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# *Modelling Premium Risk for Solvency II: from Empirical Data to Risk Capital Evaluation*

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**Gian Paolo Clemente**
gianpaolo.clemente@unicatt.it
**Catholic University of Milan**
**Dept: Mathematics, Finance Mathematics and Econometrics**

**Nino Savelli**
nino.savelli@unicatt.it
**Catholic University of Milan**
**Dept: Mathematics, Finance Mathematics and Econometrics**

**Diego Zappa**
diego.zappa@unicatt.it
**Catholic University of Milan**
**Dept: Statistical Sciences**

## ❑ THE AIM OF THIS PAPER

Considering a real case study, we derive the capital requirement for premium risk for a single line of business through a partial internal model.

We focus on the analysis of claim size distribution by exploring the performance of alternative methodologies based on the Minimum Distance Approach to fit pure, mixtures and spliced distributions.

This topic is relevant in the actuarial literature in order to analyse the impact of a threshold to separate attritional and large claims in the identification of the claim size distribution to be used for risk capital evaluation (premium risk in Solvency II).

# Aggregate Claim Amount and claim-size distribution

- Both premium rating and capital requirement for Premium Risk are based on a proper valuation of the aggregate claim amount $X$ for each LoB.

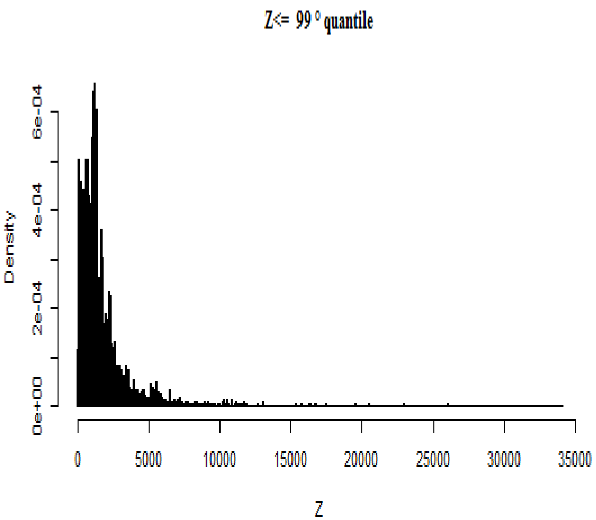
- The aggregate claim amount is well described by a compound process as the sum of a random number $K$ of random variable $Z_j$ :

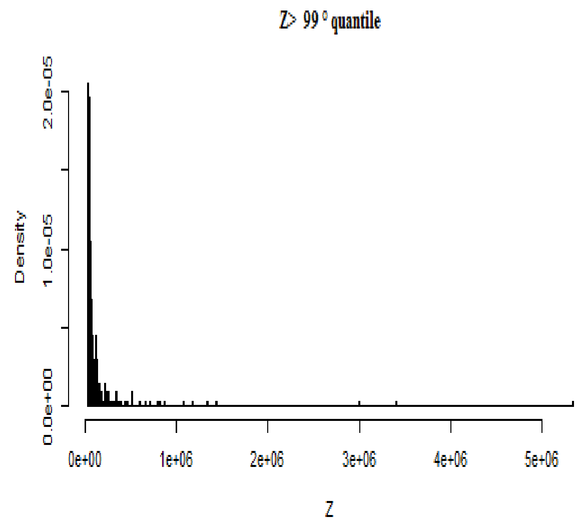
$$X = \sum_{j=1}^{K} Z_j$$

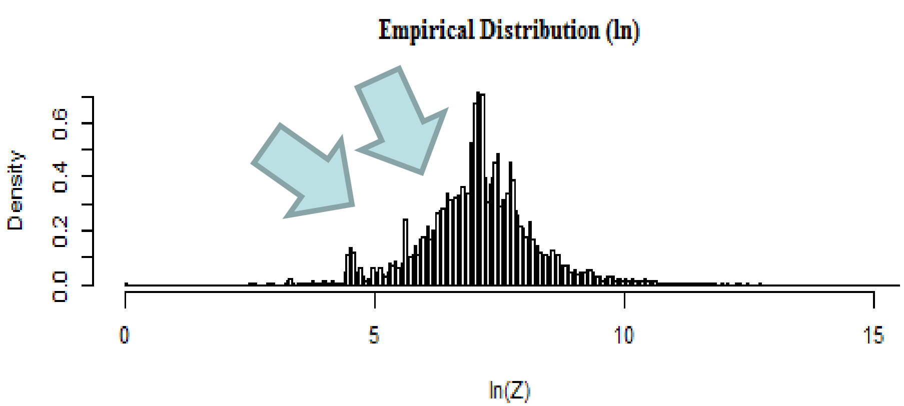
- Calibration of *claim-size distribution* ($Z_j$) is a key point in most applications:
  - no standard parametric model seems to emerge as providing an acceptable fit to both small and large claims;
  - the identification of the threshold to separate attritional and large claims is a challenge native property of spliced/mixture distributions;
  - claims are usually posted in the case reserve nearby a "round" number rather than its exact estimation leading to observe probability peaks in the empirical distribution;
  - it may be necessary to set up some constraints in parameter estimation (for example, the mean of empirical distribution equal to the mean of fitted distribution). That point is quite relevant for the due consistency with pricing analysis.

# Insurance Claims Dataset (Property LoB)

- Incurred amounts of claims (included ULAE and ALAE) of current year (2012) for a Property line of Business are reported in Figures (to consider only premium risk).
- Z represents the claim size distribution to be analysed in order to quantify the capital requirement.
- Many replicated values could be observed in empirical distribution (on log scale)
- A similar analysis has been developed by using the well-known Fire Danish claims




| Main Characteristics Empirical Distribution | |
|---|---|
| N. Obs | 33,701.00 |
| Mean | 3,616.45 |
| St. Dev | 44,029.28 |
| CV | 12.17 |
| Skewness | 80.46 |
| Kurtosis | 8,203.58 |
| 10th Percentile | 312.17 |
| 1st Quartile | 661.49 |
| Median | 1,215.32 |
| 3rd Quartile | 2,217.83 |
| 99th Percentile | 34,100.00 |
| 99.9% | 258,713.20 |
| 99.99% | 1,395,598.12 |
| Min | 1.00 |
| Max | 5,339,663.91 |

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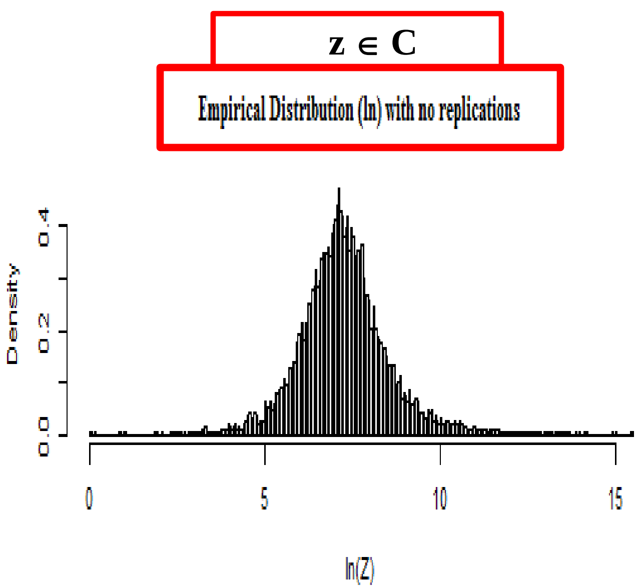
# A Mixed Distribution

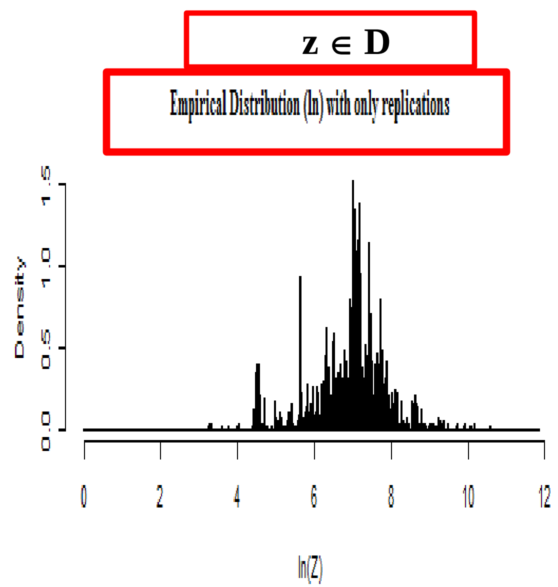
- Replicated values may condition the overall fitting process because of high densities concentrated in specific domain.

- Our proposal is to describe the distribution by using a mixed type distribution:
  - a discrete random variable with domain characterized by the peaks;
  - a continuous random variable (pure, mixtures or spliced distribution) for the remaining part.

- A random variable Z is a mixed type distribution if the domain S can be partitioned into subsets D and C with the following properties:
  - **D is countable and P(Z=z)>0 for $z \in D$**
  - **P(Z=z)=0 for $z \in C$**

- Thus, part of the distribution of Z is concentrated at points in a discrete set D, while the rest of the distribution is continuously spread over C.




**Main Characteristics**

| | No Repl. | Only Repl. |
|---|---|---|
| N. Obs | 18,370.00 | 15,331.00 |
| Mean | 5,171.47 | 1,753.19 |
| St. Dev | 59,493.41 | 3,753.37 |
| CV | 11.50 | 2.14 |
| Skewness | 59.75 | 16.15 |
| Kurtosis | 4,508.43 | 421.33 |
| 1st Quartile | 680.88 | 645.79 |
| Median | 1,328.56 | 1,167.46 |
| 3rd Quartile | 2,634.57 | 1,760.90 |
| 99.5th Perc. | 103,232.74 | 22,947.58 |
| 99.9% | 447,647.33 | 41,734.37 |
| 99.99% | 3,065,504.32 | 124,813.14 |

# ML vs MDA

- Having chosen a distribution, Maximum Likelihood (ML) is the most common method to estimate parameters.

  **ML** aims at estimating parameters such that they include the maximum information coming from the sample. The estimates drive the shape of the theoretical distribution.

- A viable alternative is represented by a Minimum Distance Approach (MDA). **The original MD method** (Parr (1985), Basu et al. (2011)) consists in solving the general unconstrained problem:

$$\min_{\theta} d(F_n(\boldsymbol{z}), F_Z(\boldsymbol{z};\theta)) \quad Z \in R_Z$$

- $\{z_1, z_2, \dots, z_n\}$ is an i.i.d. random sample from a population with cdf $F_Z(\boldsymbol{z};\theta)$
- $F_n(\boldsymbol{z}) = \frac{\sum_{i=1}^{n} I_{\{z_i \le z\}}}{n}$ is the empirical distribution (ecdf)
- $d(\cdot)$ is an appropriate distance function

- If it exists a $\hat{\theta} \in \Theta$ such that: $d\left(F_n(\boldsymbol{z}), F_Z\left(\boldsymbol{z};\hat{\theta}\right)\right) = \min_{\theta}\{d(F_n(\boldsymbol{z}), F_Y(\boldsymbol{z};\theta)); \theta \in \Theta\}$ then $\hat{\theta}$ is the minimum distance estimator of $\theta$

***Examples of d***

$$d(F_n(\boldsymbol{z}), F_Z(\boldsymbol{z};\theta)) := \begin{cases} CvM: \int [F_n(\boldsymbol{z}) - F_Z(\boldsymbol{z};\theta)]^2 dz \\ KS: \ sup|F_n(\boldsymbol{z}) - F_Z(\boldsymbol{z};\theta)| \\ AD: \dfrac{\int [F_n(\boldsymbol{z}) - F_Z(\boldsymbol{z};\theta)]^2 dz}{F_Z(\boldsymbol{z};\theta)[1 - F_Z(\boldsymbol{z};\theta)]} \end{cases}$$

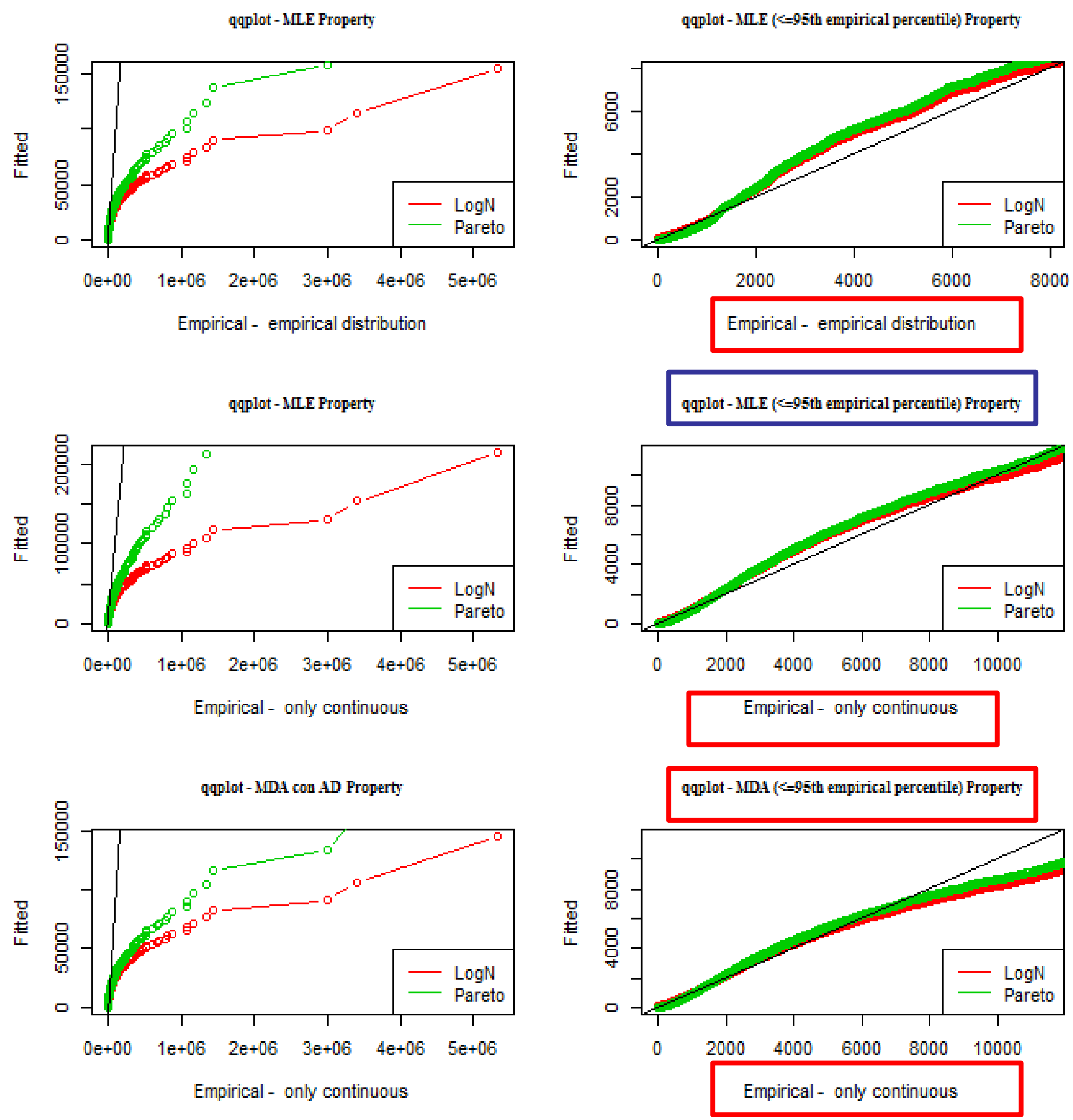
*For further distance measures see Titterington et al. (Table 4.5.1)*

# Univariate Distributions

- **Two classical univariate distributions (Pareto Type II and LogNormal)** have been fitted to:
  - the empirical distribution (Z)
  - only the distribution without replications (Z ∈ C)

  by using a classical MLE approach and original MD method with a AD loss function.

- As shown by the q-q plots, both distributions assure a discrete fitting to the distribution with no replications only on the body (until 95° percentile more or less) and a significant underestimation on the tails.

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# Main approaches proposed by actuarial literature

- A two step strategy, based on a ***separate evaluation of attritional and large claims*** is a standard way to describe claim-size distribution:
  - ***Several distributions*** for modelling positive and right-skewed data are proposed in actuarial science (see Klugman et al. (2010))
  - ***Extreme value theory*** and Generalized Pareto distributions are used to describe large claims exceeding a fixed threshold (see McNeil (1996), Embrechts et al. (1997), Gonzalez et al. (2013)).

- Other approaches are based on mixtures and composite distributions:
  - **Frigessi et al** (2002) propose a weighted mixture model based on a GPD and on a light-tailed distribution
  - **Cooray, Ananda** (2005) combine LogNormal and Pareto distributions by fixing the proportion of large claims
  - **Teodorescu, Vernic** (2007), **Teodorescu, Panaitescu** (2007), **Vernic et al.** (2009) provide different mixtures based on Exponential-Pareto, Weibull-Pareto and LogNormal-LogNormal.
  - **Scollnik** (2007) expands Cooray & Ananda paper by estimating the threshold directly by data and propose a LogNormal-GPD version.
  - **Pigeon, Denuit** (2011) extend the LogNormal-Pareto model by assuming a random threshold (Gamma or LogNormal distributed)
  - **Nadarajah, Bakar** (2012) try to improve fitting to Danish Data by using a composite distribution based on a LogNormal and various distributions for large claims. They assume the LogNormal-Burr as the best one for Danish Data.

All papers use ***maximum log-likelihood (ML) approach*** to estimate parameters.

Most of them use the public Fire Danish losses database to test the performance of their own method

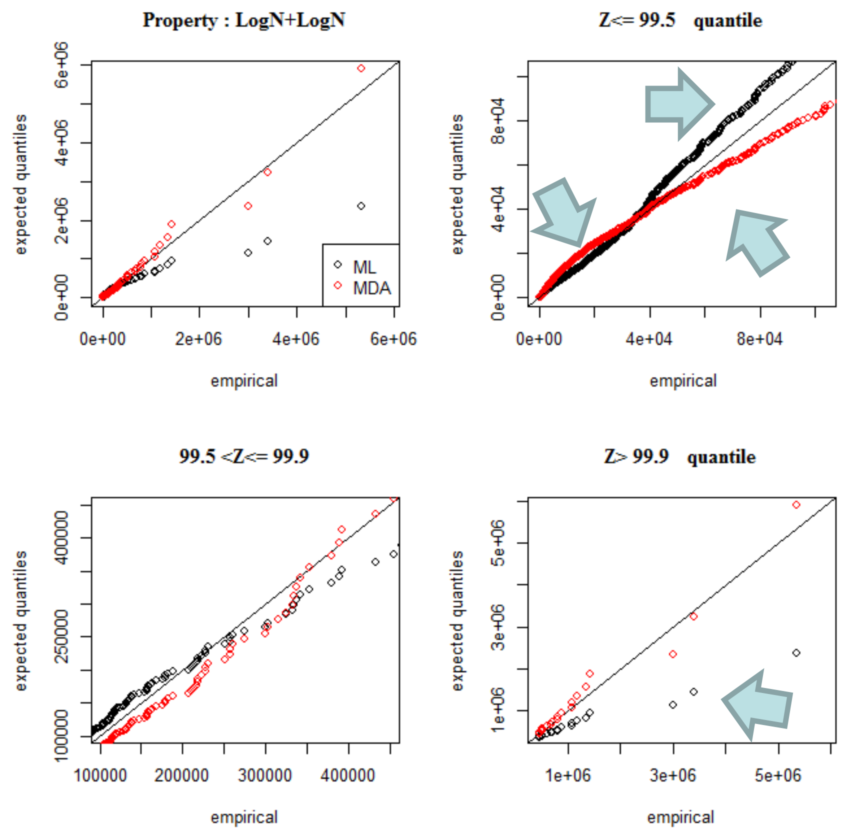
# Mixture LogNormal-LogNormal
## ML vs MDA

- **A LogNormal-LogNormal Mixture** have been applied by **using ML and MDA** (CvM Loss Function)

$$F_Z(z) = \pi F_{Z_1}(z) + (1-\pi)F_{Z_2}(z)$$
$$\text{with } 0 < \pi < 1$$

- ML estimates has been computed **by using the EM algorithm**

- ML provides an underestimation of a tail, while MDA assures a better fitting on extreme values and an overestimation on the body.




| Quantiles | Empirical | ML | MDA(CvM) |
|---|---|---|---|
| 50% | **1,329** | **1,315** | **1,484** |
| 90% | **5,975** | **6,263** | **9,341** |
| 99% | 51,988 | 60,027 | 49,310 |
| 99.50% | 103,233 | 116,006 | 84,244 |
| 99.90% | 447,647 | 370,824 | 450,898 |
| 99.99% | **3,065,504** | **1,181,437** | **2,495,439** |

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# Spliced Distribution

- In literature a modern approach is based on the estimation of spliced distributions.

The corresponding probability density function for a random variable Z with domain $(c_0, c_2)$ is defined as:

$$f_Z(z) = \begin{cases} \pi f^*_{Z_1}(z) & c_0 \leq z < c_1 \\ (1-\pi) f^*_{Z_2}(z) & c_1 \leq z < c_2 \end{cases}$$

- $\pi$ is the weight
- $c_i$ is the limit of the domains
- $f^*_{Z_i}$ is a truncated probability density function

$$f^*_{Z_i}(z) = \frac{f_{Z_i}(z)}{\int_{c_{i-1}}^{c_i} f_{Z_i}(z)dz} \text{ with } i = 1,2$$

- This distribution allows to identify a threshold of separation between the two components.

- Further conditions must be imposed if continuity and differentiability at the knots are needed (see Scollnick (2007) , Denuit et al. (2011), Nadarajah, Bakar (2012) for Composite distributions such as LogNormal-LogNormal and LogNormal-GPD)

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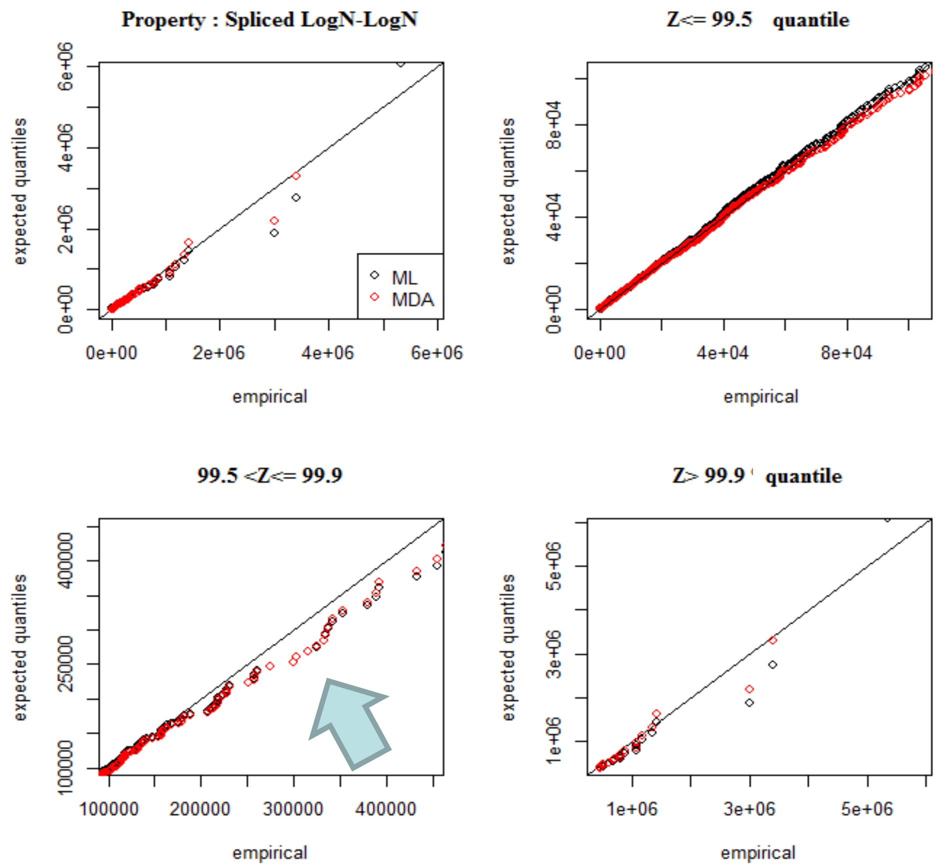
# Spliced LogNormal-LogNormal

## ML vs MDA

- **A LogNormal-LogNormal Spliced Distribution** have been applied by **using ML and MDA** (CvM Loss Function)

- Both ML and MDA provides a better fitting (w.r.t. Mixtures) on the body with an underestimation of right tail.




| Quantiles | Empirical | ML | MDA(CvM) |
|---|---|---|---|
| 50% | **1,329** | **1,315** | **1,325** |
| 90% | 5,975 | 6,208 | 5,950 |
| 99% | 51,988 | 54,306 | 52,348 |
| 99.50% | 103,233 | 100,832 | 97,776 |
| 99.90% | **447,647** | **387,046** | **395,419** |
| 99.99% | **3,065,504** | **2,021,142** | **2,357,818** |

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# Minimum Distance Approach
# weighted $L_q$ norm distances (WMDA(q,p))

- A generalization of the approach could be derived by assuming:

$$\min_{\theta} \sum_{i=1}^{n} |F_n(z_i) - F_Z(z_i, \theta)|^q \, \mathrm{w}(z_i, p)$$

where: $q>0$, $p \geq 0$ , $\mathrm{w}(z_i, p) > 0$

- If both $q=2$ and $\mathrm{w}(z_i, p) = \frac{1}{n}$, the approach leads to CvM loss distance.

  For $q=1$, $\mathrm{w}(z_i, p) = \frac{1}{n}$, we have the Wolfowitz distance.

- We have investigated different choices of $\mathrm{w}(z_i, p)$:
  - $\mathrm{w}(z_i, p = 1) \propto z_i$
  - $\mathrm{w}(z_i, p) \propto z_i^p$
  - $\mathrm{w}(z_i, p) \propto z_i \, I_{z_i \leq z_t} + z_i^p \, I_{z_i > z_t}$ ⇦ Useful to control tail estimation for risk analysis

- Further research will regard appropriate priors for $\mathrm{w}(z_i, p)$ under a bayesian framework.

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# Normalized weighted $L_q$ norm distances

- Since for different $q$, $p$, distances are not fully comparable, the choice of the best fitting for different combination of $q$, $p$ will correspond to the solution with the minimum ratio between the distance and the corresponding maximum value.

- For any q-norm and weighted q-norm, the following relations hold:
  - $\|x_i\|_q = \left(\sum_{i=1}^{n}|x_i|^q\right)^{1/q} \le \sqrt[q]{n}\max(|x_i|) \quad q \ge 1$
  - $\|x_i\|_{q,w} = \left(\sum_{i=1}^{n}|x_i|^q\, w_i\right)^{1/q} = \left(\sum_{i=1}^{n}\left|x_i \cdot {w_i}^{1/q}\right|^q\right)^{1/q} \le \sqrt[q]{n}\max\left(\left|x_i \cdot {w_i}^{1/q}\right|\right) \quad w_i \ge 0$

- We derive then the statistics :

$$\frac{\sum_{i=1}^{n}|F_n(z_i) - F_Z(z_i,\theta)|^q \cdot w(z_i,p)}{n\left(\max\left(|F_n(z_i) - F_Z(z_i,\theta)| \cdot w(z_i,p)^{\frac{1}{q}}\right)\right)^q} \le 1$$

The lower is the ratio the better is the fitting

# Diagnostics

- We introduce further naive diagnostics, other than q-q plot in order to compare different models.

  - qq- residuals

$$D = \left( \sum_{i=1}^{n} \left( z_i - F_Z^{-1}\left(F_n(z_i);\ \hat{\theta}_{p,q}\right)\right)^2 \right)^{1/2}$$

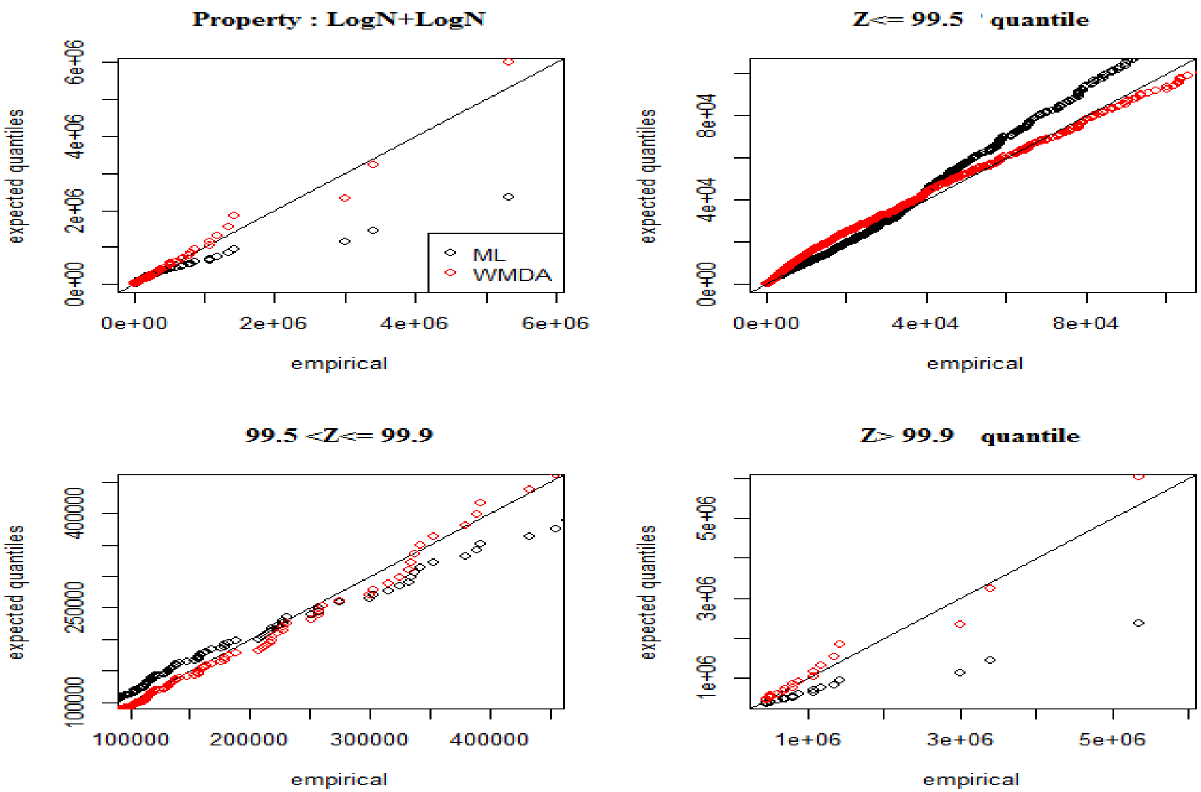
i.e. the Euclidean distance between the data and the quantiles obtained from $F_Z$ where $\hat{\theta}_{p,q}$ are the corresponding parameter estimates.

- D is heavily influenced by extreme right values.
  Alternative indexes are:
  - The mean of the raw residuals
  - The estimate of the slope, $\beta_1$, of the constrained least squares regression line:

$$z_{i(\alpha)} = \beta_0 + \beta_1 \hat{z}_{i(\alpha)} + \varepsilon_i \qquad s.t.\ \beta_0 = 0$$

where $\hat{z}_{i(\alpha)}$, $z_{i(\alpha)}$ are the α-th quantiles of the fitted model and of the empirical distribution, respectively.

# ML vs WMDA
## Mixtures (LogNormal-LogNormal)




- Weighted MDA have been initially applied to a LogNormal-LogNormal mixture
- The best combination of (q,p) was equal to (2,1.7) and it allowed to derive a behaviour better than both ML and classical MDA.

| Quantiles | Empirical | ML | MDA(CvM) | WMDA |
|---|---|---|---|---|
| 50% | **1,329** | 1,315 | 1,484 | 1,231 |
| 90% | 5,975 | 6,263 | **9,341** | 7,608 |
| 99% | 51,988 | 60,027 | **49,310** | 54,083 |
| 99.50% | 103,233 | 116,006 | **84,244** | 95,407 |
| 99.90% | **447,647** | **370,824** | 450,898 | **452,217** |
| 99.99% | **3,065,504** | **1,181,437** | **2,495,439** | **2,975,952** |
| 99.999% | **4,983,665** | **2,198,212** | **5,402,755** | **5,299,414** |

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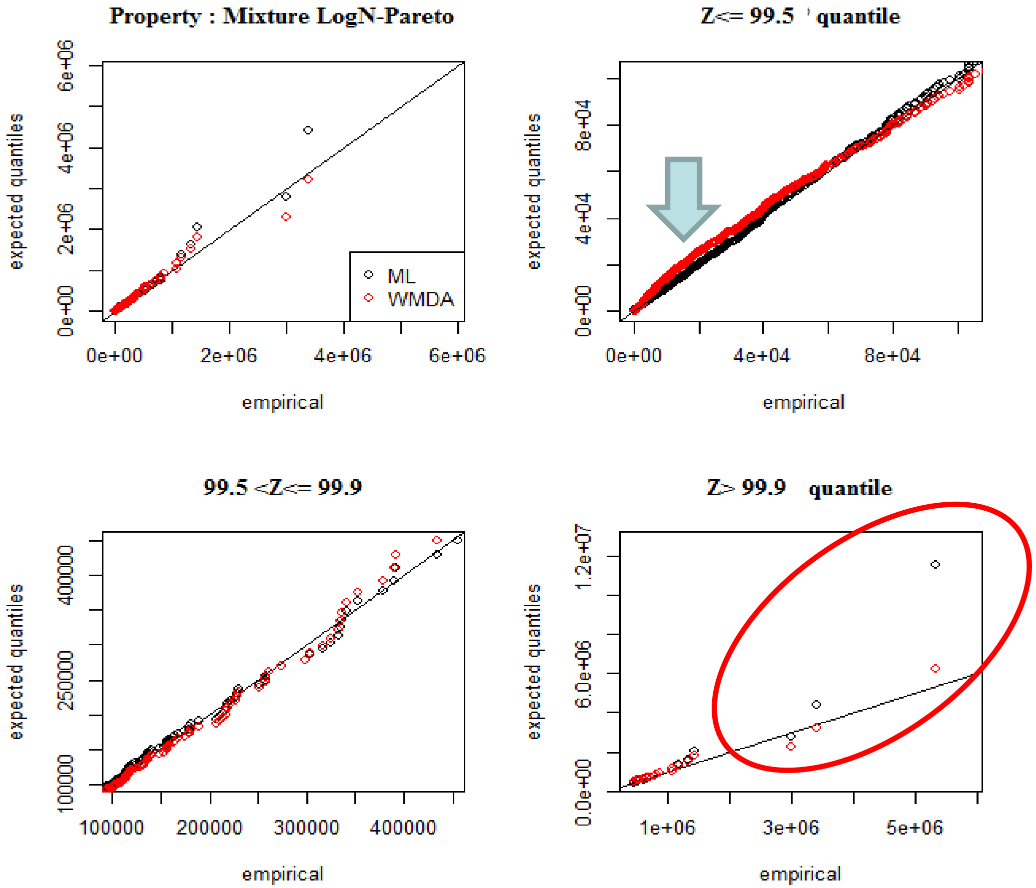
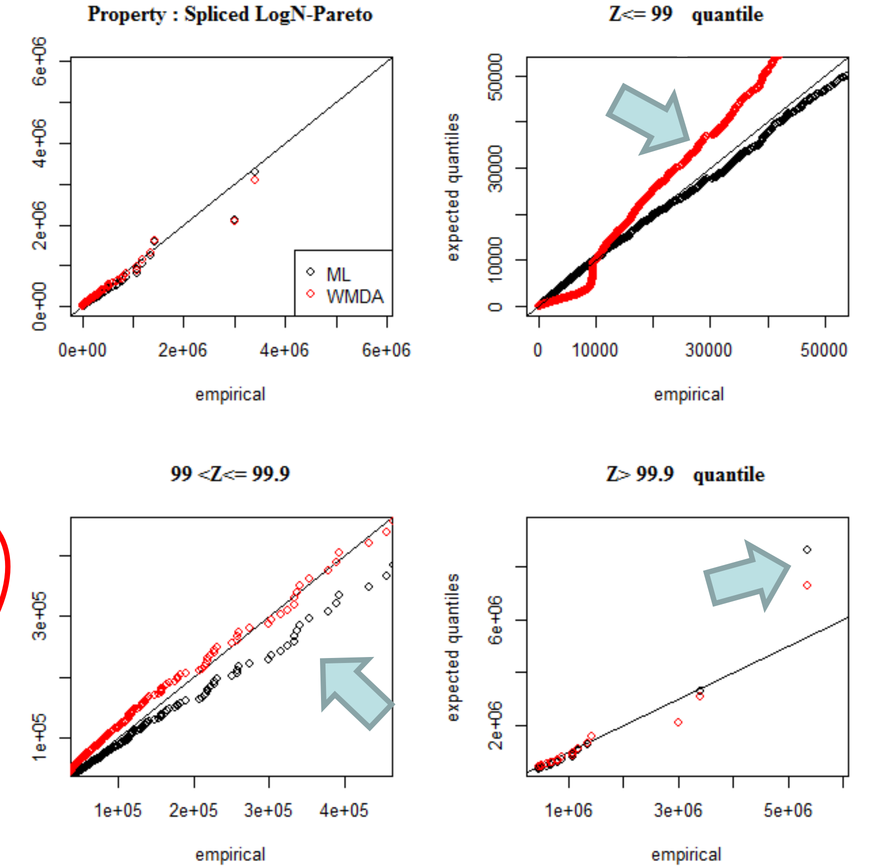
# ML vs WMDA
# (LogNormal-Pareto)

- Two alternative models (mixture and spliced) based on a LogNormal-Pareto II have been applied by using ML and WMDA

| Mixture LogNormal-Pareto | Spliced LogNormal-Pareto |
| --- | --- |

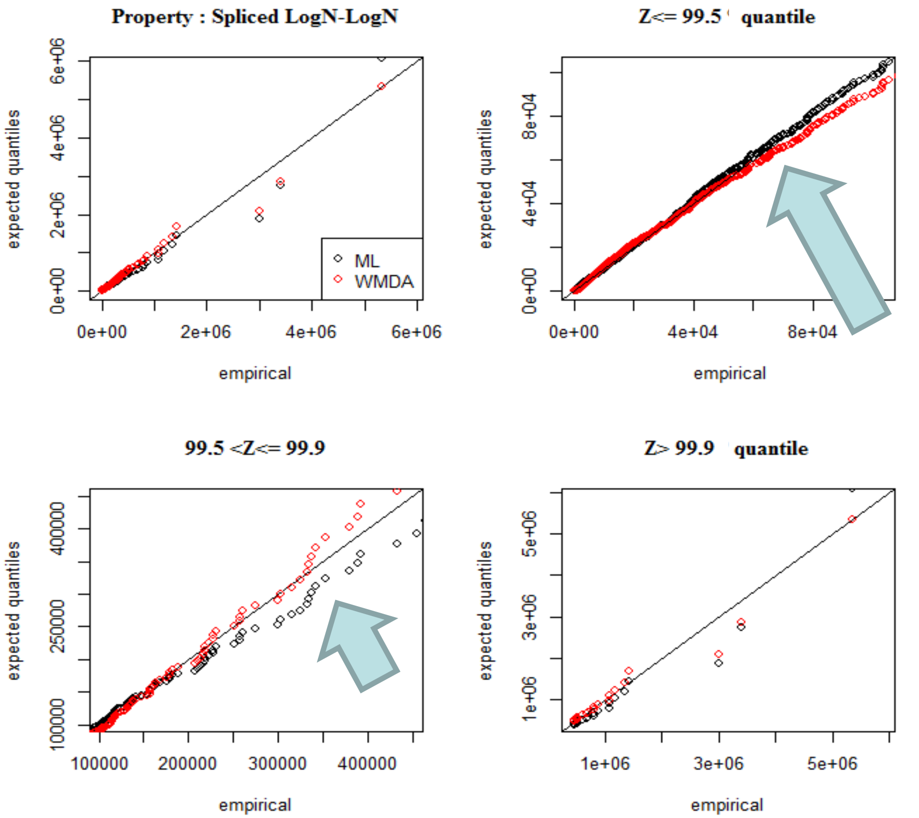


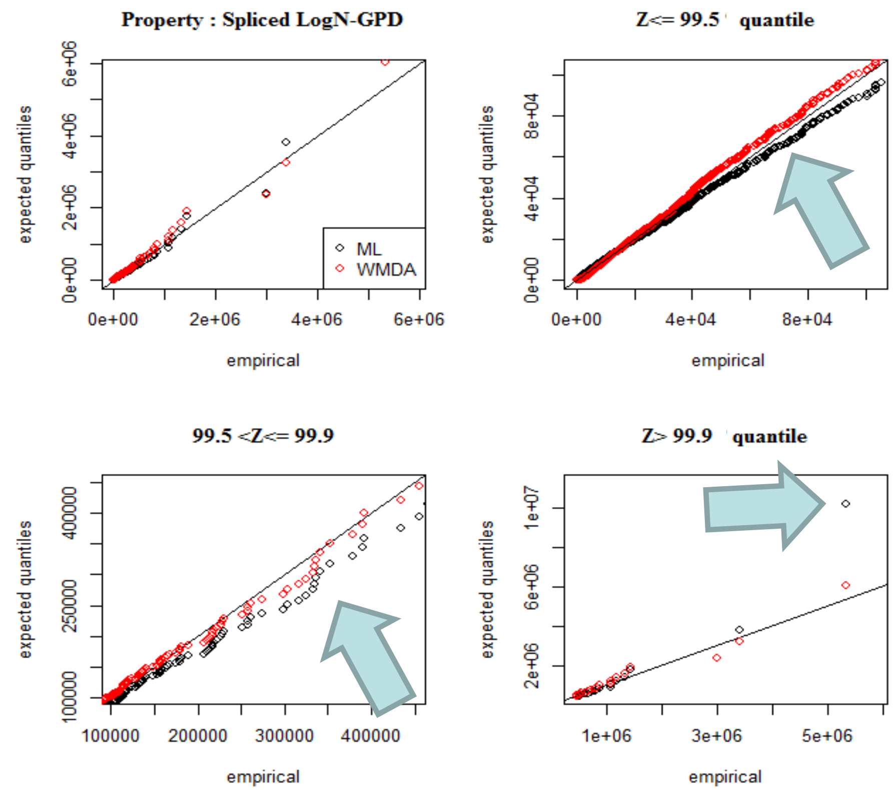
# ML vs WMDA
# Spliced

• Two alternative spliced models (LogNormal-LogNormal) and (LogNormal-GPD) have been applied by using ML and WMDA

## LogNormal-LogNormal




## LogNormal-GPD



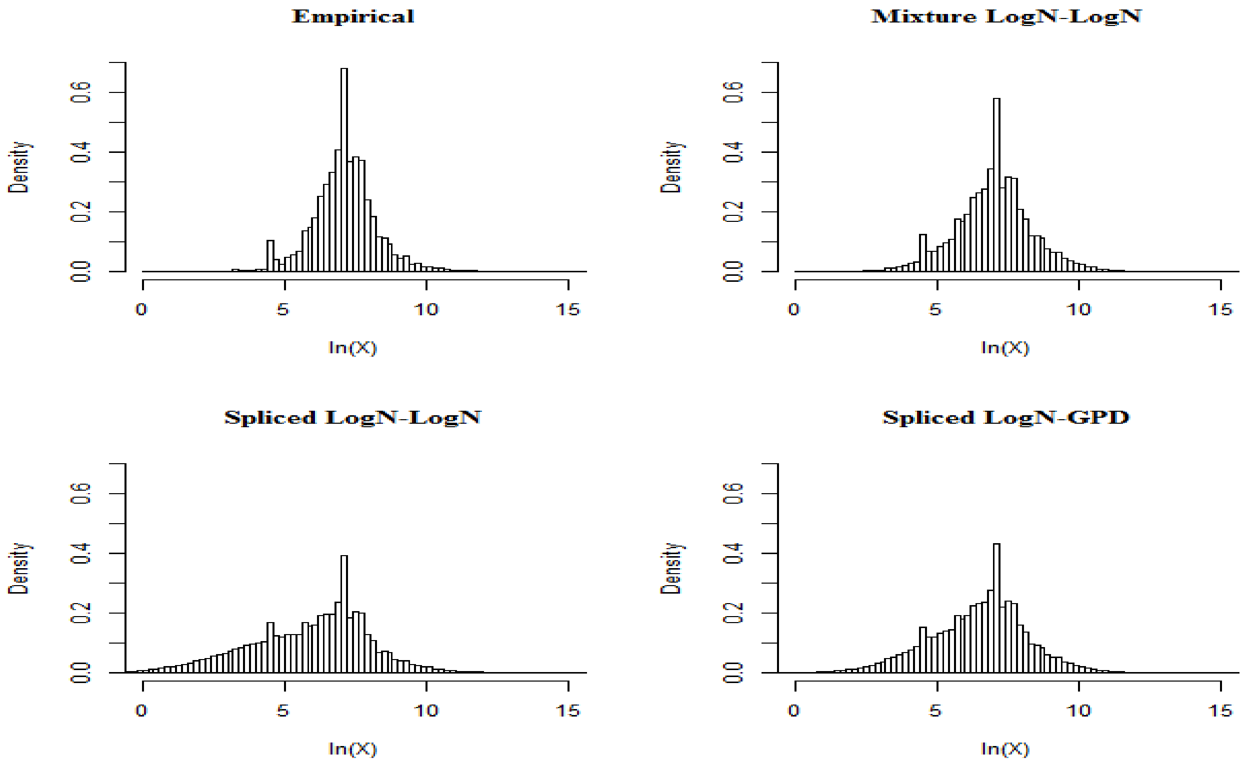
# A Comparison of the Models

- A comparison of quantiles are reported in the upper part of the Table.
- Coloured cells are the cases where the absolute differences between fitted and empirical quantiles are greater than 10%.
- In the lower part of the table main diagnostics (in green best models)

| | | | Mixture LogN-LogN | | Mixture LogN-Pareto | | Spliced LogN-LogN | | Spliced LogN-Pareto | | Spliced LogN-GPD | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Quantiles | Empirical | ML | WMDA | ML | WMDA | ML | WMDA | ML | WMDA | ML | WMDA |
| | 50% | 1,329 | 1,315 | 1,231 | 1,325 | 1,031 | 1,315 | 516 | 1,330 | 652 | 1,378 | 222 |
| | 75% | 2,635 | 2,877 | 3,068 | 2,693 | 3,124 | 2,859 | 1,898 | 2,659 | 1,170 | 2,649 | 1,098 |
| | 90% | 5,975 | 6,263 | 7,608 | 5,965 | 8,552 | 6,208 | 6,163 | 6,252 | 2,328 | 6,151 | 4,635 |
| | 99% | 51,988 | 60,027 | 54,083 | 54,192 | 55,597 | 54,306 | 51,056 | 48,564 | 66,467 | 49,489 | 56,414 |
| | 99.50% | 103,233 | 116,006 | 95,407 | 103,145 | 97,802 | 100,832 | 92,891 | 89,117 | 119,832 | 92,303 | 104,422 |
| | 99.90% | 447,647 | 370,824 | 452,217 | 442,870 | 465,344 | 387,046 | 470,291 | 358,989 | 431,033 | 386,932 | 435,807 |
| | 99.99% | 3,065,504 | 1,181,437 | 2,975,952 | 3,048,699 | 2,432,283 | 2,021,142 | 2,516,518 | 2,301,645 | 2,252,203 | 2,621,880 | 2,501,671 |
| | 99.999% | 4,983,665 | 2,198,212 | 5,299,414 | 10,268,613 | 5,692,661 | 5,466,665 | 4,879,742 | 7,649,107 | 6,491,477 | 9,037,325 | 5,516,829 |
| Diagnostics | Full Domain | D | 4,188,560 | 1,004,668 | 6,372,603 | 1,282,696 | 1,590,089 | 1,108,581 | 3,472,141 | 4,193,313 | 4,960,263 | 1,148,225 |
| | | E(res) | 482.32 | -343.17 | -456.27 | -434.62 | 139.87 | 548.55 | 151.95 | 565.99 | -81.21 | 830.42 |
| | | slope | 1.77 | 0.99 | 0.68 | 0.93 | 1.01 | 1.05 | 0.76 | 0.85 | 0.65 | 0.94 |
| | Without last 3 extremes | D | 1,178,883 | 540,566 | 715,043 | 505,115 | 547,733 | 334,682 | 657,504 | 857,504 | 540,207 | 632,960 |
| | | E(res) | 112.30 | -352.10 | -73.39 | -434.62 | 83.53 | 469.48 | 376.00 | 604.22 | 172.67 | 824.03 |
| | | slope | 1.26 | 0.97 | 0.97 | 0.93 | 1.11 | 0.96 | 1.12 | 0.89 | 1.03 | 0.90 |

# Claim-Size Distribution

- Previous models have been fitted considering only the distribution without replications ($Z \in C$).
- Full claim size distribution is then easily derived for each model by using the cdf of the mixed variable where:
  - for $Z \in C$ we use the fitted model
  - for $Z \in D$ we use the empirical distribution
- Some examples of the pdf of claim-size distributions are reported in Figures.



# Premium Risk capital charge and Aggregate Claim Amount

- A Collective Risk Simulation Model is here applied with the aim to quantify the required capital *for premium risk of the LoB at the end of year t.*

- We denote, for simplicity, with the r.v. $\tilde{X}_{t+1} = \tilde{X}_{t+1}^{paid,CY} + \widetilde{BE}_{t+1}^{CY}$ the amount of incurred claims (both paid and reserved) in the current year t+1.
  (NB: *from now on the superscript tilde indicates random variables*).
  Furthermore we assume that acquisition and management expenses are deterministic.

- Following the collective approach, for each line of business the aggregate claims amount is given by a **mixed compound process**

$$\tilde{X}_{t+1} = \sum_{j=1}^{\tilde{K}_{t+1}} \tilde{Z}_{j,t+1}$$

where:

- $\tilde{K}_{t+1} \sim Poi(n_{t+1}\tilde{q})$: **the number of claims distribution** ($\tilde{K}_{t+1}$) **is the Poisson law**, with a parameter n increasing year by year by the *real growth rate g* **($n_{t+1}=n_t \cdot (1+g)$)** and with a ***structure variable*** $\tilde{q}$ distributed as a Gamma with mean equal to 1
- **the claim size amounts $Z_{j,t+1}$ are assumed <u>i.i.d.</u>** and scaled by *the claim inflation rate i:* **$E(Z^r_{t+1})=(1+i)^r \cdot E(Z^r_t)$.**
  **We will compare the fitted models properly scaled.**

# Parameters

| Number of Claims | |
|---|---|
| $n_{t+1} = E(\tilde{K}_{t+1})$ | 34,375 |
| $\sigma_{\tilde{q}}$ | 6% |
| **Claim Size Distribution** | |
| $m_{t+1} = E(\tilde{Z}_{t+1})$ | 3,724.95 |
| $c_{\tilde{Z}} = CoV(\tilde{Z}_{t+1})$ | 12.17 |
| $\gamma_{\tilde{Z}} = Skew(\tilde{Z}_{t+1})$ | 80.46 |
| **Loadings** | |
| $\lambda_{t+1}$ (safety loading) | 5% |
| $c_{t+1}$ (exp. loading) | 30% |
| **Gross Premiums** | |
| $B_{t+1}$ | 192,067,510.97 |
| $B_t$ | 182,816,972.18 |

- Expected Gross Premiums of next year $B_{t+1}$ are derived as:

$$B_{t+1} = (n_{t+1}m_{t+1}) \cdot \frac{(1+\lambda_{t+1})}{(1-c_{t+1})}$$

where:
- $(n_{t+1}m_{t+1})$ is the risk premium
- $\lambda_{t+1}$ is the safety loading coefficient (as a percentage of risk premium)
- $c_{t+1}$ is the expenses loading coefficient (as a percentage of gross premium)

For sake of simplicity that *written premiums are* assumed to be *equal to earned premium*

As regard to parameter calibration:
- The expected number of claims for next year considers a growth rate of roughly 2%, while the expected average claim amount has been scaled by a 3% inflation rate.
- Variability coefficient and skewness of claim size distribution reported in Table are derived by the empirical distribution. We will use directly fitted models in aggregate claim amount evaluation.

# Capital Requirement

- Capital requirement for premium risk has been obtained as a difference between the 99.5% quantile and the expected value of the aggregate claim amount distribution:

  $SCR_{t+1} = F_{\tilde{X}}^{-1}(0.995) - E(\tilde{X})$

- We report both the amount of the capital requirement ($SCR_{t+1}$) and the ratio between the capital requirement and initial gross premiums: $\frac{SCR_{t+1}}{B_t}$.

  As a reference, the EU Solvency I requirement is around 16-20% of (net) written premiums for the overall risk of a non-life company

- The aggregate claim amount distribution is derived by a compound process **using alternative models for severity distribution** fitted previously.

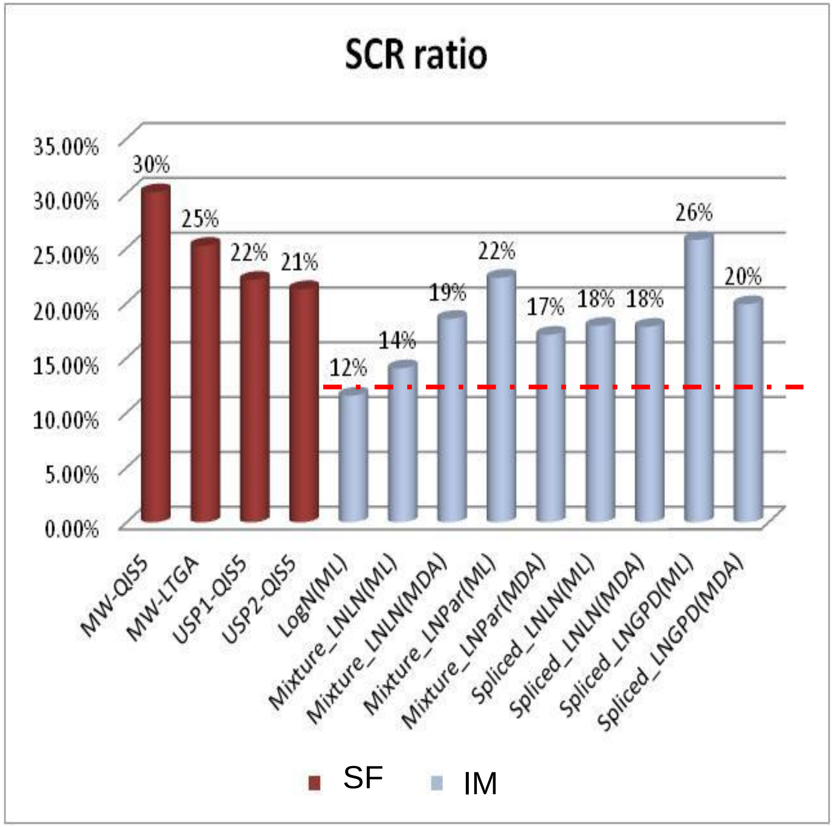
| | LogNormal | | Mixture LogN-LogN | | Mixture LogN-Pareto | | Spliced LogN-LogN | | Spliced LogN-GPD | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Method of Moments | ML | ML | WMDA | ML | WMDA | ML | WMDA | ML | WMDA |
| CoV($\tilde{X}$) | 8.91% | 6.13% | 7.28% | 9.11% | 10.74% | 8.37% | 8.79% | 8.96% | 11.56% | 9.86% |
| $\gamma(\tilde{X})$ | 0.16 | 0.12 | 0.18 | 0.35 | 0.43 | 0.26 | 0.34 | 0.28 | 0.64 | 0.32 |
| $F_{\tilde{X}}^{-1}(0.995)$ (x $10^6$) | 157.10 | 149.17 | 153.72 | 161.96 | 168.82 | 159.31 | 160.80 | 160.64 | 175.14 | 164.40 |
| $SCR_{t+1}$ (x $10^6$) | 29.05 | 21.12 | 25.67 | 33.91 | 40.78 | 31.22 | 32.75 | 32.60 | 47.10 | 36.36 |
| $\frac{SCR_{t+1}}{B_t}$ | 15.89% | 11.59% | 14.04% | **18.55%** | 22.30% | 17.11% | **17.92%** | **17.83%** | 25.76% | 19.89% |

# A comparison
# between Internal Model and Standard Formula

- We compare the SCR ratio for premium risk derived by either Internal Model (IM) and EU Solvency II Standard Formula (SF), obtained as previously mentioned as the ratio between the capital requirement and initial gross premiums: $\frac{SCR_{t+1}}{B_t}$

- Final version of SF is still under review at the moment for the final calibration, but recent Quantitative Impact Studies (QIS5 and LTGA) provide the capital requirement for premium risk for a single LoB as:
  - $SCR_{t+1}^{SF,QIS5} = \rho(\sigma)B_{t+1}$ where $\rho(\cdot)$ measures the difference between the 99.5 quantile and the mean of a LogNormal distribution with standard deviation $\sigma$.
  - $SCR_{t+1}^{SF,LTGA} = 3\sigma B_{t+1}$.
  - In QIS5, $\sigma$ can be a fixed value (market wide approach) or calibrated by using internal data and fixed methodologies (undertaking specific approaches).

    LTGA provided only a market wide approach, without any undertaking specific methodologies.
  - Volatility factors $\sigma$ are equal to 10% in QIS5 and 8% in LTGA.




- LogNormal distribution underestimates SCR ratio
- It is necessary to define how to select the alternative distribution to describe the insurer's risk profile

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# Conclusion and Further Research

- A partial internal model has been developed in order to obtain capital requirement for premium risk.

  In particular, it has been analysed the effects of a different calibration of the severity distribution on the aggregate claim amount

- WMDA seems to assure a better fit than ML due to its property to adapt the distribution to the data.

- A good fit of extreme values is assured when weights are used. In some cases, the drawback is that it can produce an underestimation of the body of the distribution.

- Further developments will regard an analysis of variability of estimators via *bootstrap procedures*, a bayesian approach for weights.

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