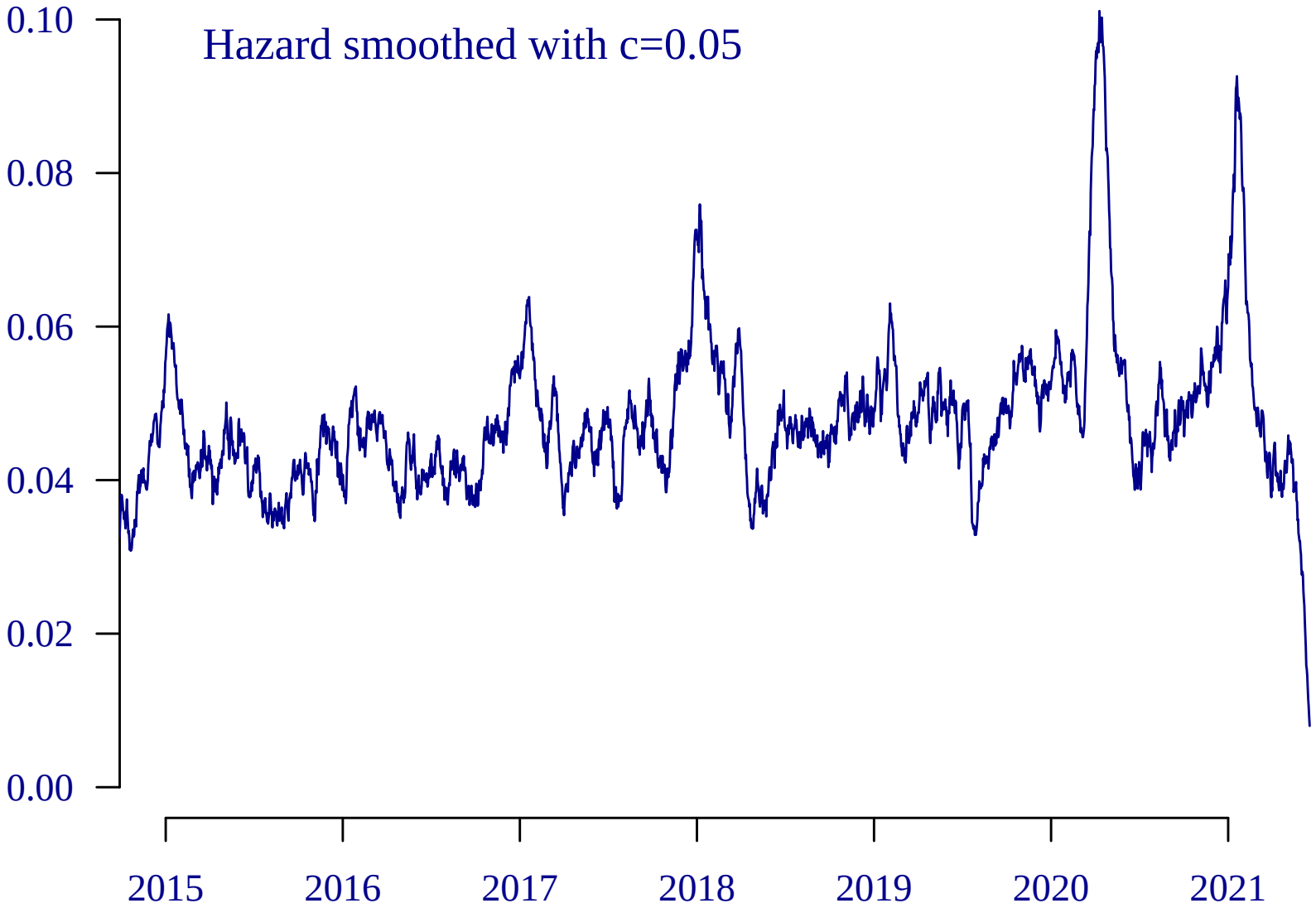
Daily hazard for UK annuities



Daily hazard for UK annuities



Daily hazard for UK annuities



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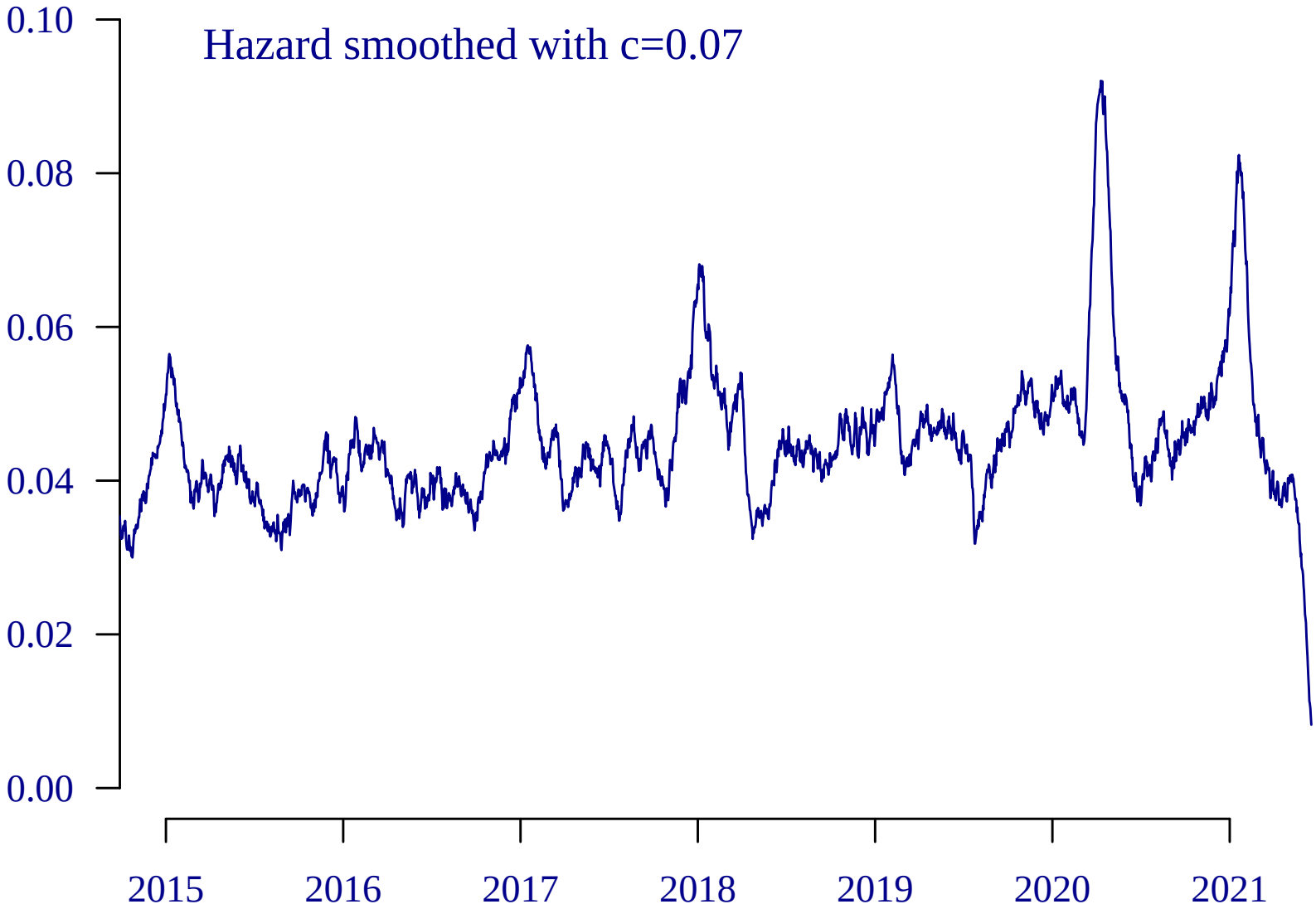
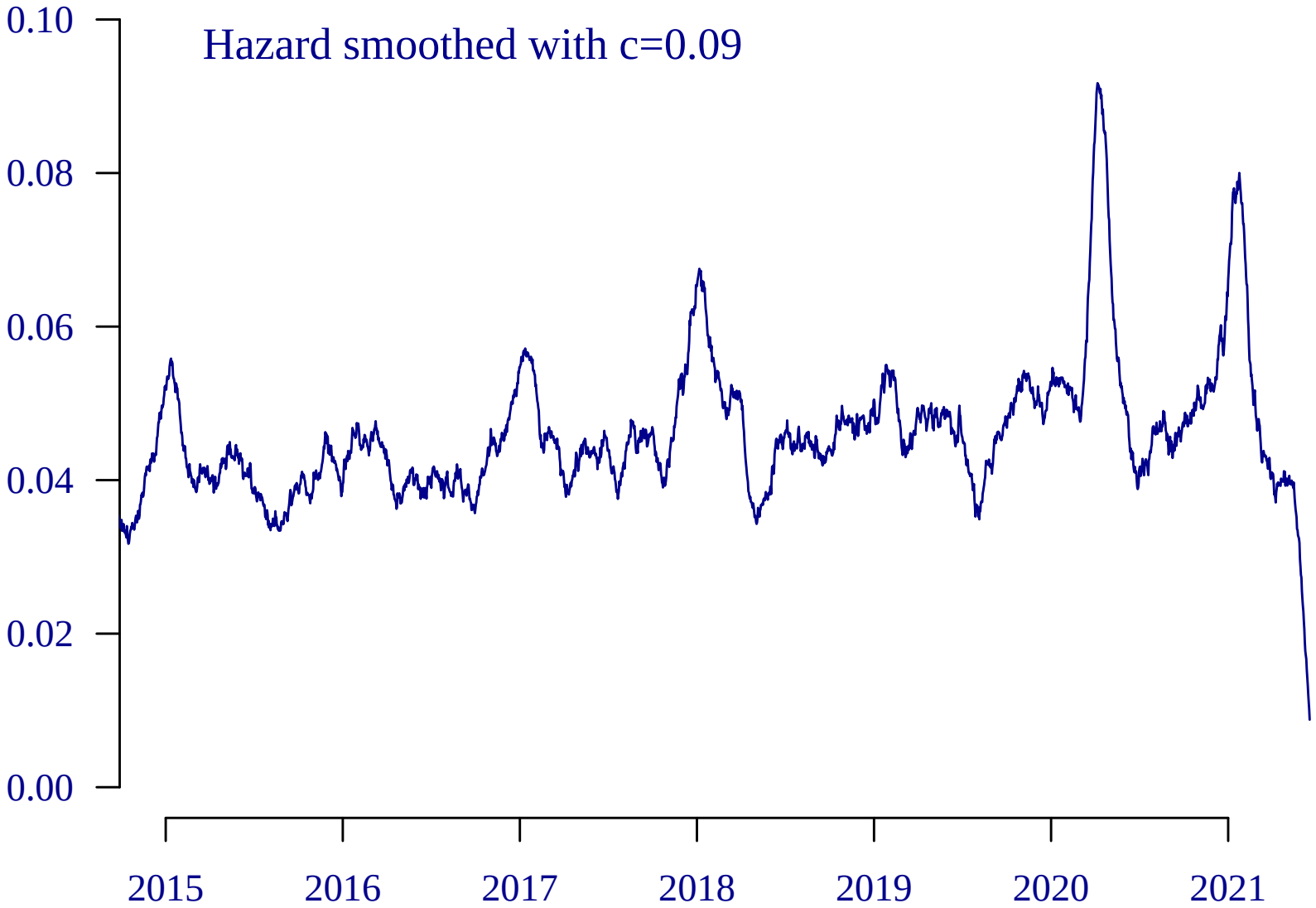
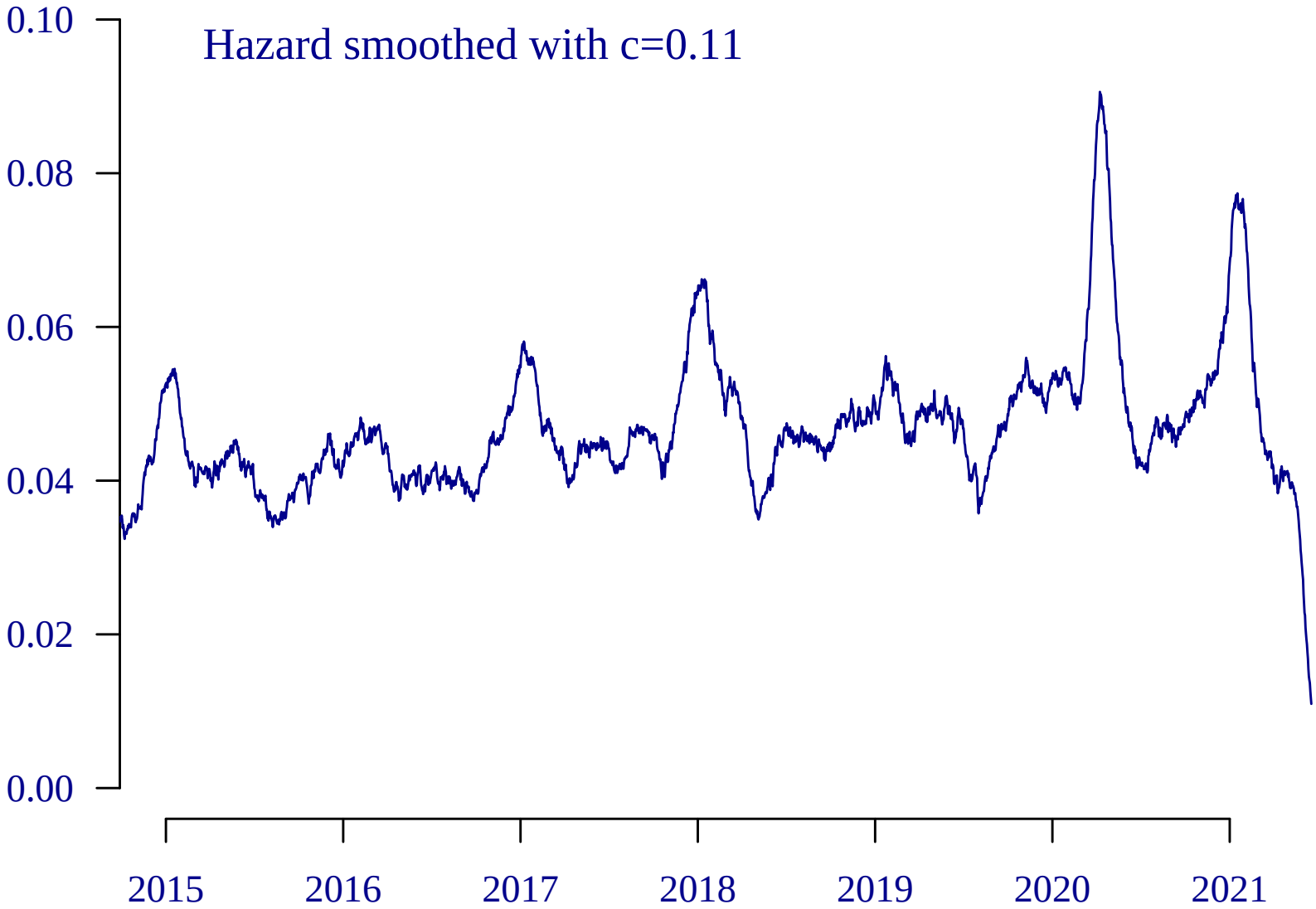


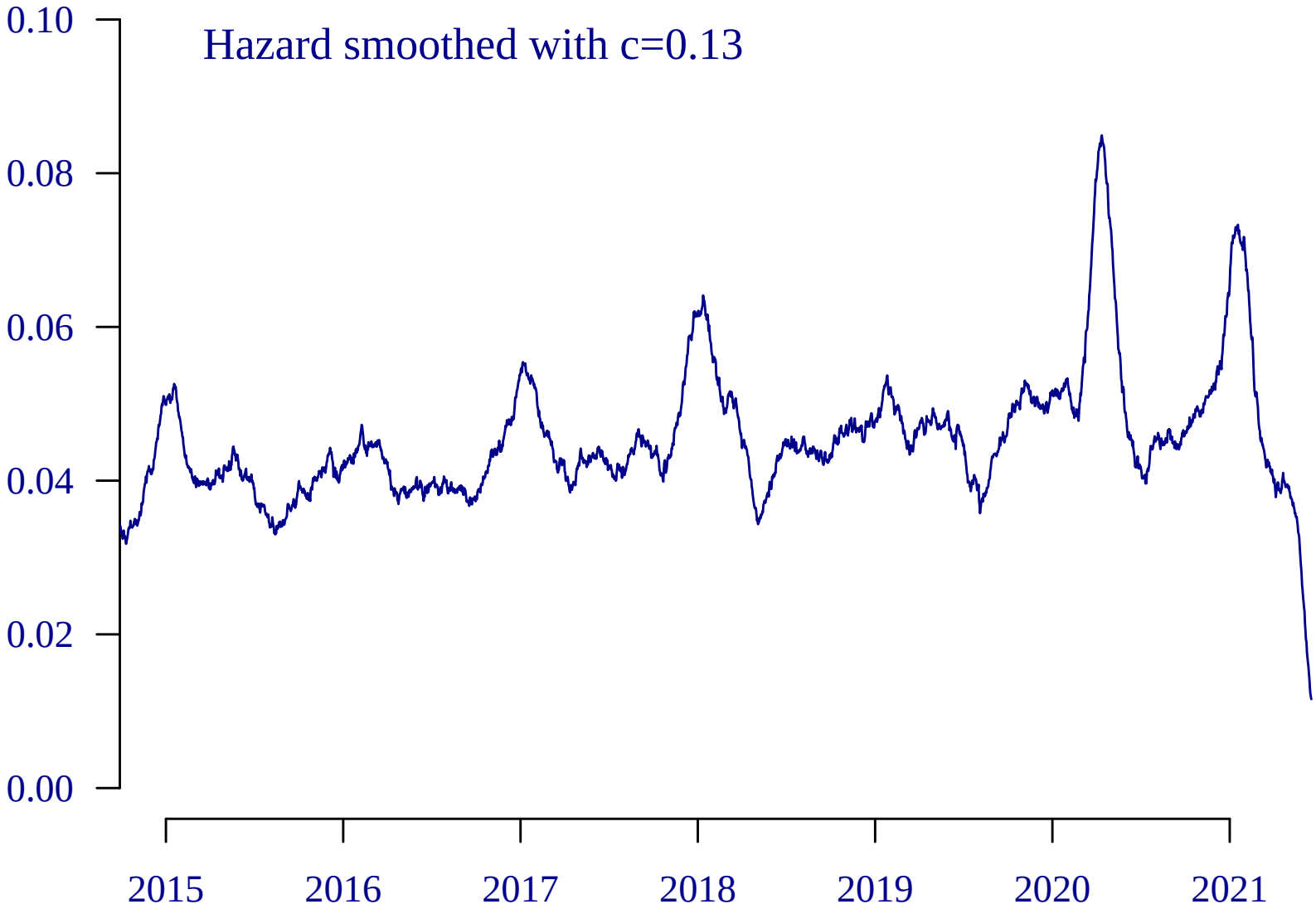
Daily hazard for UK annuities



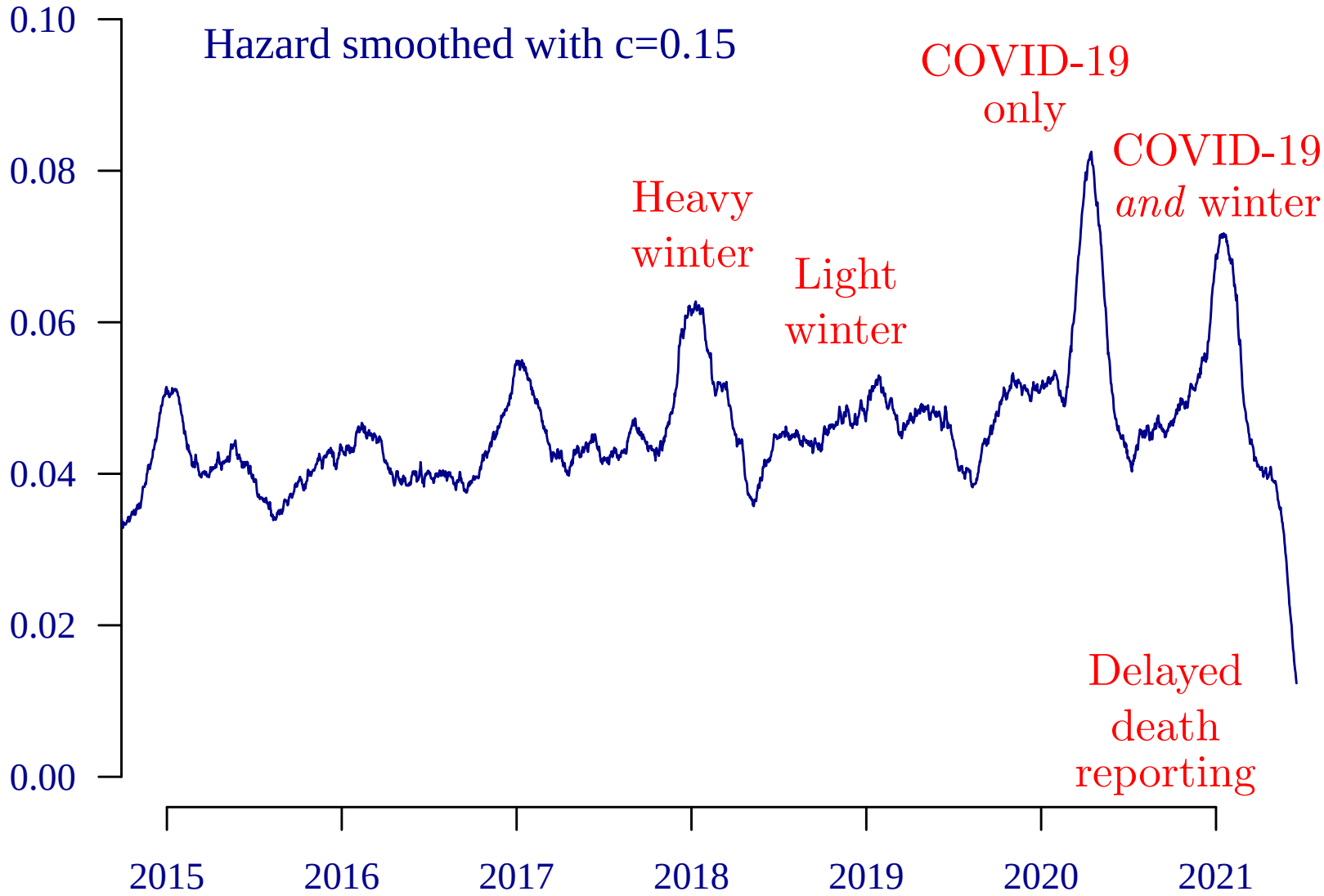
Daily hazard for UK annuities



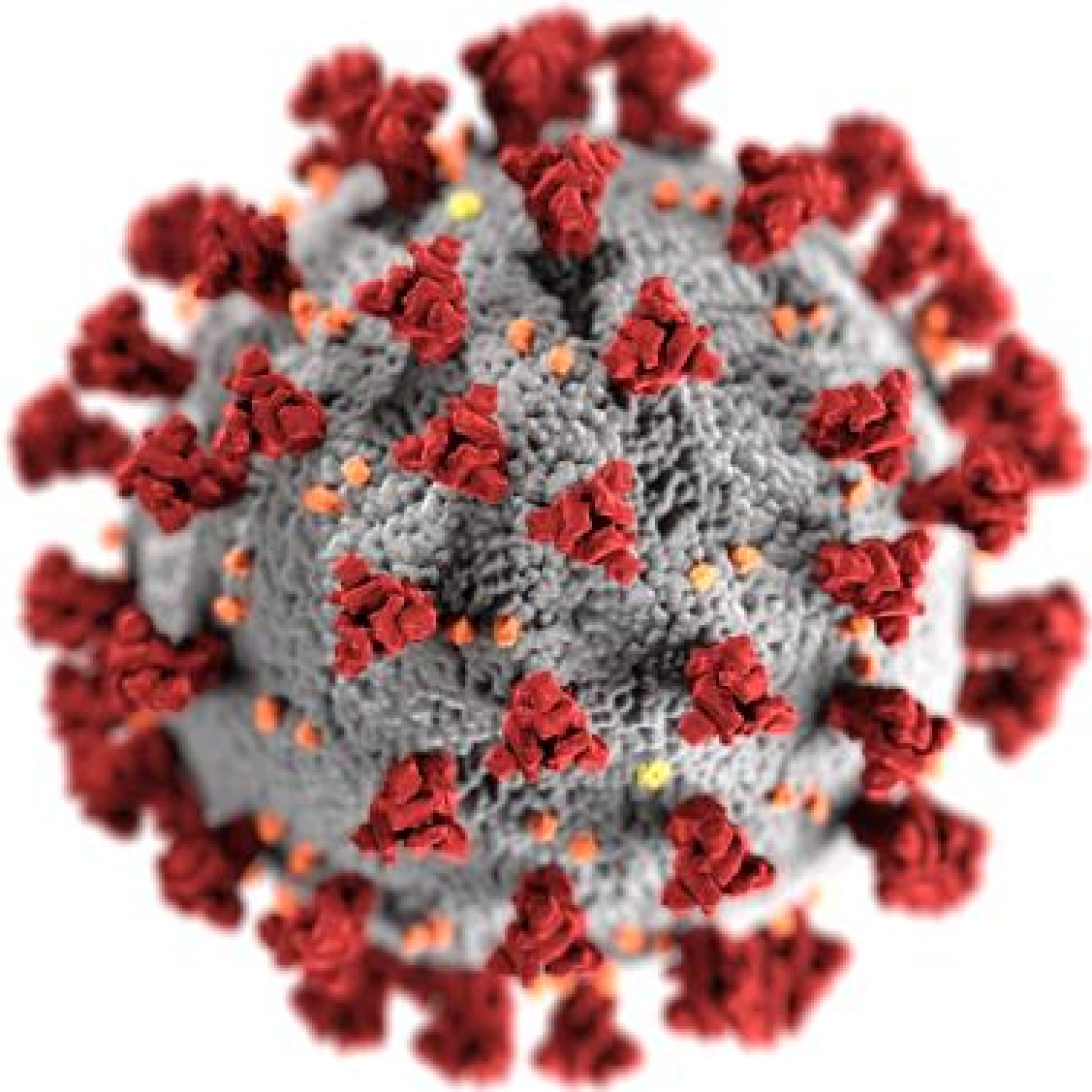
Daily hazard for UK annuities



Daily hazard for UK annuities



4 Reporting delays



Reporting delays

- Delays in reporting deaths to administrator.
- Consider two extracts at times $u_1 < u_2$.
- Calculate ratio, R, of $\hat{\mu}_{y+t}$ using each extract.

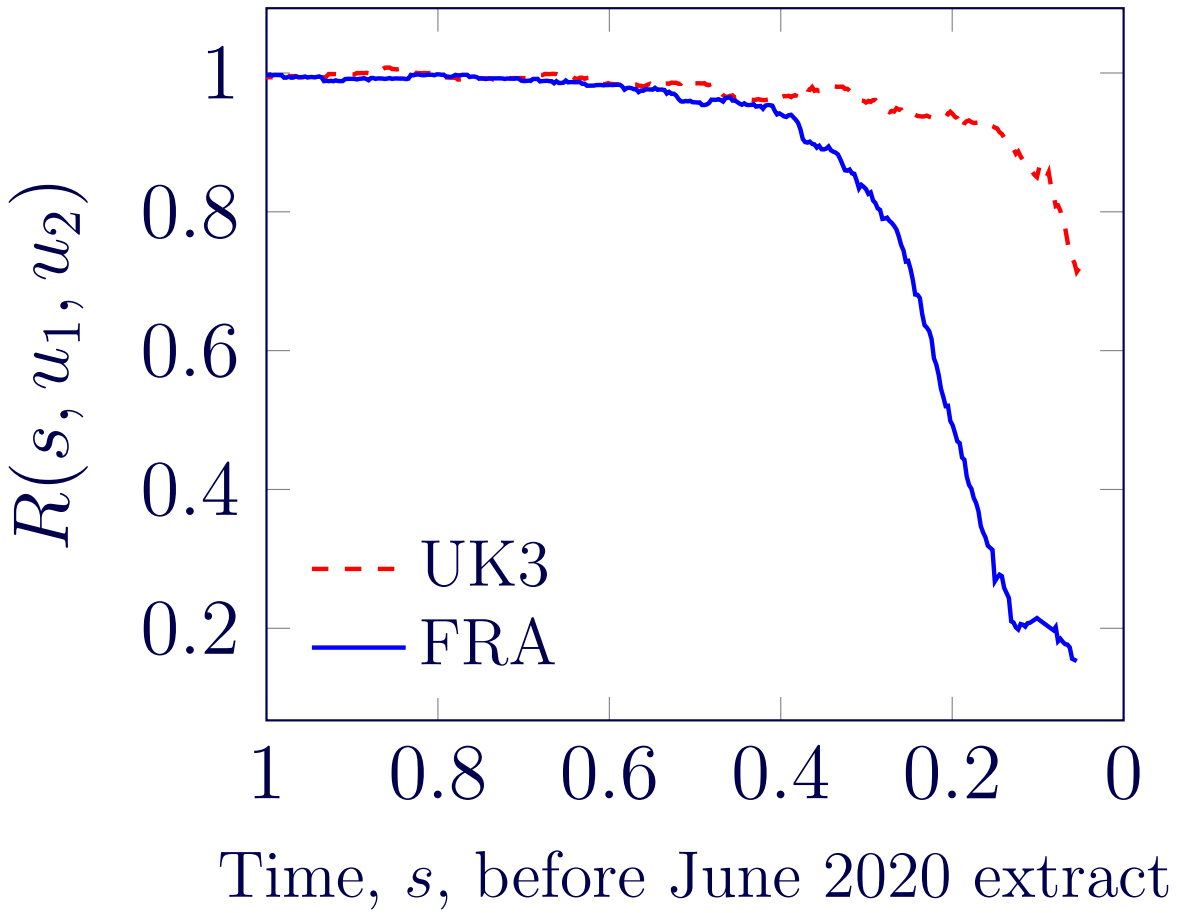
Reporting delays

$$R(s, u_1, u_2) = \frac{\hat{\mu}_{u_1-s} \text{ using extract at time } u_1}{\hat{\mu}_{u_1-s} \text{ using extract at time } u_2} \tag{3}$$

Source: Richards [2022a, equation (3)].

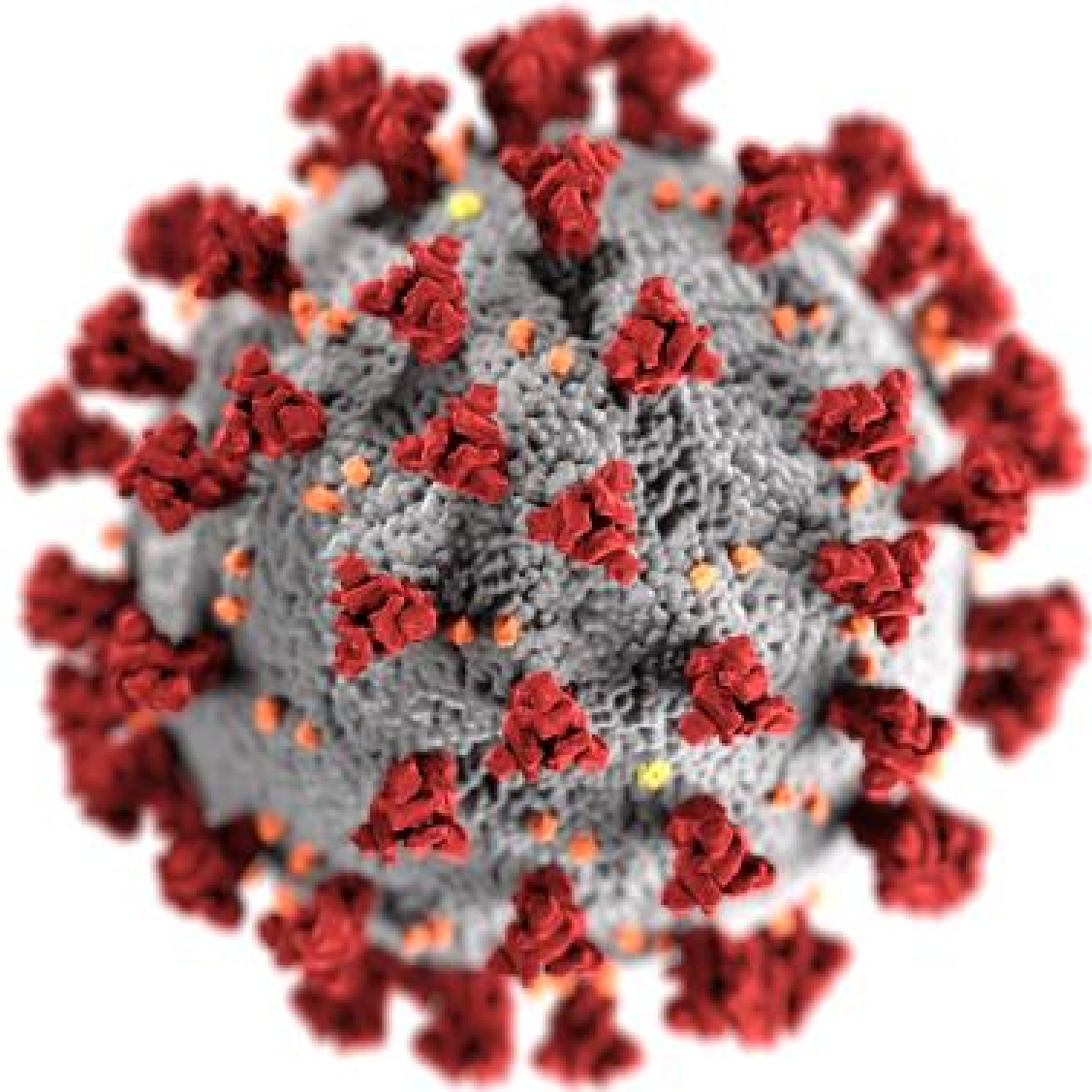
Reporting delays

Proportion of deaths reported by June 2020:



Source: Richards [2022a, Figure 7].

5 Mortality by age



Mortality modelling

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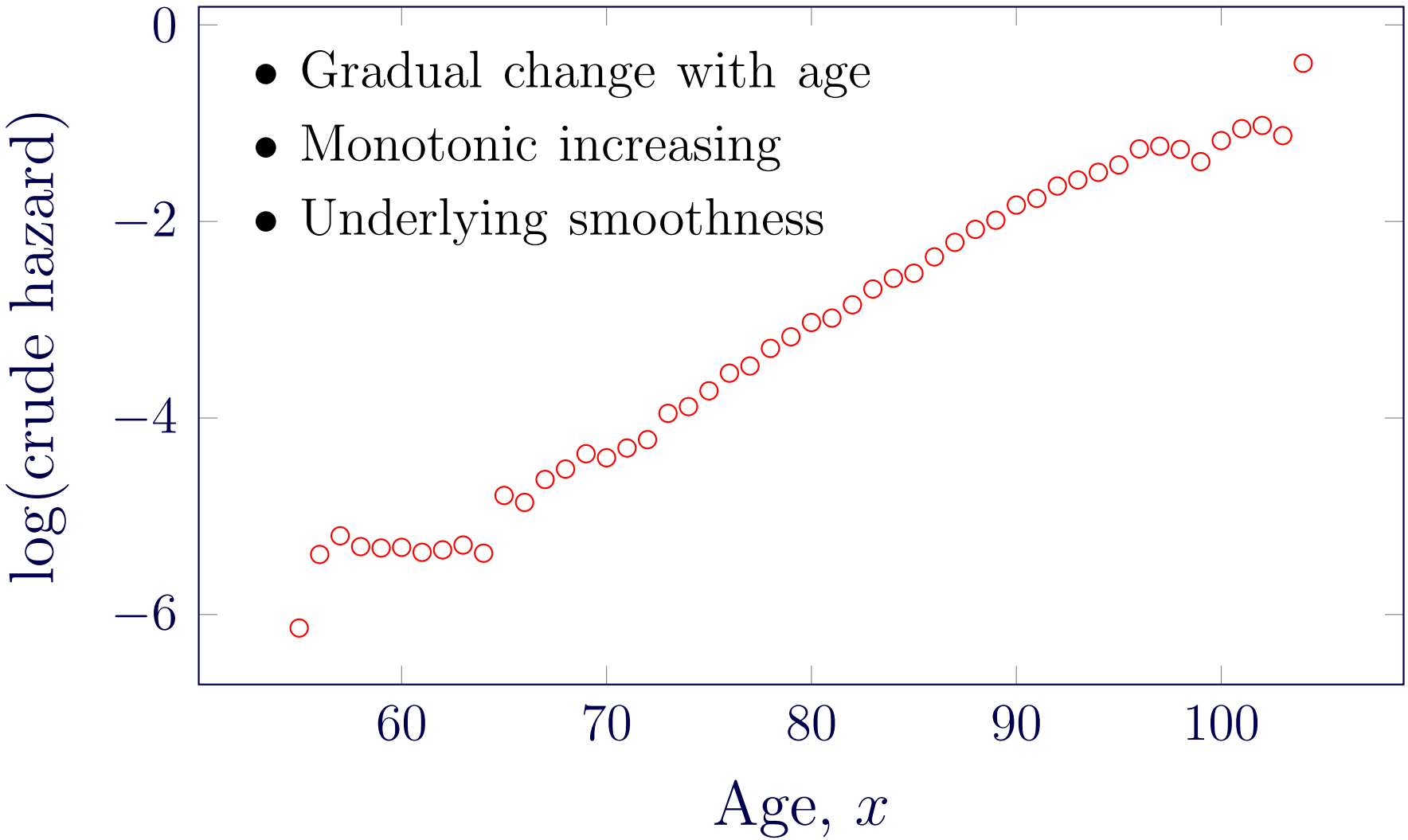
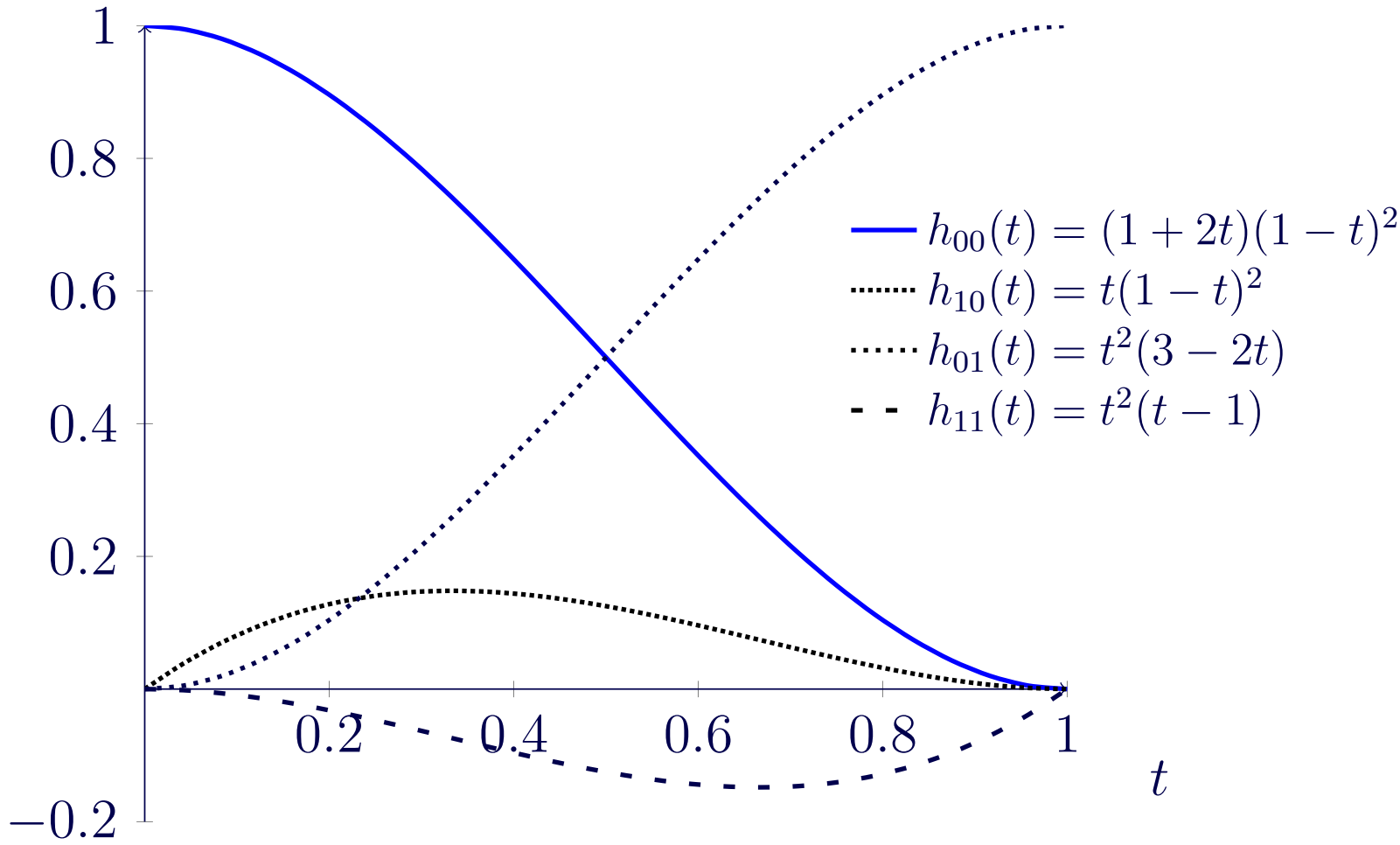
- Actuaries needs to take acount of risk factors:
  - age,
  - gender,
  - annuity size,
  - and others...
- We need a model that allows for these, but also allows for mortality varying by time.

Mortality by age



Source: Richards [2022b, Figure 5(a)]. UK3 data set, 2015–2019.

A basis of Hermite splines



Source: Richards [2020].

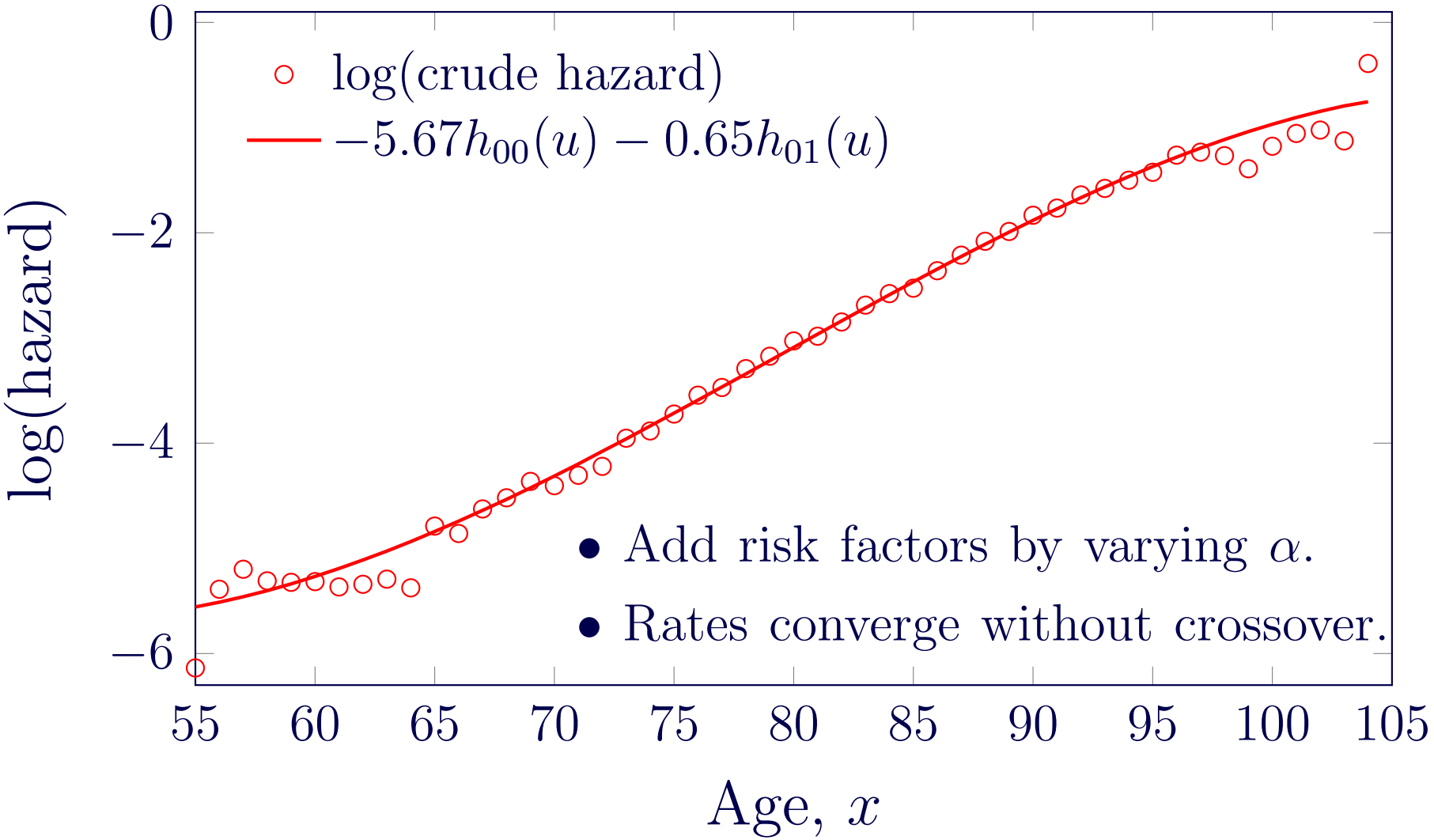
Hermite-spline mortality model

- Age $x \in [x_{\min}, x_{\max}]$.
- Define $u = \dfrac{(x - x_{\min})}{(x_{\max} - x_{\min})}$, so $u \in [0, 1]$.
- $\log \mu_x = \alpha h_{00}(u) + \omega h_{01}(u)$

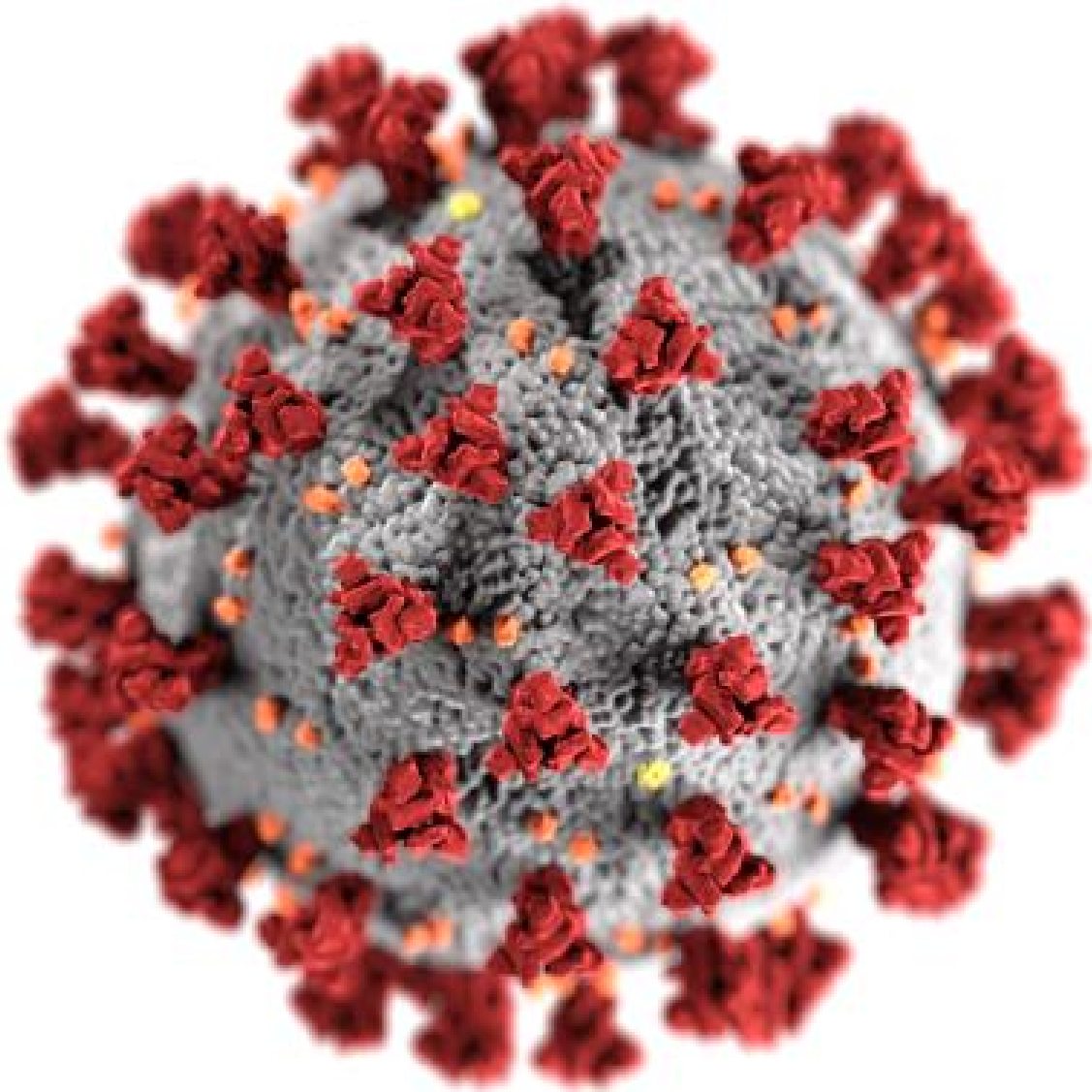
Parameters α and ω can be estimated from data.

Source: Richards [2020].

Hermite-spline mortality model

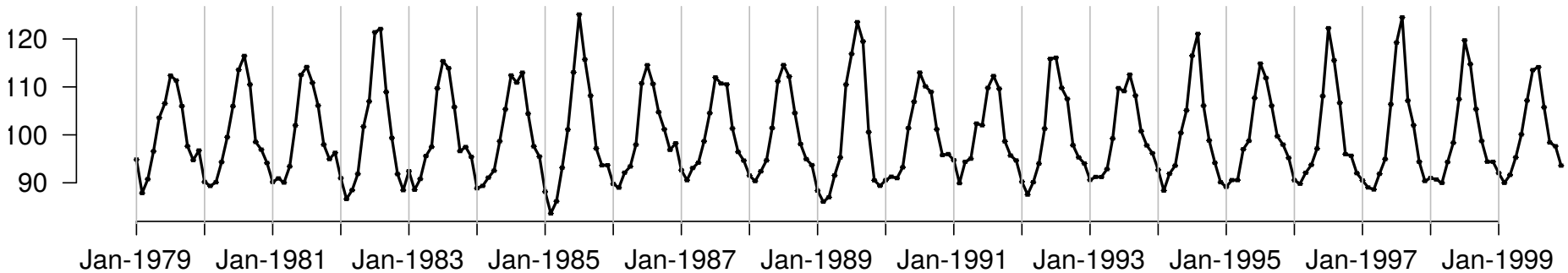


6 Mortality across time



Mortality across time: seasons

Percentage of average daily number of deaths in Australia, all causes, 1979–1999.



Source: de Looper [2002].

Mortality levels fluctuate *even without a pandemic*.

Mortality in time

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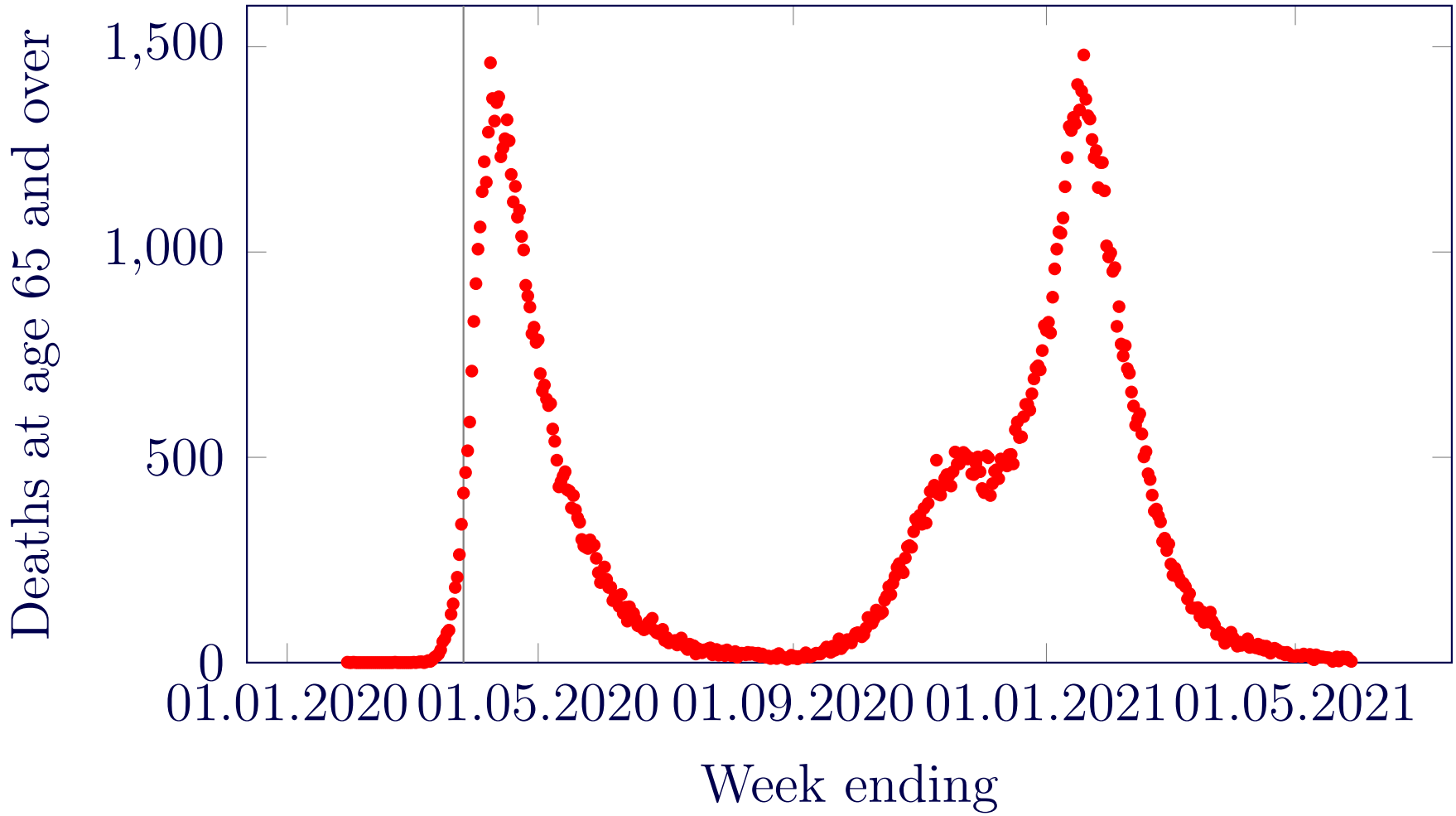
- Not monotonic (ever).
- But smooth on a day-to-day basis, even during a pandemic...

COVID-19 deaths

UK deaths where COVID-19 was listed as one of the causes.



Source: ONS data.

Modelling requirements

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Mortality by age

- Slow, monotonic changes.
- Little flexibility needed.

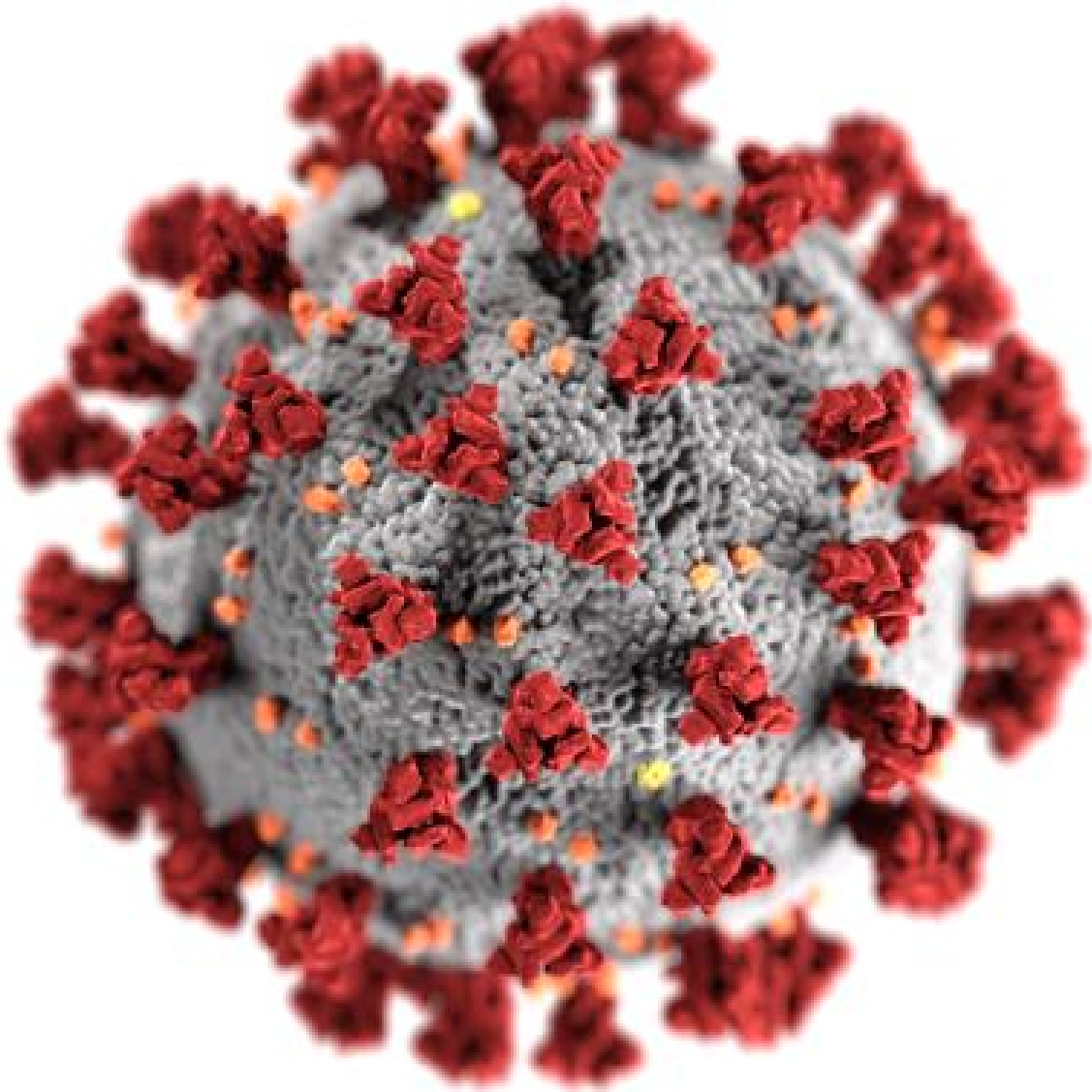
Mortality by time

- Fast, non-monotonic changes.
- Greater flexibility needed.

Continuous age-period model

$$\log \mu_{x,y} = \underset{\text{age component}}{\text{Monotonic}} + \underset{\text{period component}}{\text{Locally flexible}}$$

7 Flexible modelling over time



Schoenberg splines

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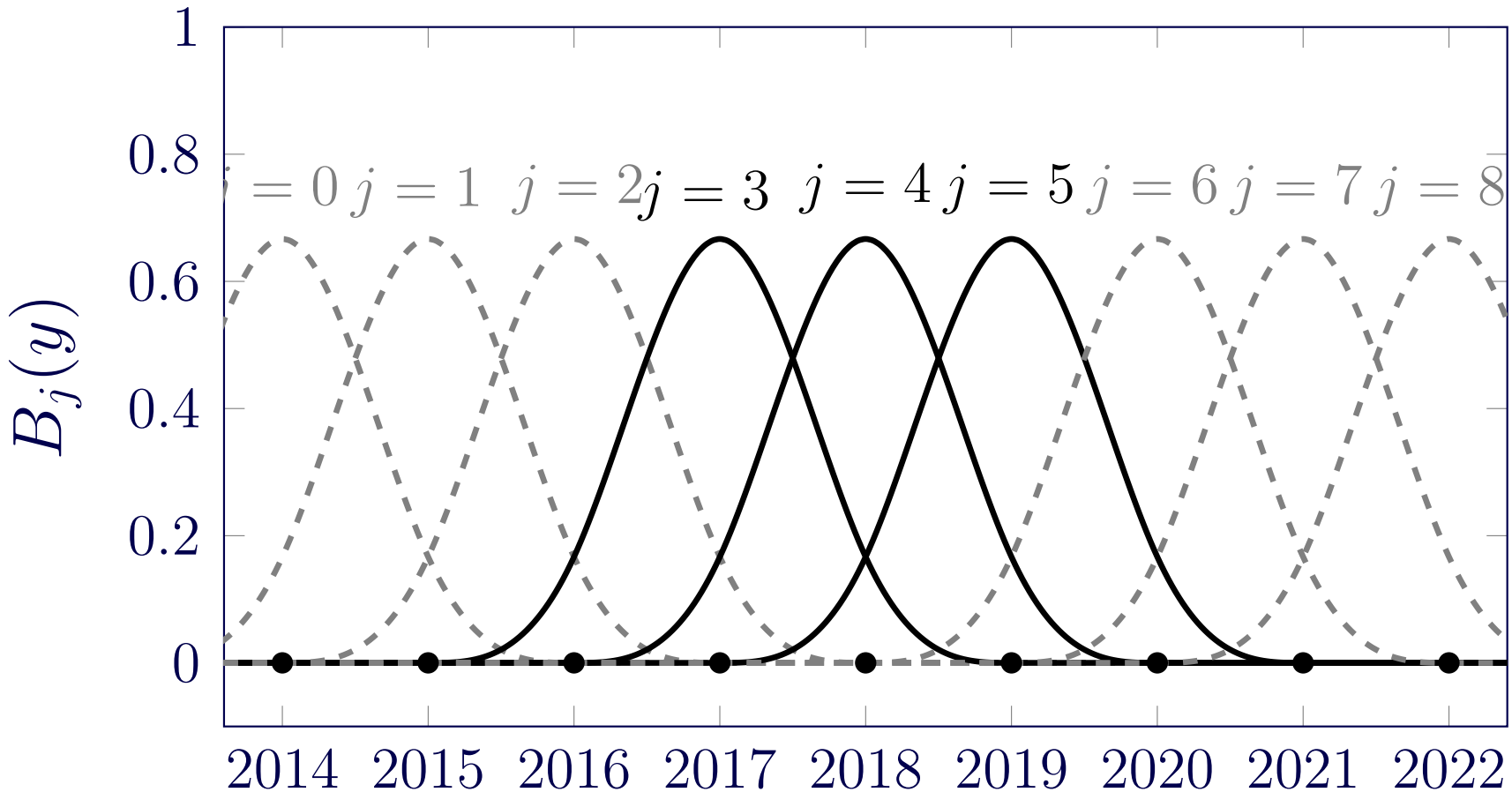
- Proposed by Schoenberg [1964].
- Series of polynomials joining *knot points*.
- Also known as basis splines or B-splines.

A basis of cubic B-splines

A basis of nine equally-spaced cubic B-splines spanning 1st January 2015 to end-2020, indexed $j = 0, 1, \ldots, 8$.



Schoenberg v. Hermite

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- Hermite splines span the entire interval of interest.
- In contrast, B-splines are local functions.
- B-splines permit greater local flexibility...

B-splines

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Define:

- $B_j(y)$ as the j^{th} basis spline at time y.
- $\kappa_{0,j}$, the coefficient of spline B_j.
- $\mu_{x,y}$, the mortality hazard at age x and time y.
- μ_x, the Hermite-spline model for mortality by age.

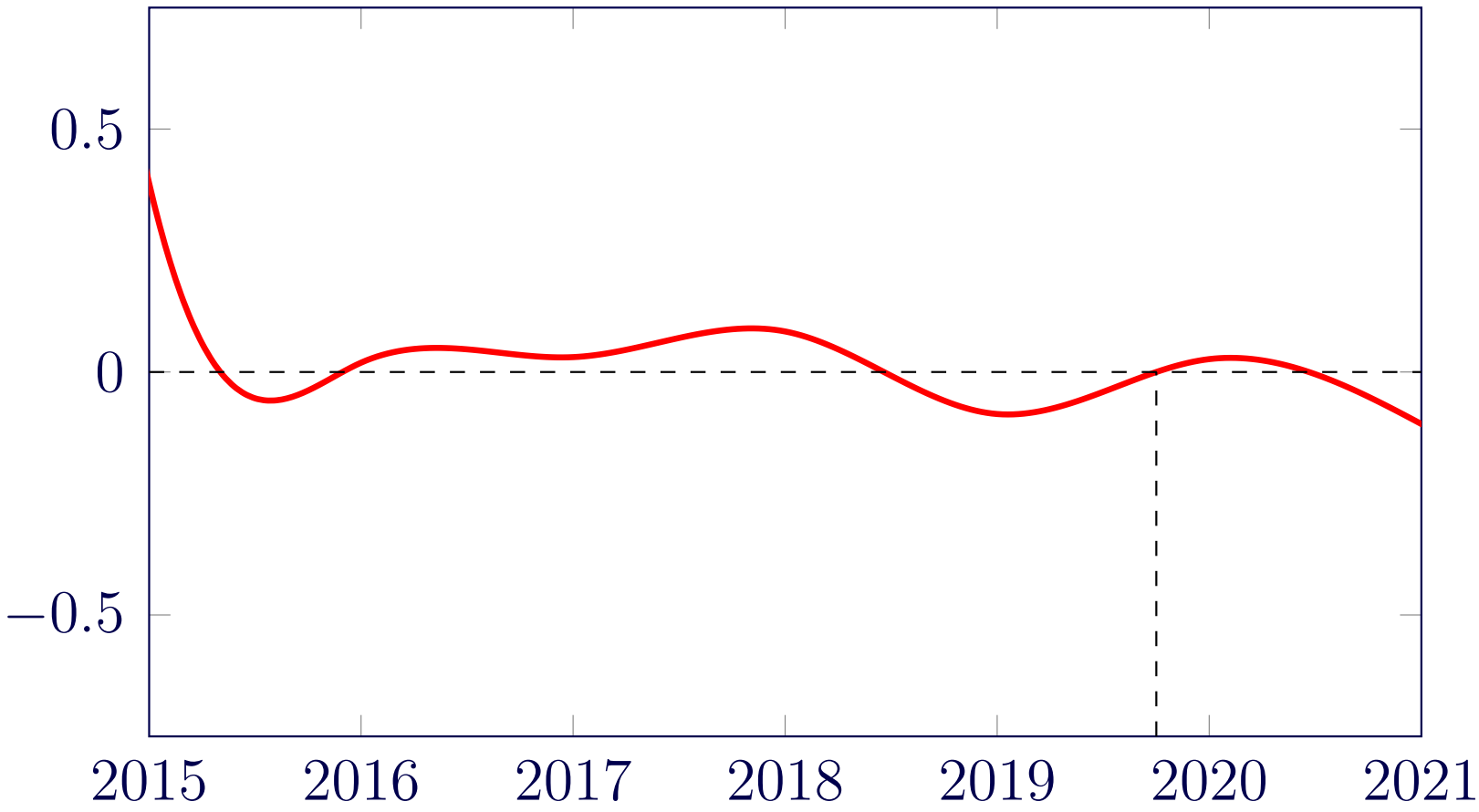
Continuous age-period model

LONGEVITAS

$$\log \mu_{x,y} = \underbrace{\log \mu_x}_{\text{Hermite age component}} + \underbrace{\sum_{j \geq 1} \kappa_{0,j} B_j(y)}_{\text{Schoenberg time component}}$$

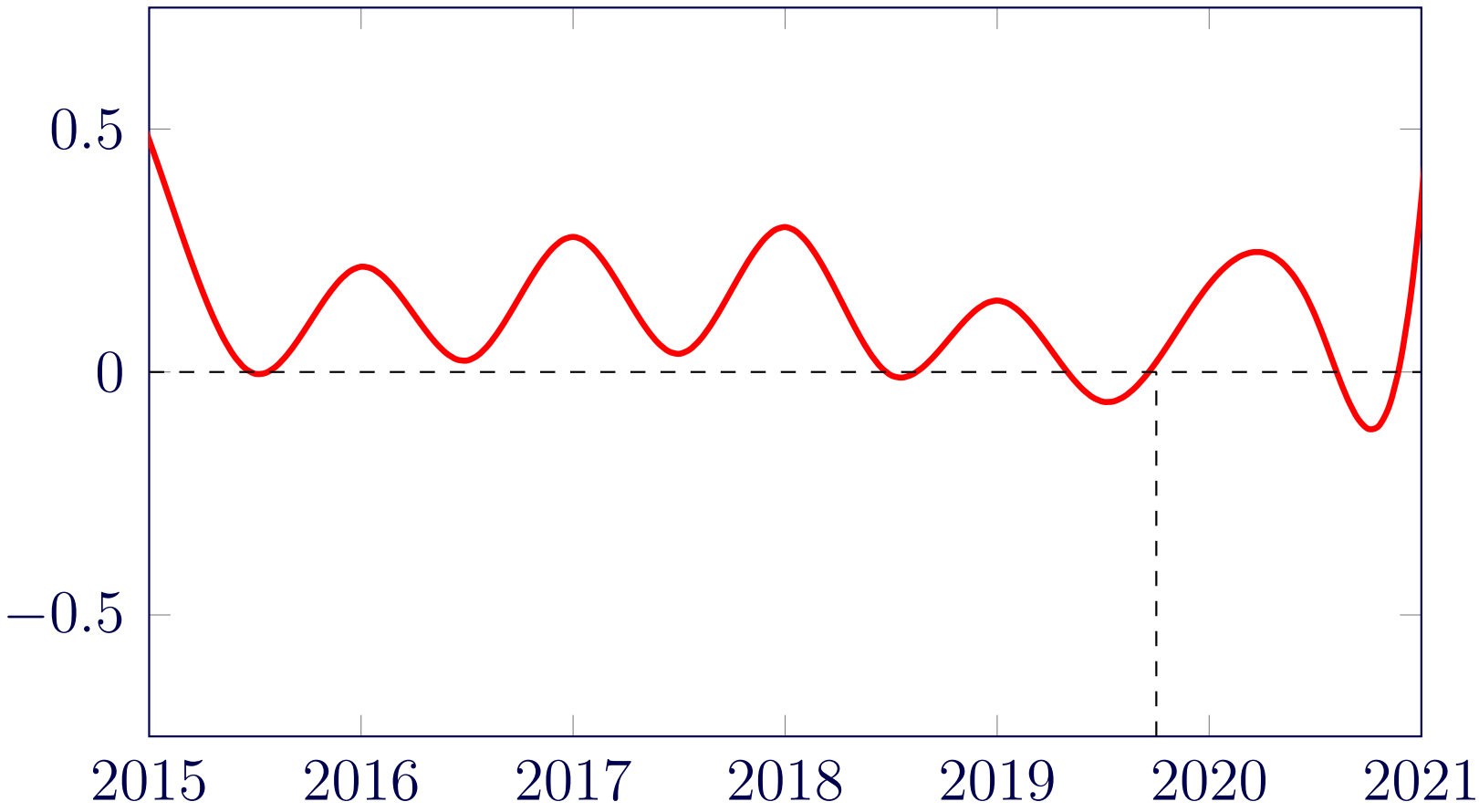
UK3, one knot per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



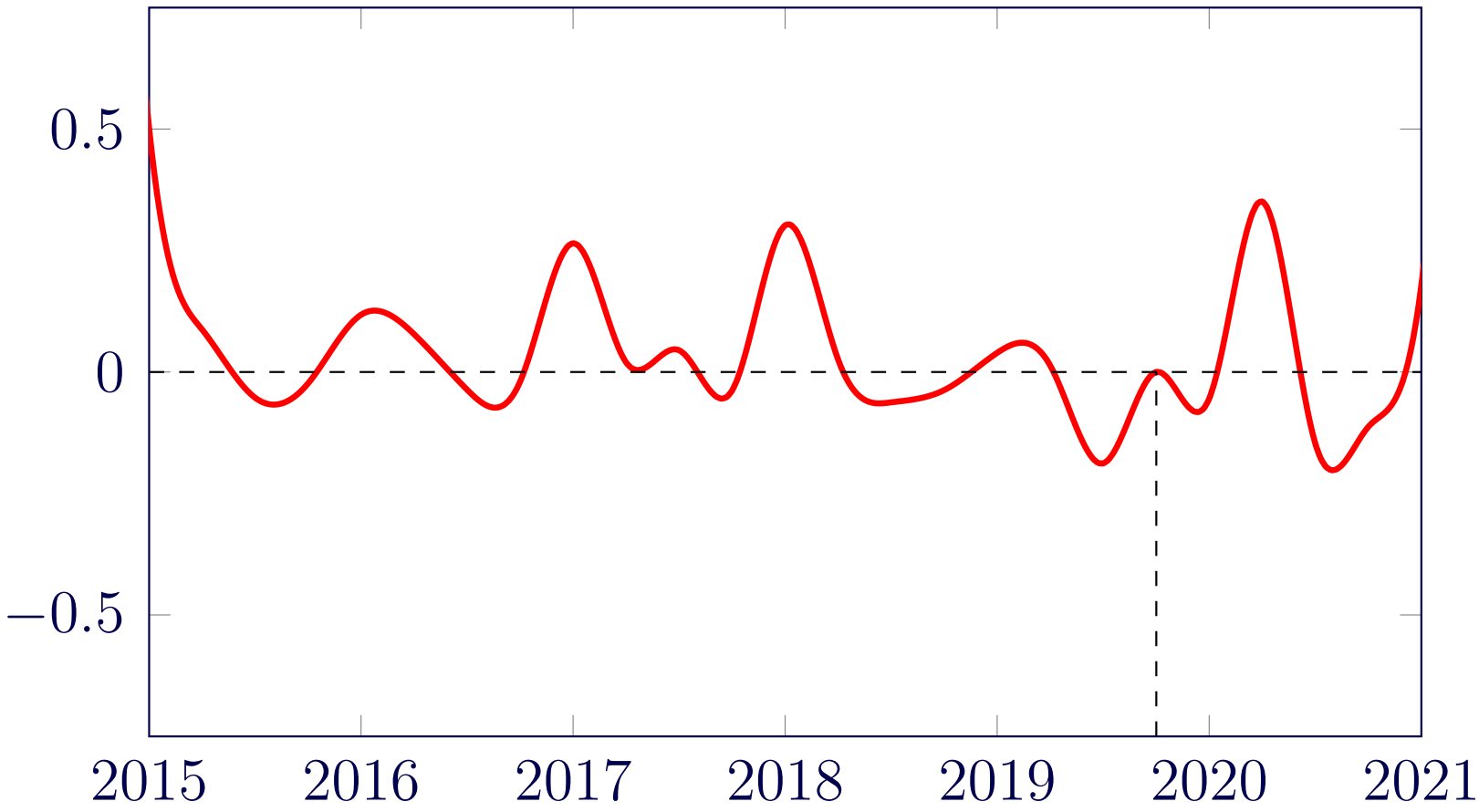
UK3, two knots per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



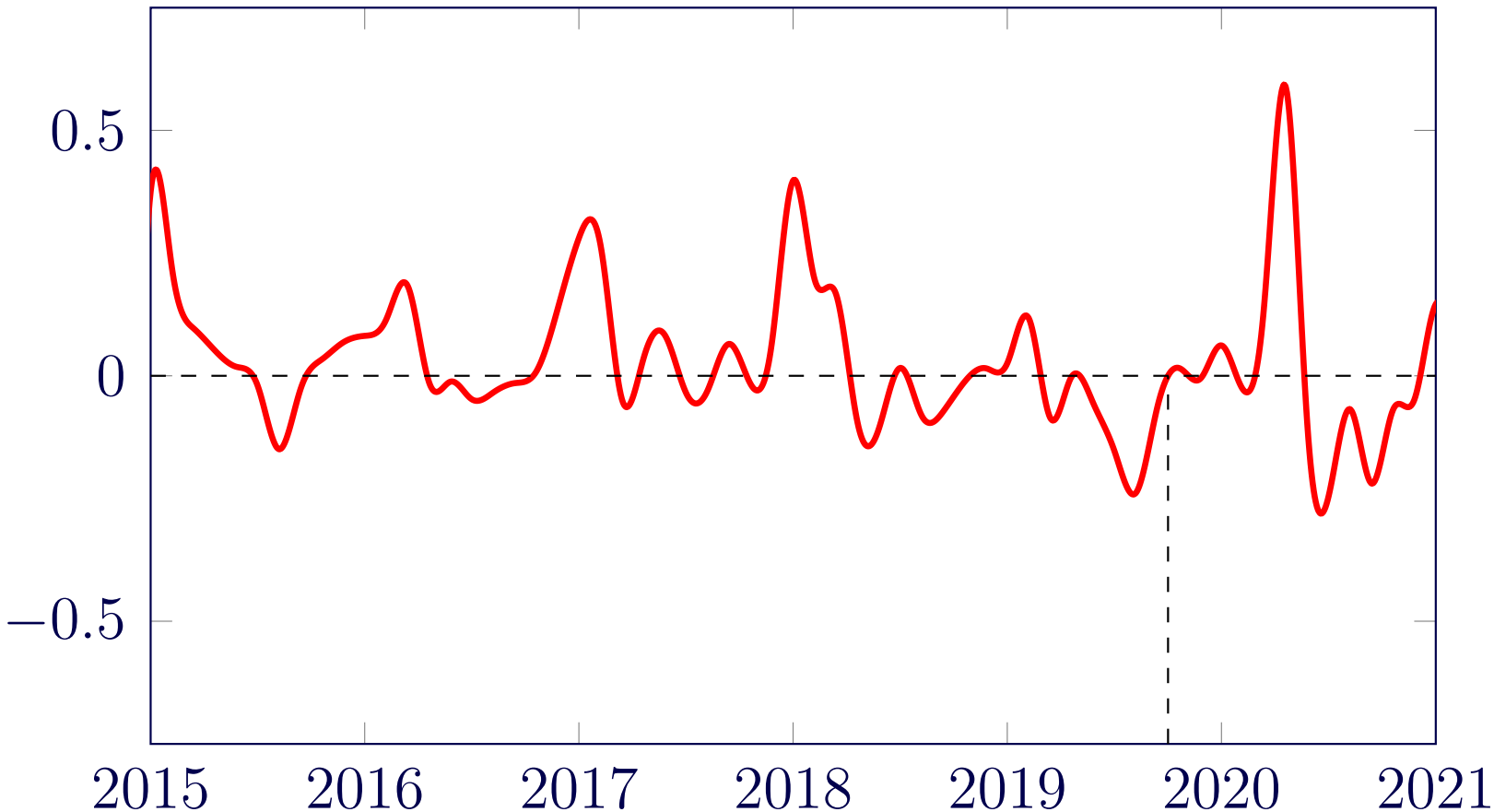
UK3, four knots per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



UK3, ten knots per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



Variable knot spacing

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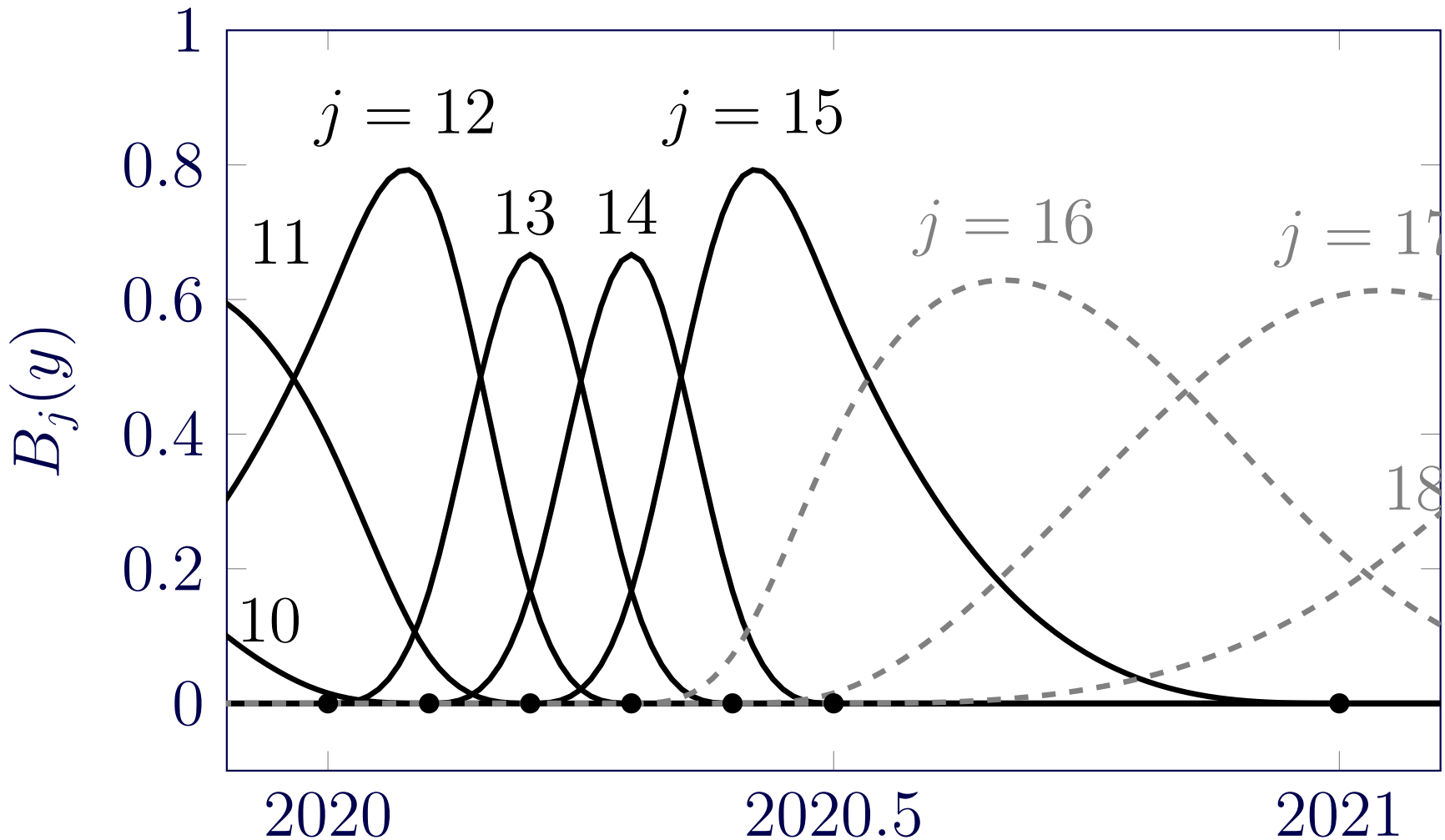
- Knots don't have to be equally spaced [Kaishev et al., 2016].
- Use two knots per year for seasonal variation...

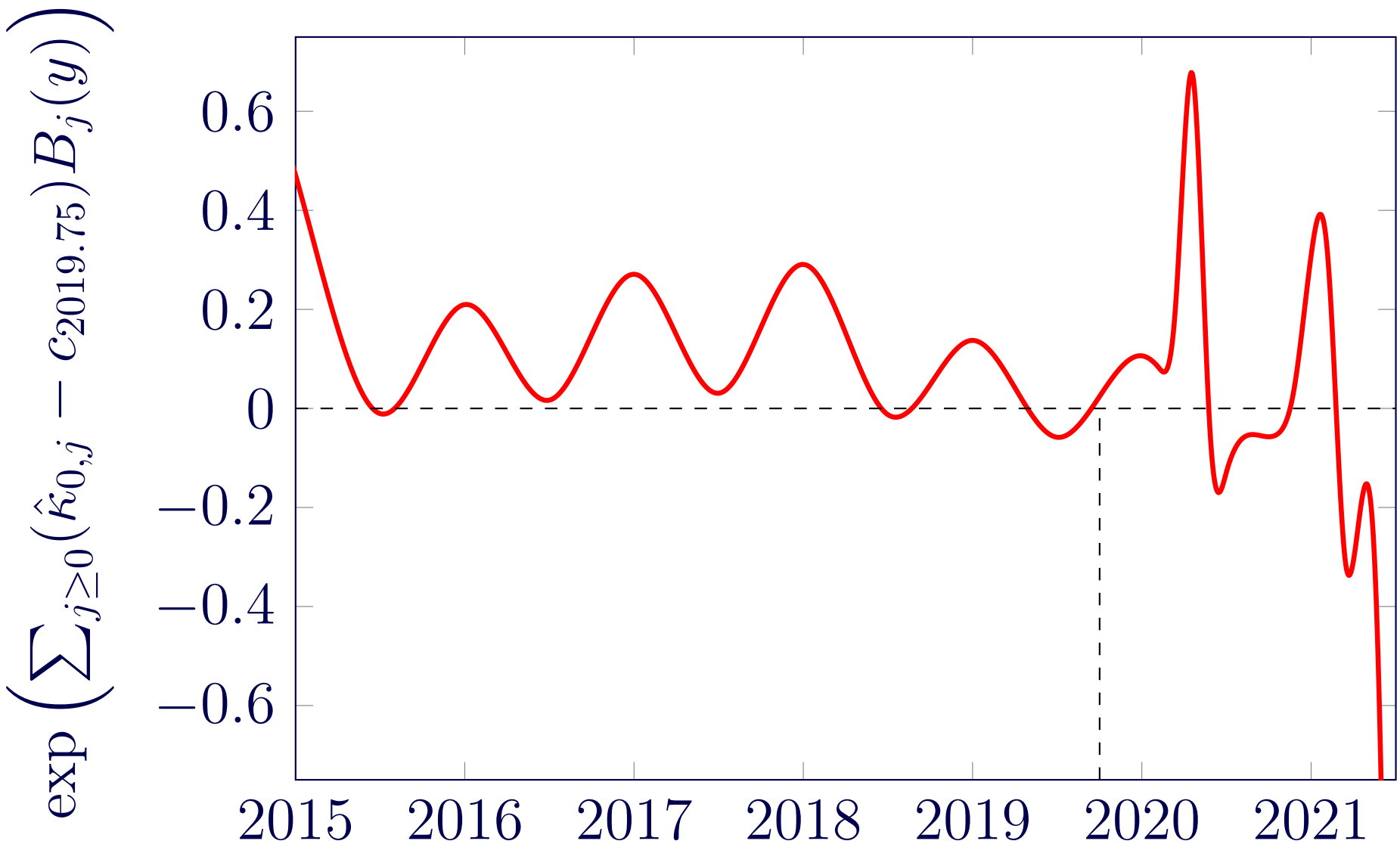
  ...and add knots where we know the shocks are.

Variable knot spacing

Part of a basis of nineteen variably-spaced cubic B-splines.



UK3 variable knots



Change of scale

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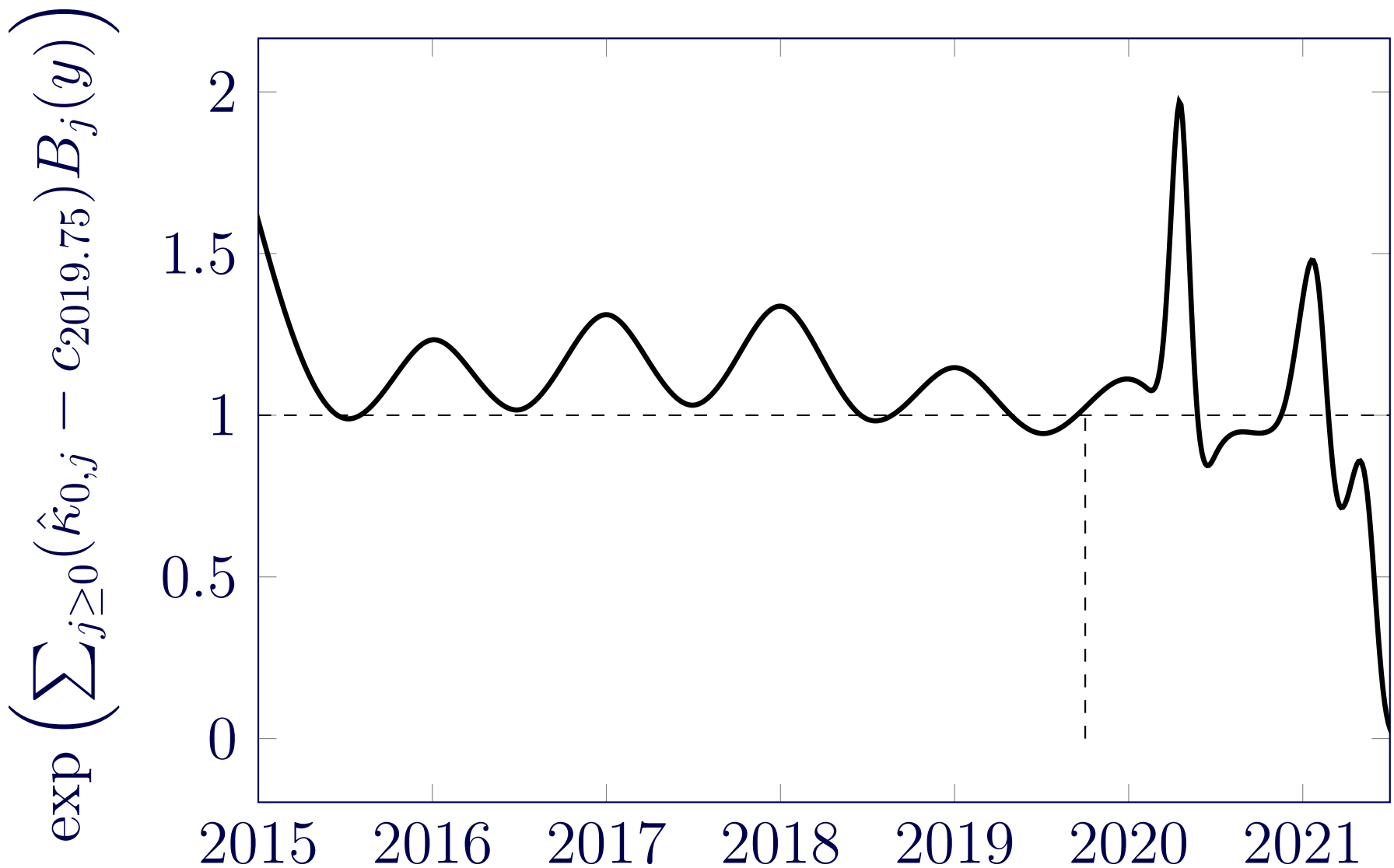
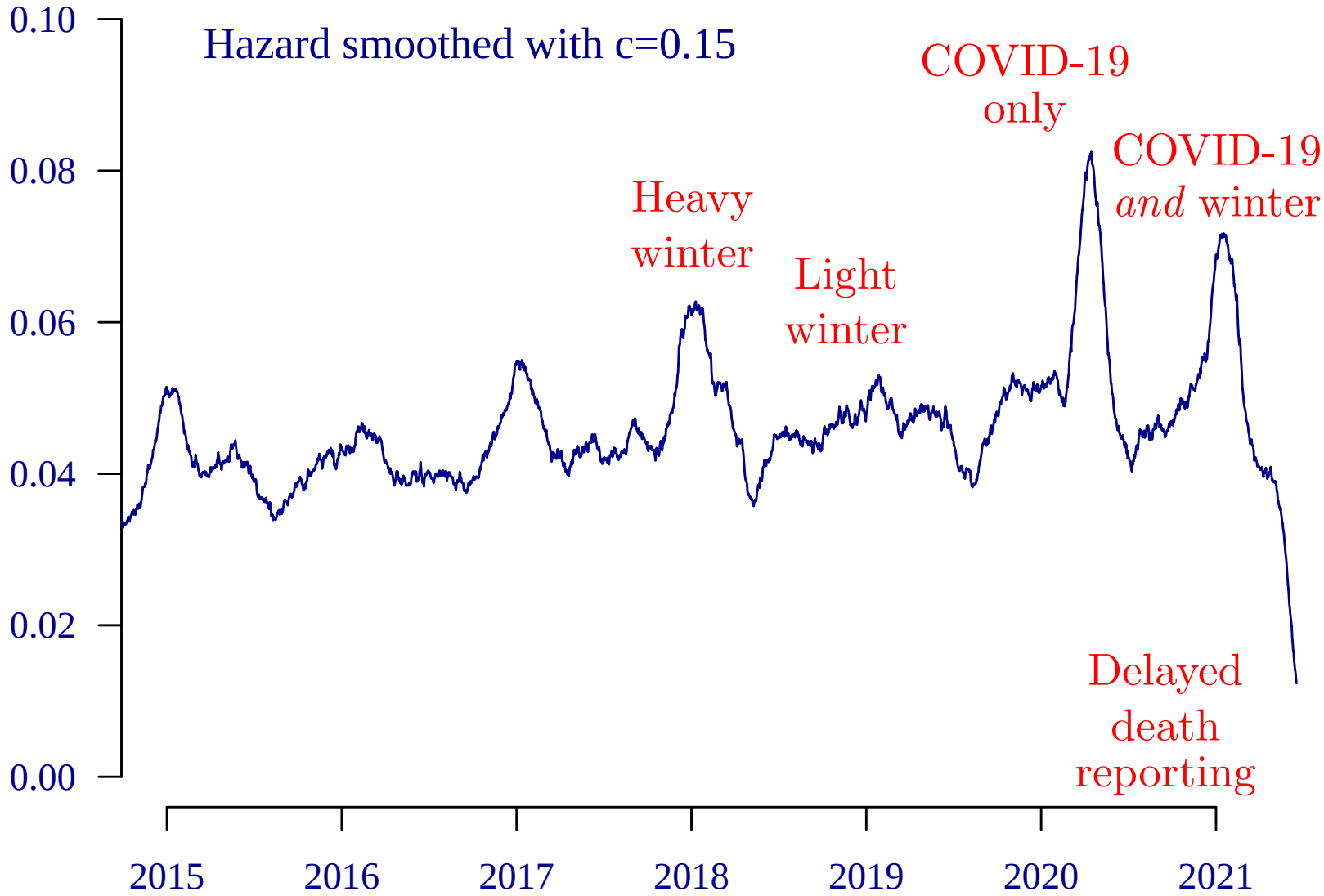
- Previous graphs showed addition to $\log \mu_x$.
- We can instead show the multiplier of μ_x ...

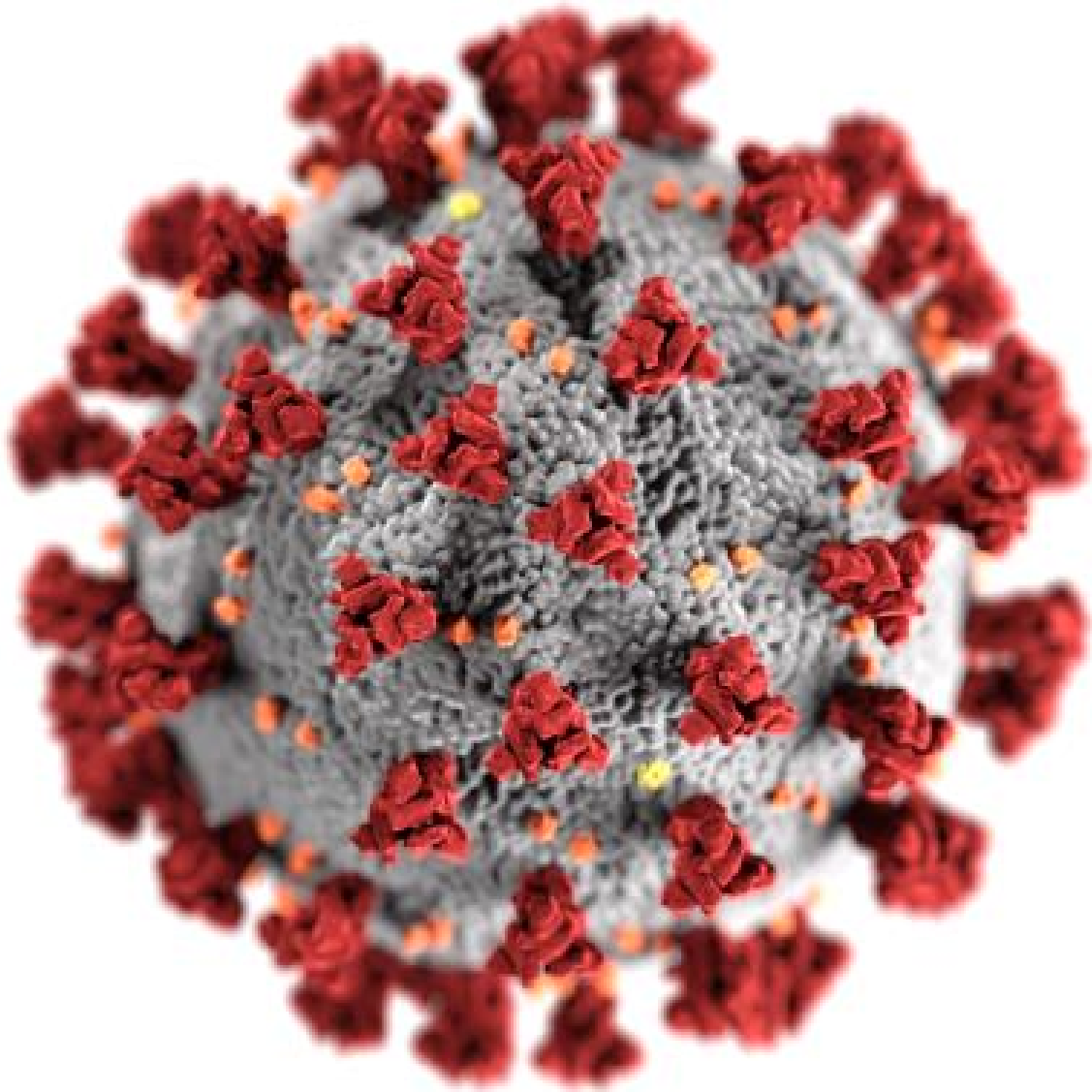
UK3 mortality multiplier



Daily hazard for UK annuities



8 Conclusions



Conclusions — I

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- Mortality shocks intense and short-term.
- Use continuous-time methods to handle rapid changes in time.

Conclusions — II

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Continuous age-period model:

Modelling by age

- Needs little flexibility.
- Use Hermite splines.

Modelling in time

- Needs lots of flexibility.
- Use Schoenberg [1964] splines.

Conclusions — III

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Continuous age-period model:

- Add knots around pandemic shocks.
- Exercise judgement as to normal mortality level.

References I

M. de Looper. *Seasonality of death*, volume Bulletin No. 3. Australian Institute of Health and Welfare, 2002. ISBN 978–1–74024–209–7.

V. K. Kaishev, D. S. Dimitrova, S. Haberman, and R. J. Verrall. Geometrically designed, variable knot regression splines. *Computational Statistics*, 31(3): 1079–1105, 2016. doi: 10.1007/s00180-015-0621-7.

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References II

S. J. Richards. Real-time measurement of portfolio mortality levels in the presence of shocks and reporting delays. *Annals of Actuarial Science*, 2022a. doi: 10.1017/S1748499522000021.

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I. J. Schoenberg. Spline functions and the problem of graduation. *Proceedings of the American Mathematical Society*, 52:947–950, 1964. doi: 10.1073/pnas.52.4.947.

References III

Coronavirus graphic  from CDC

More on longevity risk at www.longevitas.co.uk

9 Legal matters

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