ORDINE NAZIONALE DEGLI ATTUARI

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"Undertaking Specific Parameters or a Partial Internal Model under Solvency 2?"

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Agenda

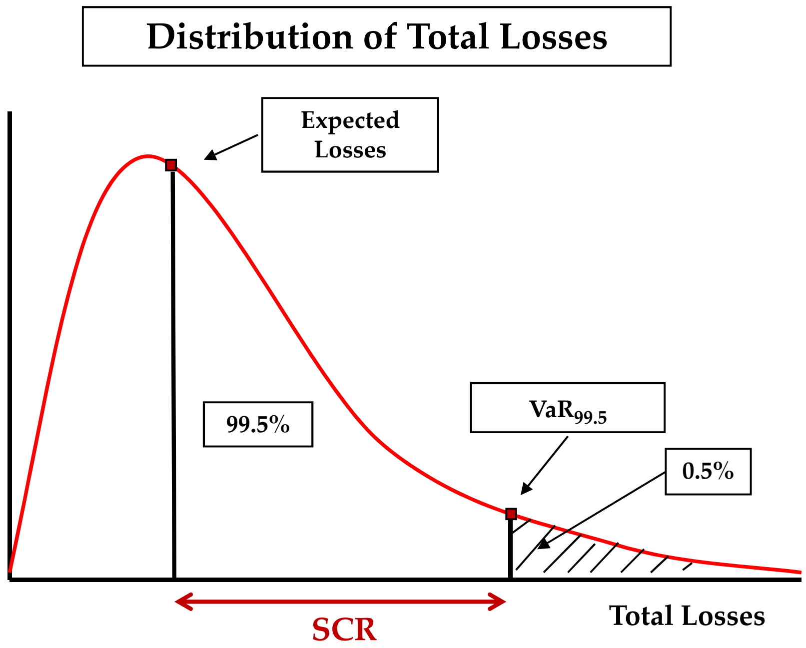
- Introduction
- USPs within Non-Life Premium Risk
- Towards a Partial Internal Model for Premium Risk
- GLM, GAM or Mixture of them?
- Case study: USPs versus PIRM
- How about volatility?
- Conclusions
- References

Introduction

Solvency 2 directive represents a complex project for reforming the present vigilance system of solvency for European insurance companies.

- **What?**

  A definition of a **Solvency Capital Requirement** ("SCR") as an economic capital to reflects the **true risk profile** of the undertaking, taking account of **all quantifiable risks**, as well as the net impact of **risk mitigation techniques**.



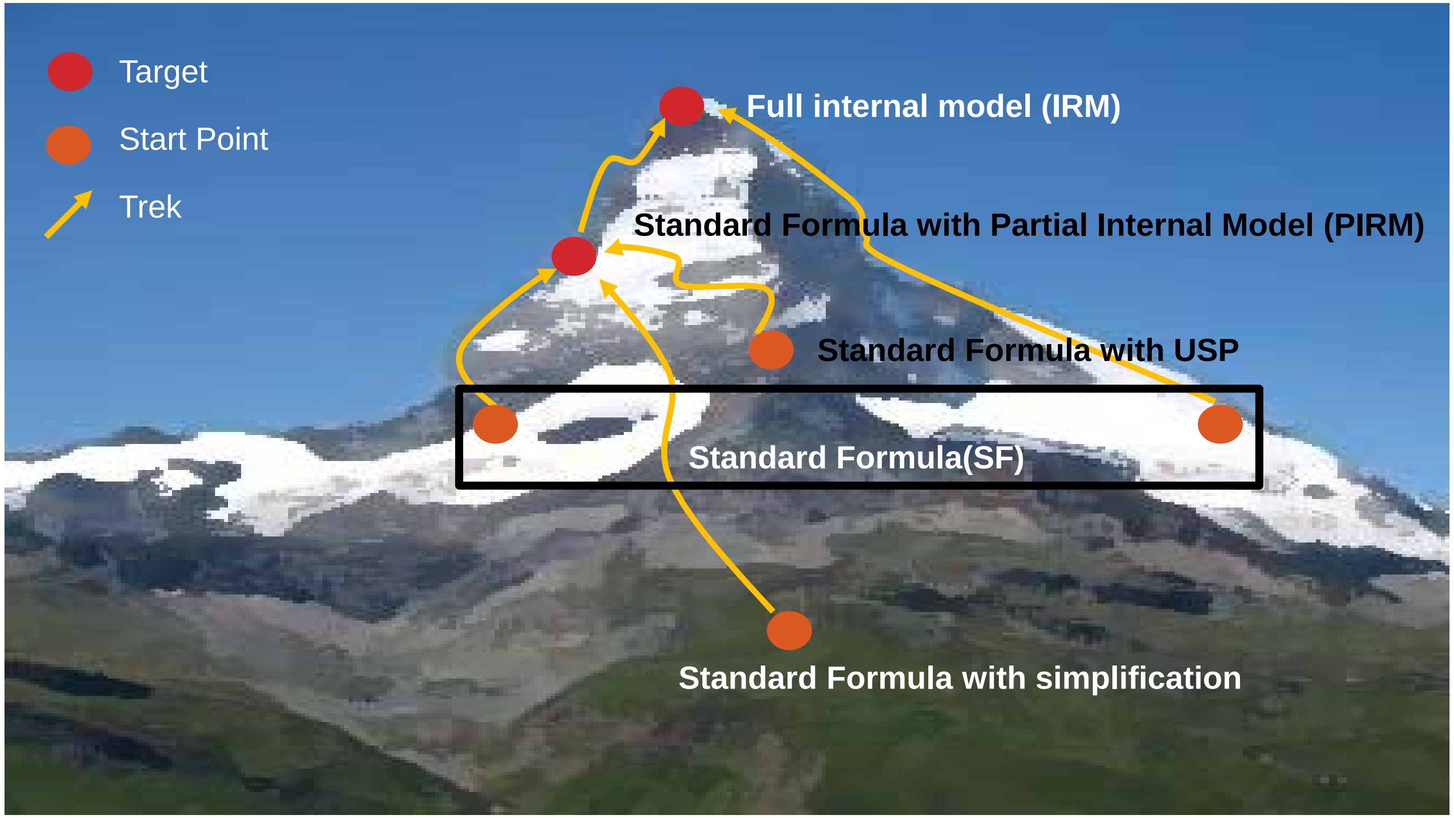
SCR

- **Time Horizon:** 1 year
- **Risk Measure:** Value at Risk
- **Probability of Ruin:** 0.5%

Introduction

- **How?**

In principle, Solvency 2 provides a range of methods to calculate the SCR which allows undertakings to choose a method that is **proportionate to the nature, scale and complexity of the risk** that are measured.

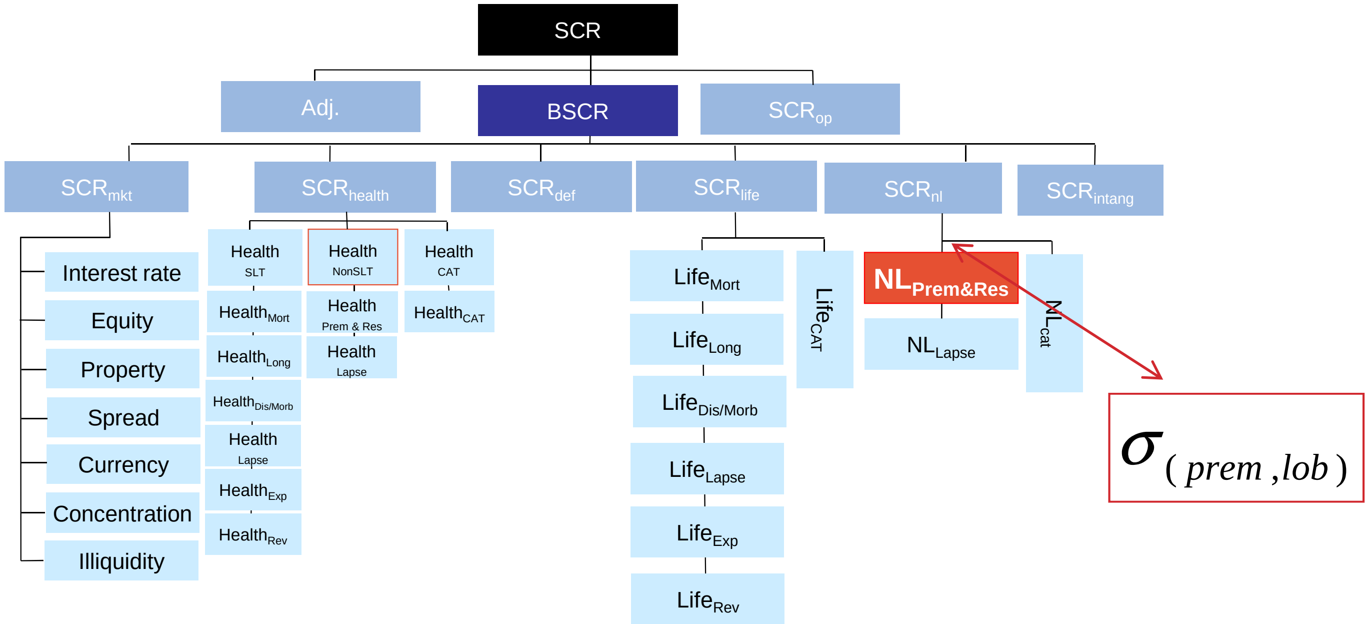


Introduction

The scope of this work is to compare the USPs' methodologies proposed in QIS5 with a PIRM for **premium risk**.

In particular we introduce this approach:

- for Personal Line insurance and/or for each product priced using regression techniques
- in order to stress the value of the model used from Pricing Staff



The premium risk is defined in the TS of QIS5: [2] *"Premium risk results from fluctuations in the timing, frequency and severity of insured events (…). Premium risk includes the risk that premium provisions turn out to be insufficient to compensate claims or need to be increased. Premium risk also includes the risk resulting from the volatility of expense payments.(…)"*.

USPs within Non-Life Premium Risk

$$\sigma_{(prem,LoB)} = c \cdot \sigma_{(U,prem,LoB)} + (1-c) \cdot \sigma_{(M,prem,LoB)}$$

Undertakings can replace a part of standard parameters with specific parameters (USP):

- According a criterion of credibility that depends on LoB and the length of the time series Nlob used for the estimation:

For GTPL, MTPL, Credit and Suretyship:

N_{lob}	5	6	7	8	9	10	11	12	13	14	≥15
C	34%	43%	51%	59%	67%	74%	81%	87%	92%	96%	100 %

For the other LoBs:

N_{lob}	5	6	7	8	9	≥10
C	34%	51%	67%	81%	92%	100%

- The data used for the calculation of undertaking-specific parameters should be **complete, accurate and appropriate**.

Which USPs to choose?

	Method 1	Method 2	Method 3
Assumptions	• The expected loss is proportional to the premium • The company has a different but constant expected loss ratio ("ELR") • The least squares fitting approach is appropriate	*In addition to the assumptions of Method 1*: • The distribution of the loss is lognormal • The maximum likelihood fitting approach is appropriate	• A separate analysis of the random variables number of claims and cost per claims • Based on the Swiss Solvency Test approach (Gisler, 2009)
Approach	• This method use the Ultimate Cost after one year by accident year • The Volatility depends on volatility year by year of Earned Premium or ELR • One year of adverse claim experience can produce material effects on the volatility • The company tends to reserve prudently in the first accident year	• It is a method similar to the previous	• The approach is significantly influenced by the variability in the exposure and in the number of claims • Requiring a greater number of information than the other two methods • If the company has reserved less prudently in the first development year, probably it has a volatility higher than the values obtained with Methods 1 and 2.

Towards a Partial Internal Model for Premium Risk

- **Why?**

With a (Partial) Internal Risk Model, an Insurance Company can **calibrate** the volatility of its business and risk profiles.

$$\boxed{\sigma^{SF}_{(prem,lob)}} \overset{?}{\geq} \boxed{\sigma_{(prem,lob)} = c \cdot \sigma_{(U,prem,lob)} + (1-c) \cdot \sigma^{SF}_{(M,prem,lob)}} \overset{?}{\geq} \boxed{\sigma^{PIRM}_{(prem,lob)}}$$

Premium Risk

Undertakings, therefore, will have to evaluate the **error in the assumptions, models or methods** used to **solve** a **pricing problems**.

$$X = \sum_{i=1}^{N} Y_i$$

Aggregate Claim Amount («ACA»)

$$P = E(X) = E(N) \cdot E(Y)$$

Pure Premium or Expected Value of ACA

$$\sigma^{PIRM}_{(prem,lob)} \overset{?}{=} \sigma(X)$$

Volatility or Premium Risk or standard deviation of ACA

Different prospective

The **new idea** of this presentation is represented by:

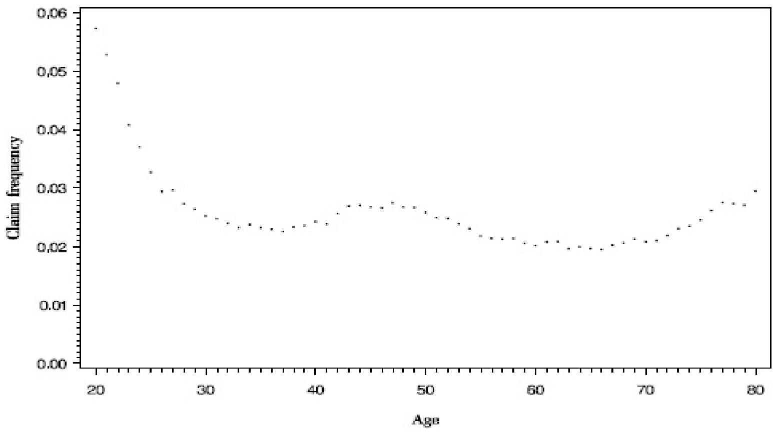
$$\boxed{\sigma^{PIRM}_{(prem,lob)} = \sigma(X)}$$

This seems to be in contrast with the definition of SCR, but in a PIRM: "*Insurance and reinsurance undertakings may use a different time period or risk measure (…) to calculate the Solvency Capital Requirement in a manner that provides policy holders and beneficiaries with a level of protection equivalent to that set out in Article 101*" (art. 122(1) S2 directive)

	Pricing	Premium Risk
Pricing Staff	i. are usual to do a GLM pricing exercise to determine the risk cost (**Pure Premium**); ii. make a number of adjustments to reflect IBNR, inflation trends, etc, to produce a **Total Premium**.	▪Using **GLM** for all risk profiles ▪**Updating the observed ACA at ultimate 1-year** ▪**Integrated Methods** to calculate Premium and its volatility.
Risk Management		

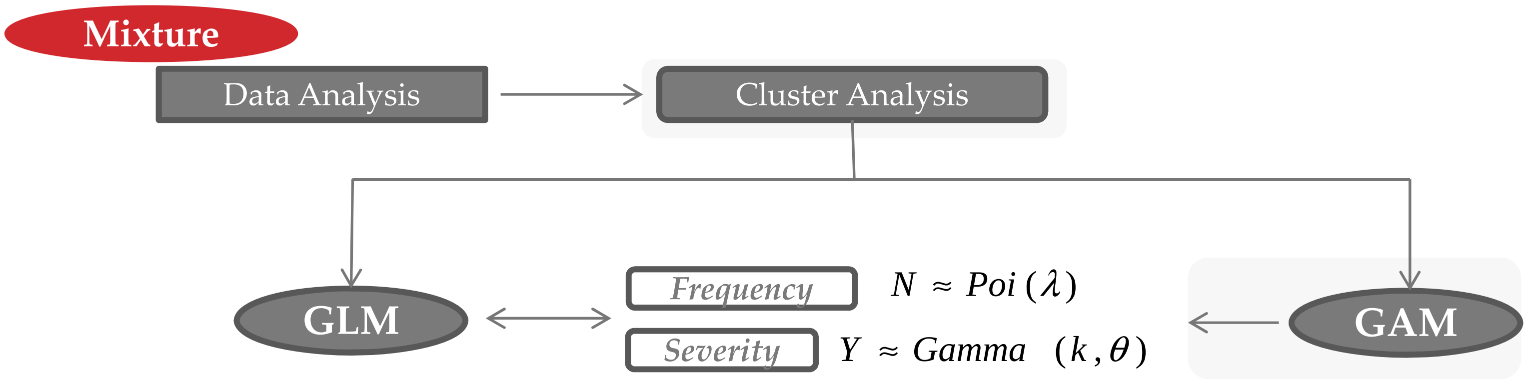
GLM, GAM or a Mixture of these?

GLM is a benchmark within this technical framework:



- How to manage continuous rating variables?
- Is Cluster Analysis a good solution?
- An obvious disadvantage is that the premium for two policies with different but close values for the rating variable may have substantially different premiums if the values happen to belong to different intervals

This is what we propose:



Case study: USPs versus PIRM

Perimeter

Hypothetical portfolio - Car

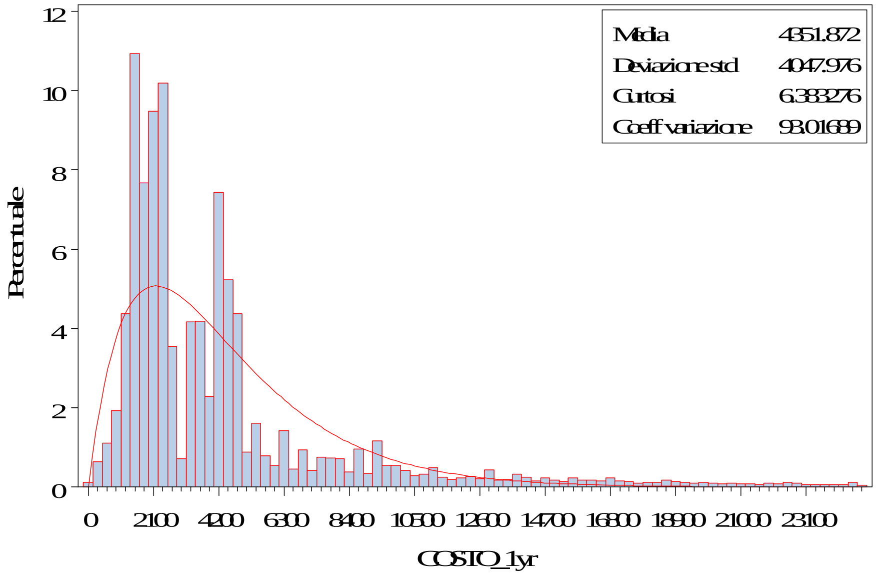
- Size: Medium (in term of Volume)
- Lob: Motor Third Party Liability solo
- Nlob: 15 years (*full credibility - USPs*)
- Insurance Portfolio: all risks which are associated claims and any (*ultimate 1-year*) costs incurred by year (*2009-2011 - PIRM*).

Purposes

- Estimating the impact of the use of USPs
- Defining the «Best Model» with goodness of fit analysis between a GLM or GLM after a GAM analysis («Mixture Model», «GLM(GAM)»)
- A comparison between the SF market parameters, USPs and the standard deviation of the model

Insurance Portfolio

With a *one-way* analysis we can appreciate the probabilistic assumptions:

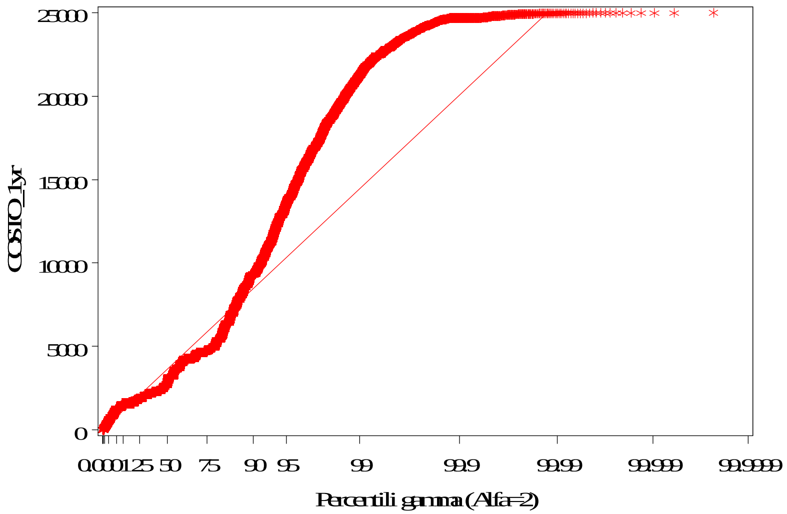


Moments

Total Portfolio	6,003,860	**Exposure**	3,525,452
N	271,602	**Mean Std Error**	8
Mean	4,352	**Std Dev**	4,048
CoV (%)	93	**Variance**	16,386,113
Skewness	2	**Kurtosis**	6

Distrib. Gamma

Parameter		Estimate
Treshold	Theta	0.9
Scale	Sigma	2172
Shape	Alfa	2
Means		4532
Std Dev		3076



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GLM vs Mixture Model

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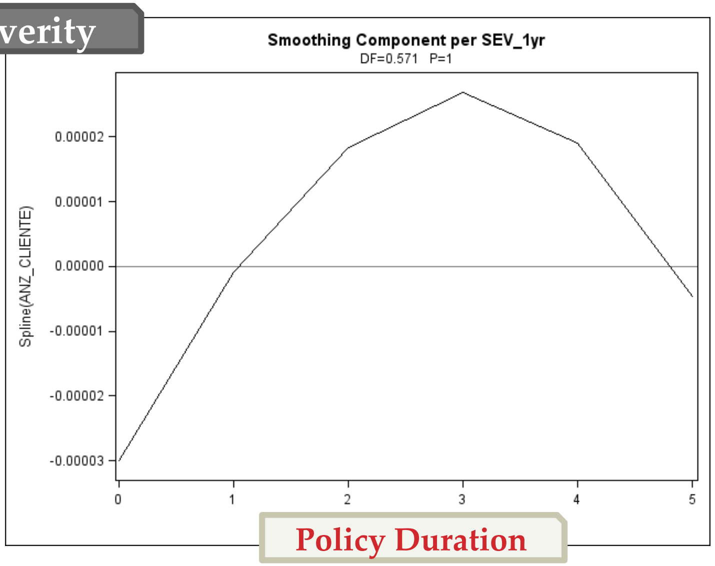
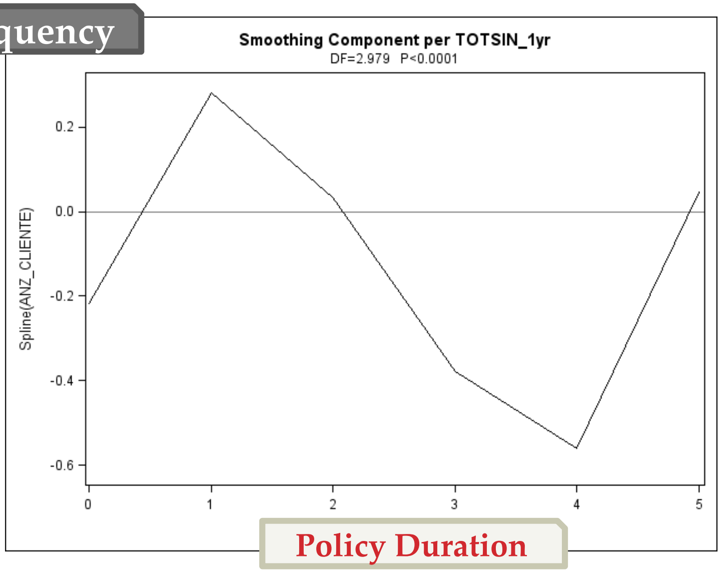
Severity



	GLM	GLM (GAM)
Rating Factors	9	9
Deviance	444,633	-1.0%
Scaled Deviance	782,667	0.0%
Dev(s) / GdL	1	0.0%
Chi-Squared	995,642	-1.0%
Scaled Chi-Sq.	1,752,581	-1.3%
Chi-SQ(s)/ GdL	2	9.0%
AIC	13,338,237	-2.1%
AICC	13,338,237	-2.2%
BIC	13,338,897	0.0%

Frequency



	GLM	GLM (GAM)
Rating Factors	9	9
Deviance	1,084,599	-1.5%
Scaled Deviance	866,789	0.0%
Dev(s) / GdL	1	0.0%
Chi-Squared	3,533,096	-1.3%
Scaled Chi-Sq.	2,823,576	-1.4%
Chi-SQ(s)/ GdL	3.3	-0.5%
AIC	1,196,920	-3.0%
AICC	1,196,920	- 2.8%
BIC	1,197,586	-1.0%

Premium Model

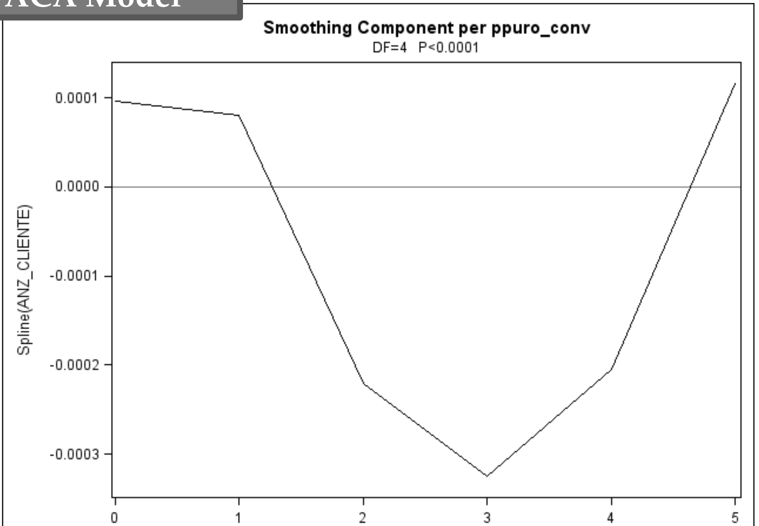
Before a convolution of the frequency/severity model:

	Observed	GLM	GLM(GAM)
Claim Cost (Y)	1,657,230,759	- 0.5%	0.1%
Number of Claim (N)	782724	0.0%	0.0%

- No difference for frequency
- Mixture Model is better than GLM

After the convolution between frequency and severity model in case of the *Gamma distribution*:

ACA Model



Policy Duration

	GLM	GLM (GAM)
Rating Factors	9	9
Deviance	138,501	-0.5%
Scaled Deviance	866,806	0.0%
Dev(s) / GdL	1	0.0%
Chi-Squared	194,889	-1.1%
Scaled Chi-Sq.	1,219,712	-1.5%
Chi-SQ(s)/ GdL	1	-0.0%
AIC	10,683,615	-0.6%
AICC	10,683,615	-0.0%
BIC	10,684,081	-0.7%

How about volatility?

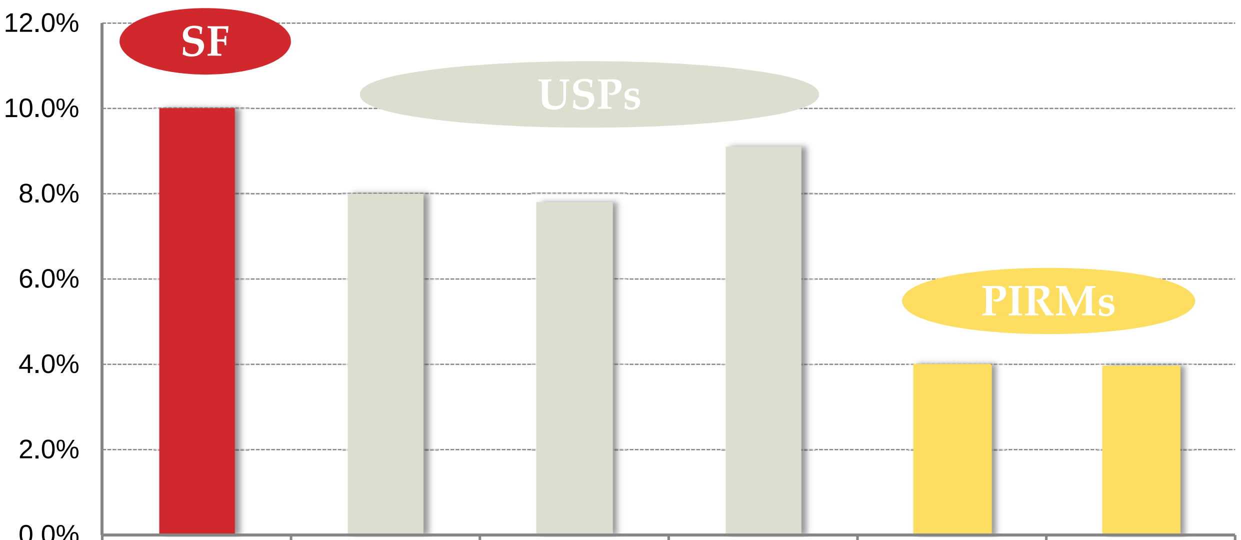
Perimeter

Case Study

- Size: Medium (in term of Volume)
- Lob: Motor Third Party Liability solo
- Nlob: 15 years

[2] →
$$\begin{cases} \sigma^{SF}_{(M,prem,lob)} = 10\% \\ c = 100\% \end{cases}$$

PIRMs allow a considerable saving in term of SCR for the *Premium Risk* thanks to a model already used by Pricing Staff



Market (SF)	USP Meth.1	USP Meth.2	USP Meth.3	GLM	GLM(GAM)
10.0%	8.0%	7.8%	9.1%	4.0%	3.9%

$$\sigma^{PIRM}_{(prem,lob)} = \sqrt{\left(\frac{E\left(ACA_{1-yr}\right)^2}{scale}\right)}$$

In this case study
→Small difference between the two PIRMs

Conclusions

	Standard Formula	USPs	PIRM
Pro	▪ Factor based or scenario based ▪ Quite simple to deploy	▪ Risk based on the historical data ▪ The volatility could be lower than SF	▪ Strength connection → Pricing/Premium Risk ▪ The volatility could be lower than USPs
Contro	▪ It couldn't take into account the real risk profile	▪ Data: complete, accurate and appropriate ▪ Supervisor Pre-Approval process	▪ More detailed Pre-Application process than USPs

Future Developments - PIRM

- Determine the 99.5% percentile of the ACA distribution
- Explore other statistical models to evaluate the random effects (e.g. GEE and GLMM)
- Evaluate different models for Attritional Losses and Large Losses (e.g. GLM within a Quasi-Likelihood approach)
- Use an (Ultimate 1-year) Aggregate Claim Amount net of the reinsurance
- Check the model for a total MTPL business (car, motorcycle, moped, etc.)
- Define a way to aggregate different LoBs and discover the correlation with Reserve/CAT risk

Thank you

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Main References

[1] CEIOPS (2010), "*QIS5 Technical Specifications*", Brussels, 5 July 2010

[2] Cerchiara, R.R. (2013), "*Come migliorare l'assorbimento di capitale?*", Proceedings of Congress "Quali sono le leve per un impiego più efficiente del capitale nel business danni", Towers Watson, Milano, Italy, 26 March 2013

[3] Daykin, C.D., Pentikainen, T., Pesonen, M. (1994), "*Practical Risk Theory for Actuaries*", 2nd edition, Boca Raton, Chapman & Hall/CRC

[4] EIOPA (2012), "*Technical Specifications for the Solvency II valuation and Solvency Capital Requirements calculations (Part I)*", EIOPA-DOC-12/362, 18 October 2012

[5] European Parliament (2009), "Solvency II Directive 2009/138/EC"

[6] Gigante, P., Picech, L., Sigalotti, L. (2010), "*La tariffazione nei rami danni con modelli lineari generalizzati*", Trieste: EUT editore

[7] Hastie, T.J., Tibshirani, R.J. (1990), "*Generalized Additive Models*", New York, Chapman & Hall

[8] Klugman, S. A., Panjer, H. H., Willmot, G.E. (2012), "*Loss Models: From Data to Decisions*", 4th edition, Hoboken (New Jersey), Whiley - SOA

[9] Nelder J.A., Wedderburn, R.W.M. (1972), "*Generalized Linear Models*", Journal of the Royal Statistical Society, Series A, 135, 370–384

[10] Ohlsson, E., Johansson, B. (2010), "*Non-Life Insurance Pricing with Generalized Linear Models*", EAA Series Textbook, Berlin, Springer