



Gruppo di lavoro percettori rendite

Gli scenari di proiezione della sopravvivenza dei pensionati

Applicazione dei modelli Lee-Carter e Renshaw-Haberman ai dati INPS

Susanna Levantesi

Sapienza Università di Roma

susanna.levantesi@uniroma1.it

Massimiliano Menzietti

Università della Calabria

massimiliano.menzietti@unical.it

I modelli impiegati per proiezione della mortalità

- ▶ Appartengono ai principali modelli di proiezione della mortalità proposti in letteratura

Modello	Formula
Lee-Carter (LC)	$\log(m_{x,t}) = \beta_x^{(1)} + \beta_x^{(2)} k_t^{(2)}$
Renshaw-Haberman (RH)	$\log(m_{x,t}) = \beta_x^{(1)} + \beta_x^{(2)} k_t^{(2)} + \beta_x^{(3)} \gamma_{t-x}^{(3)}$

- ▶ Il modello Lee-Carter utilizzato è di tipo Poisson log bilineare (decessi distribuiti come una Poisson): rispetto al modello originario tiene conto della diversa variabilità del fenomeno alle varie età
- ▶ Il modello Renshaw-Haberman prevede un fattore di coorte $\gamma_{t-x}^{(3)}$
- ▶ **Impiego del «bootstrap»:** tecnica statistica di ricampionamento che permette di ottenere un certo numero di campioni della stessa numerosità di quello originario *estraendoli con ripetizione dal campione dei dati originali*

Osservazioni sulle proiezioni 2010-2012: backtesting

- ▶ Confronto tra mortalità osservata negli anni 2010-2012 e mortalità proiettata in base ai modelli Lee-Carter (LC) e Renshaw-Haberman (RH) per gli anni 2010-2012 e relativi intervalli di confidenza (5%-95%)
- ▶ Osservazioni sul dato storico 2010-2012
 - La mortalità non ha un andamento monotono decrescente
 - La mortalità ad alcune età ha subito un aumento nel 2012
- ▶ Le proiezioni in base al modello RH risultano contenere bene i tre anni di dati storici
- ▶ Qualche criticità sui novantenni e risultati migliori per dipendenti rispetto agli autonomi

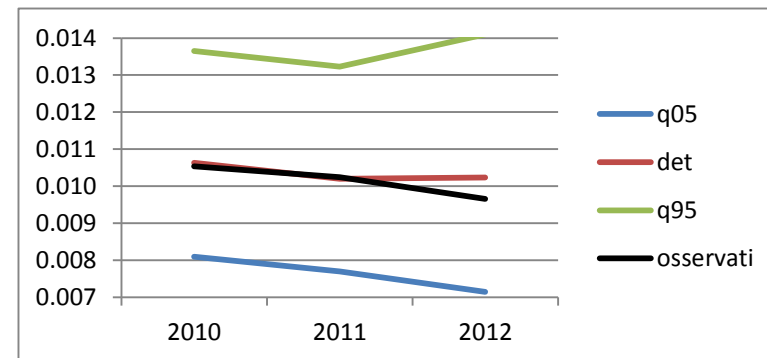
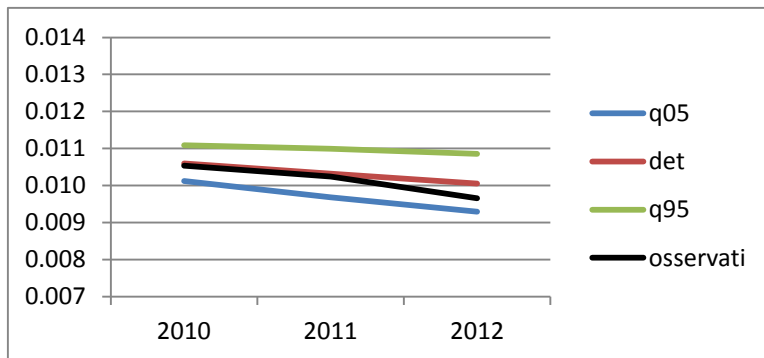
Backtesting 2010-2012: **Inps dip.**

Maschi

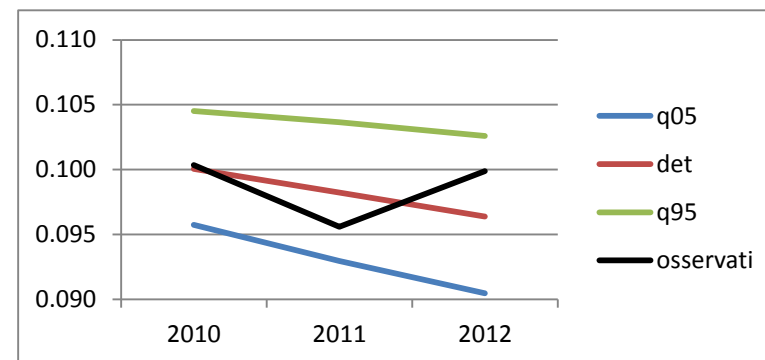
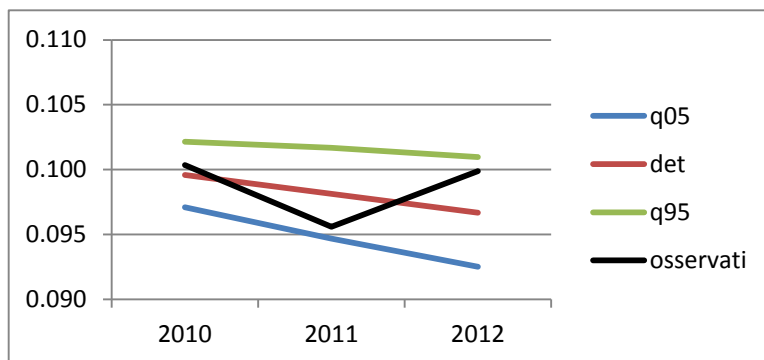
Lee-Carter

Renshaw-Haberman

Età
65



Età
85



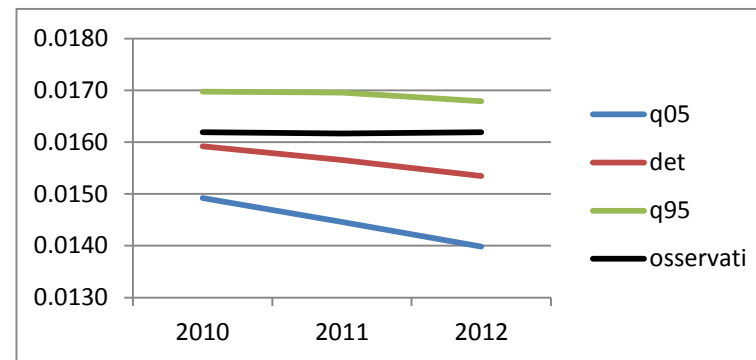
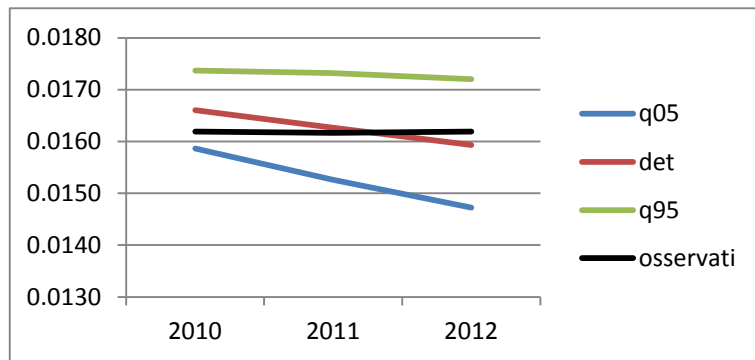
Backtesting 2010-2012: Inps aut.

Maschi

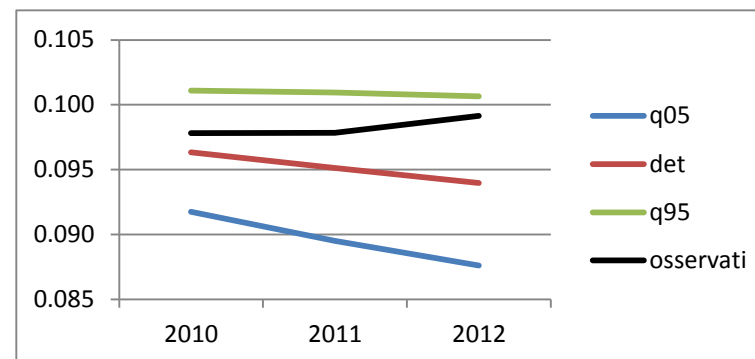
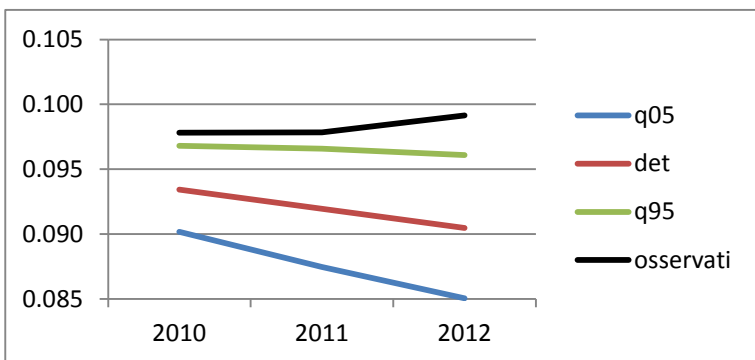
Lee-Carter

Renshaw-Haberman

Età
65



Età
85



Dati INPS

▶ **Lavoratori dipendenti**

- Dipendenti (cat: A)
- Trasporti, telefonici, elettrici (cat: E)

Età Maschi: 60-95
Età Femmine: 60-95

Periodo: 1980-2012

▶ **Lavoratori autonomi**

- CDCM (cat: B)
- Artigiani (cat: C)
- Commercianti (cat: R)

Età Maschi: 65-95
Età Femmine: 60-95

Periodo: 1980-2012

▶ **Totale**

- Dipendenti
- Autonomi

Età Maschi: 65-95
Età Femmine: 60-95

Periodo: 1980-2012

Heatmap dei tassi di variazione della mortalità: **Inps dip.**

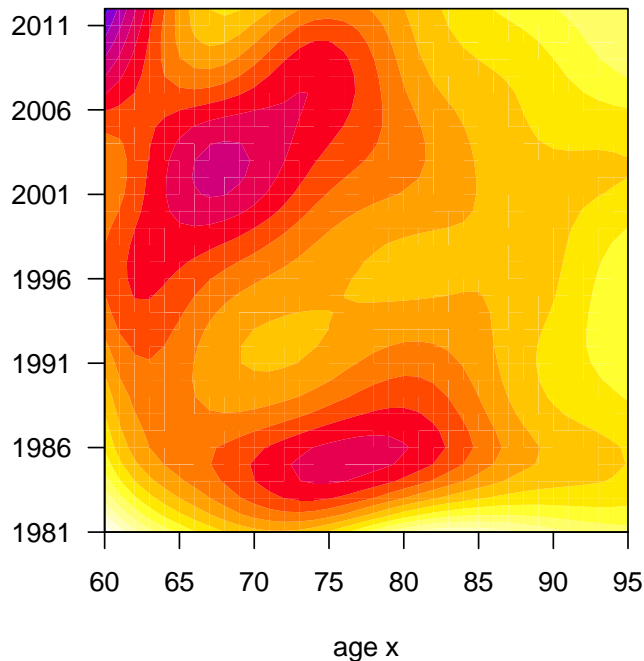
Maschi

$$1 - \frac{q_{x,t+1}}{q_{x,t}}$$

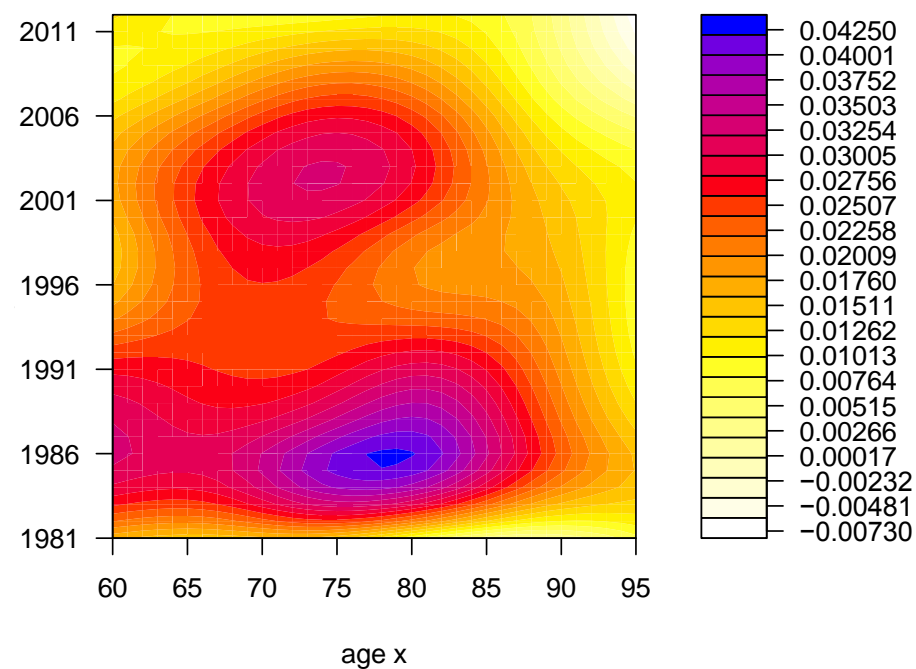
dati con smoothing

Femmine

Tassi di variazione della mortalità



Tassi di variazione della mortalità



Heatmap dei tassi di variazione della mortalità: **Inps aut.**

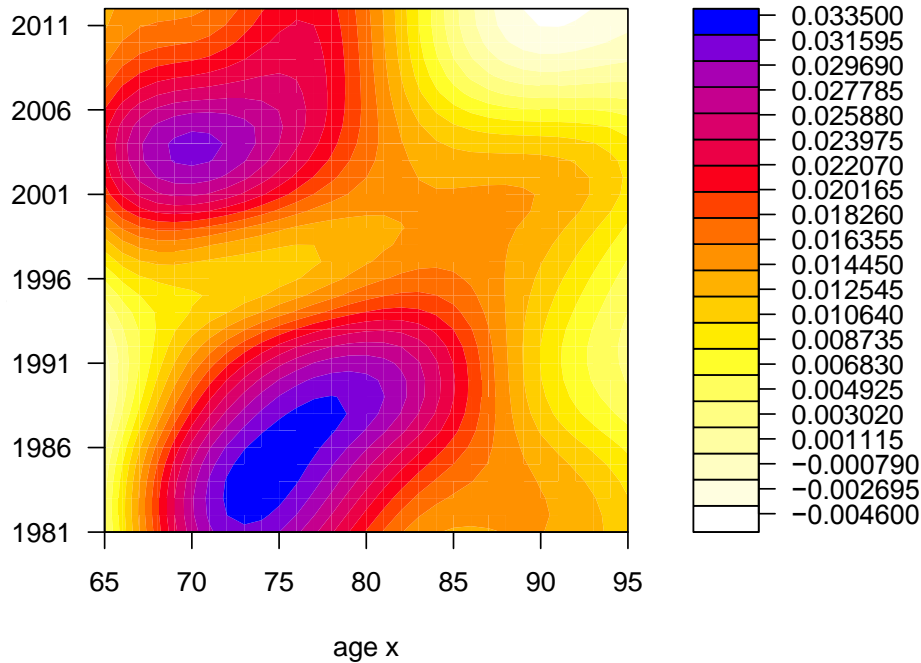
Maschi

$$1 - \frac{q_{x,t+1}}{q_{x,t}}$$

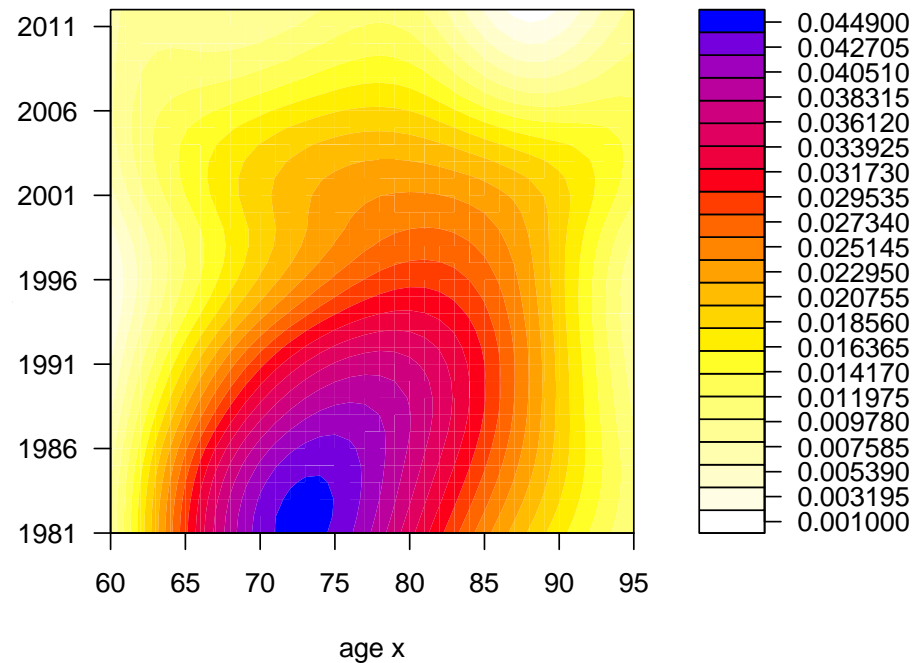
dati con smoothing

Femmine

Tassi di variazione della mortalità

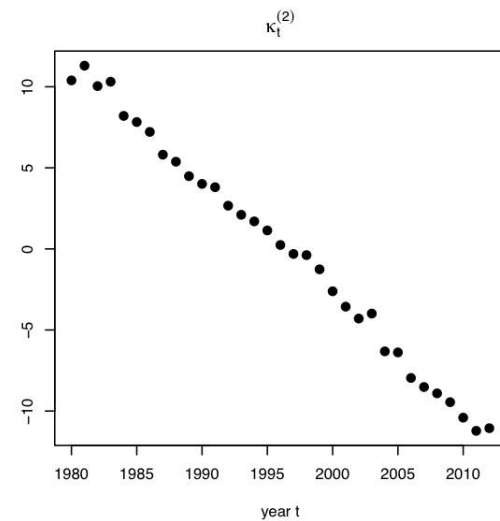
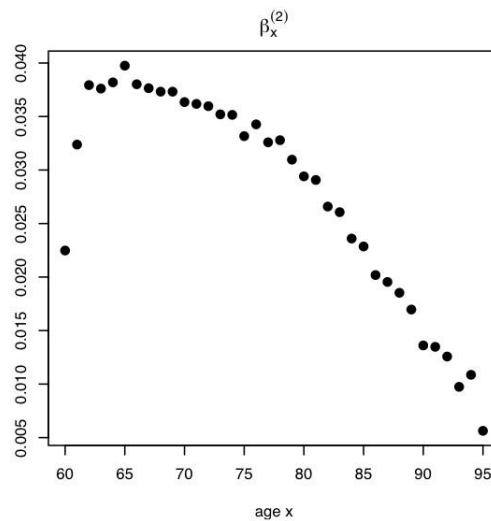
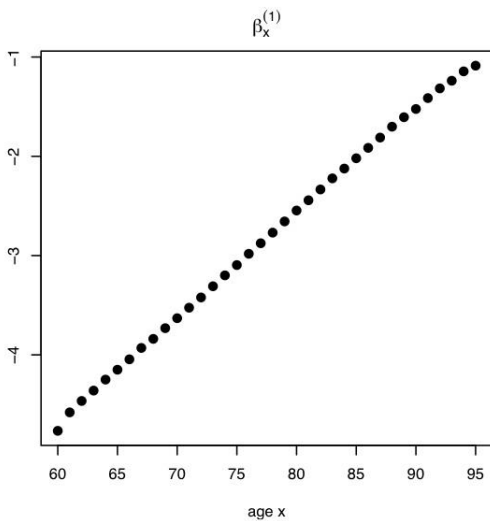


Tassi di variazione della mortalità

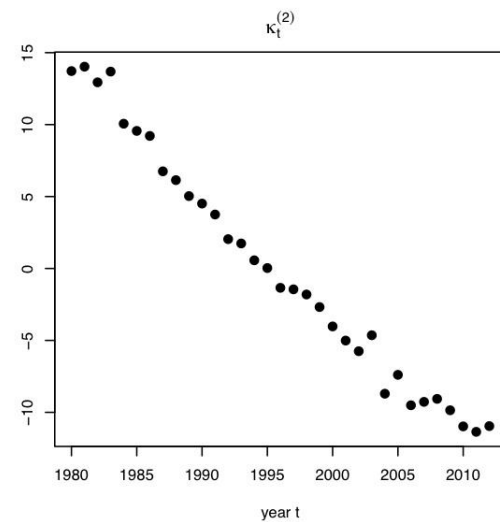
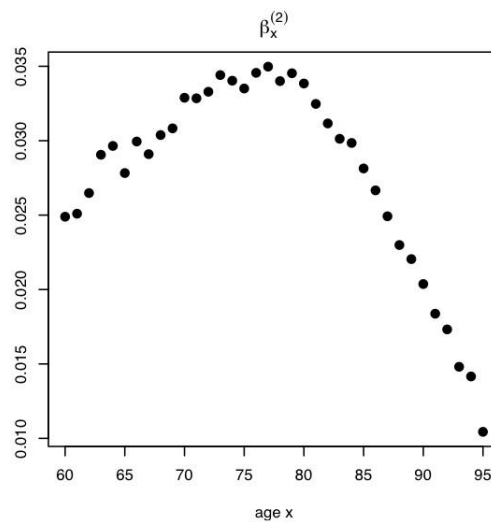
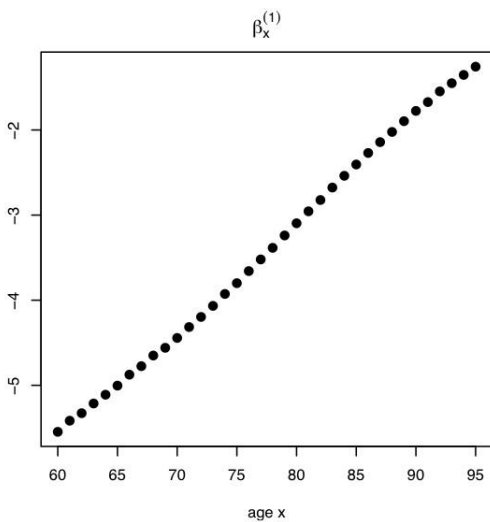


Parametri del modello Lee-Carter: **Inps dip.**

Maschi

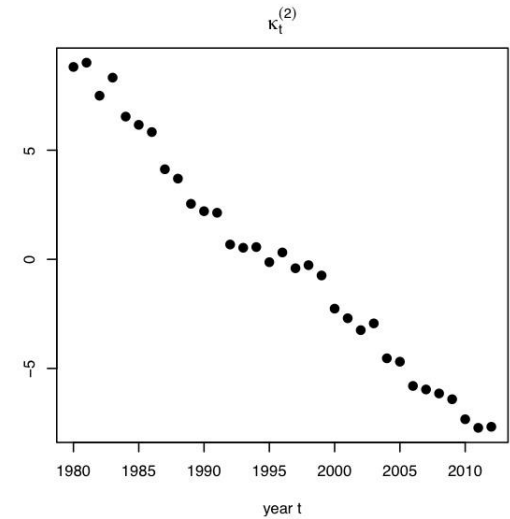
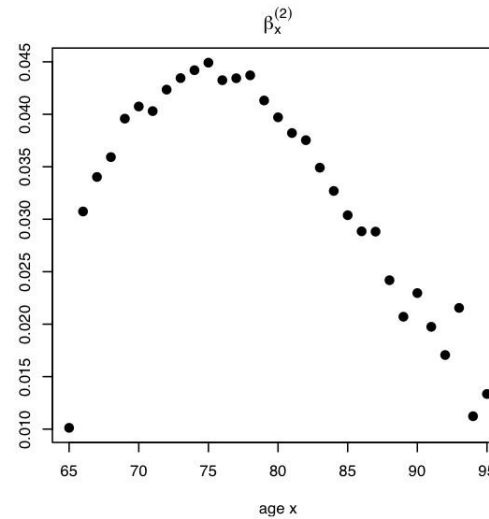
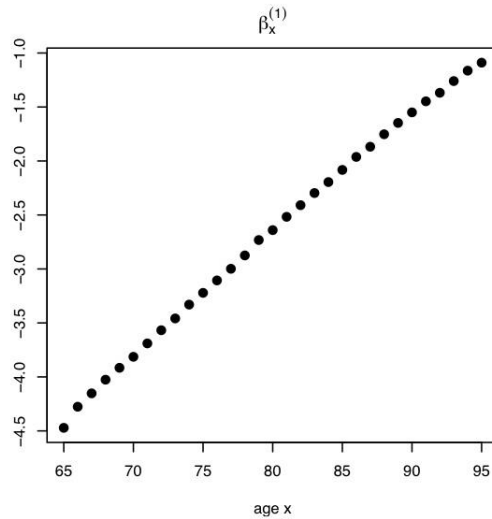


Femmine

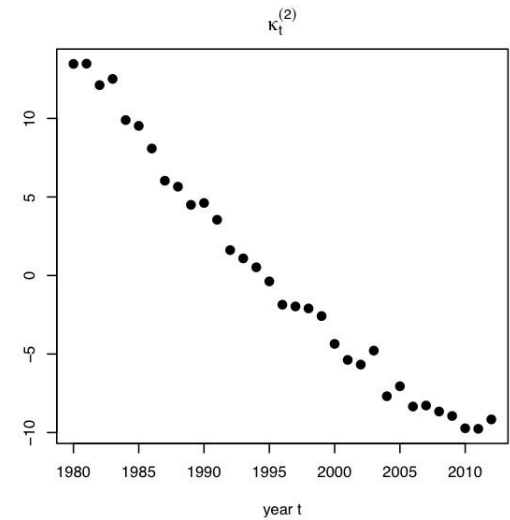
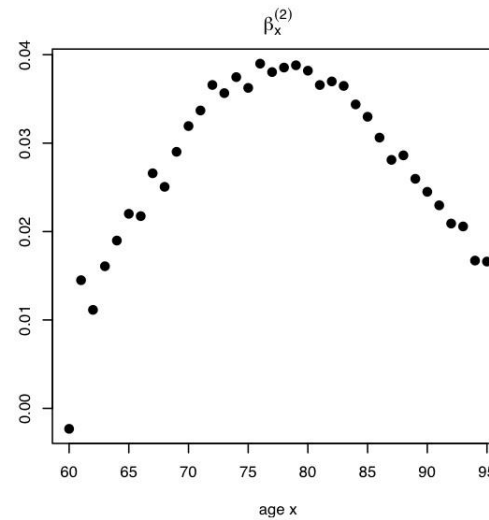
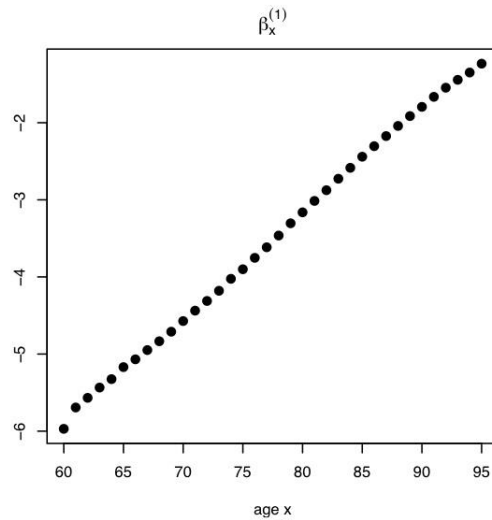


Parametri del modello Lee-Carter: **Inps aut.**

Maschi

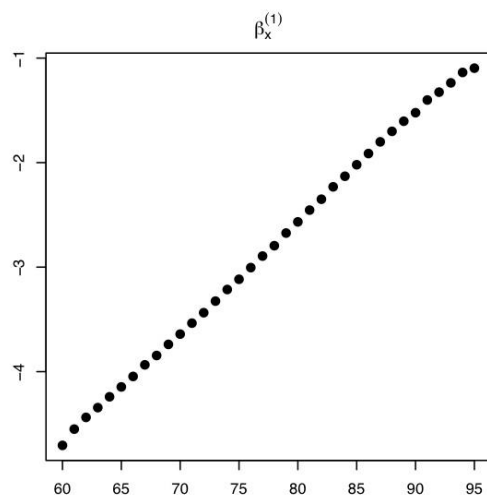


Femmine

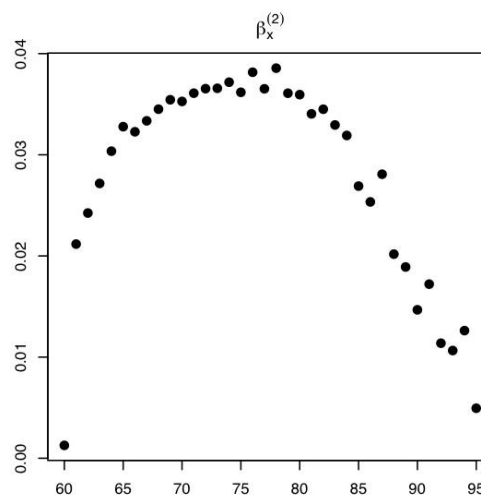


Parametri del modello RH: **Inps dip.**

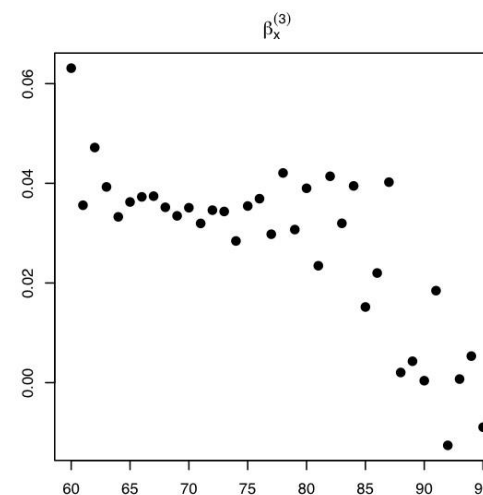
Maschi



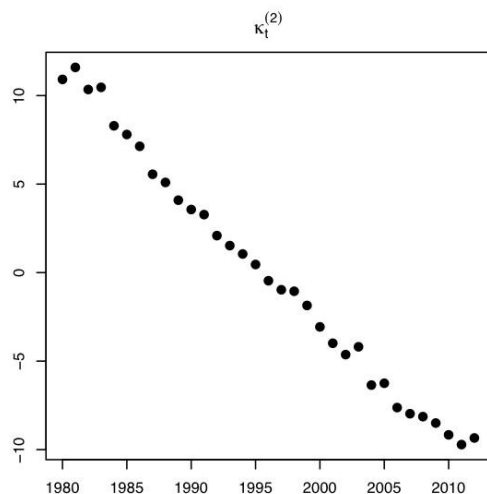
age x



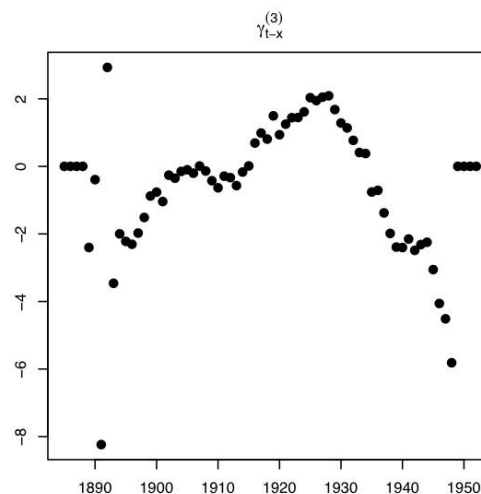
age x



age x



year t

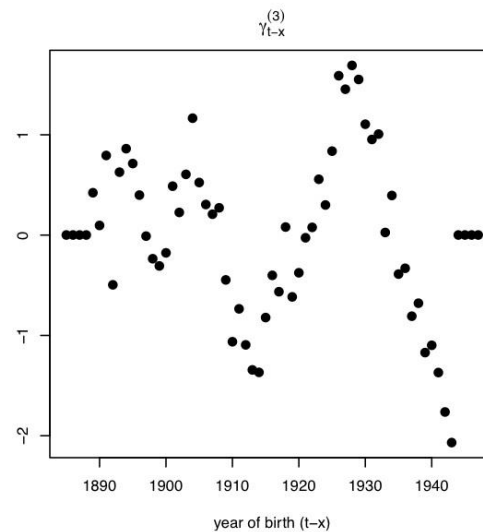
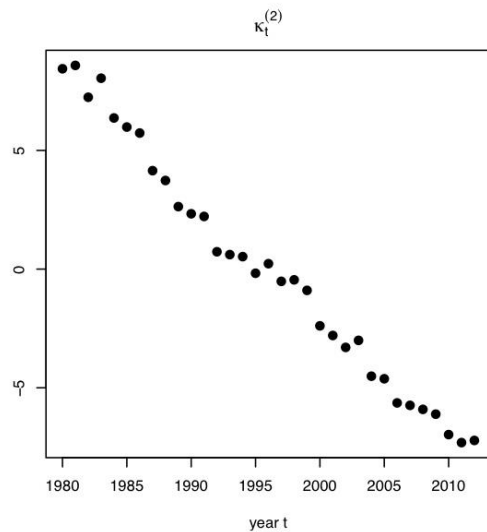
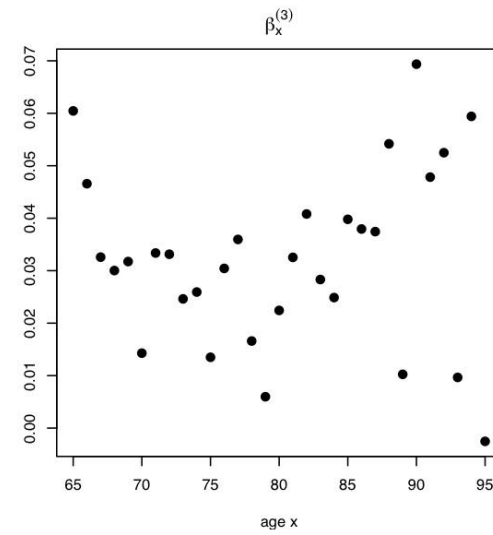
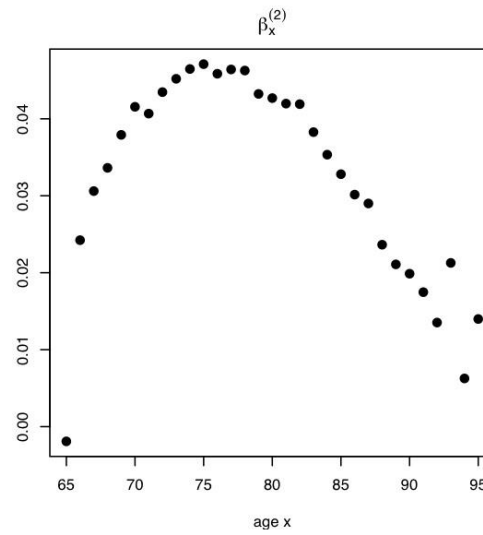
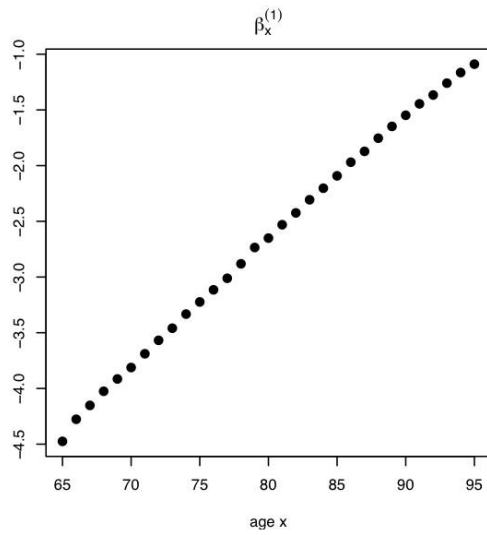


year of birth $(t-x)$



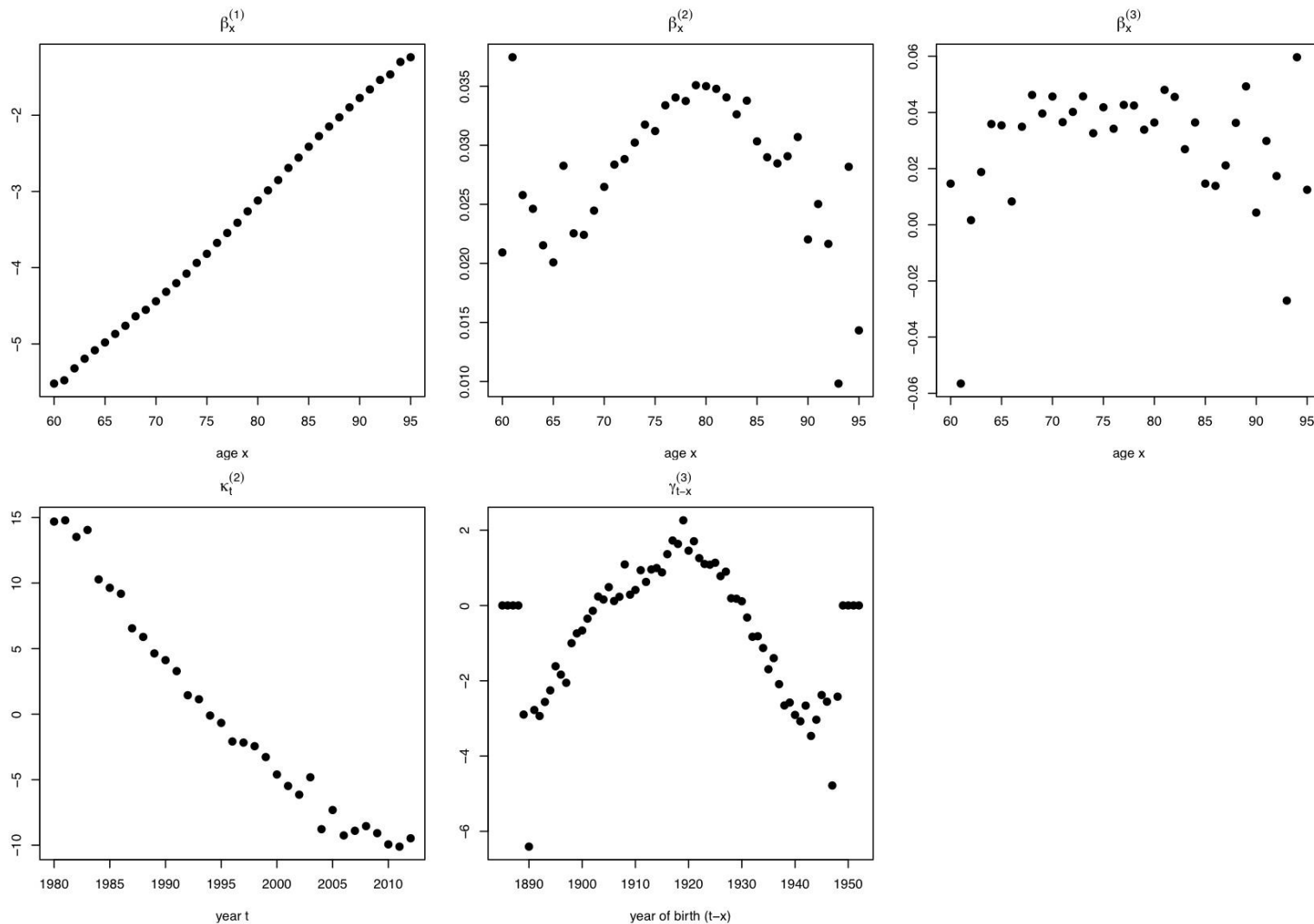
Parametri del modello RH: **Inps aut.**

Maschi



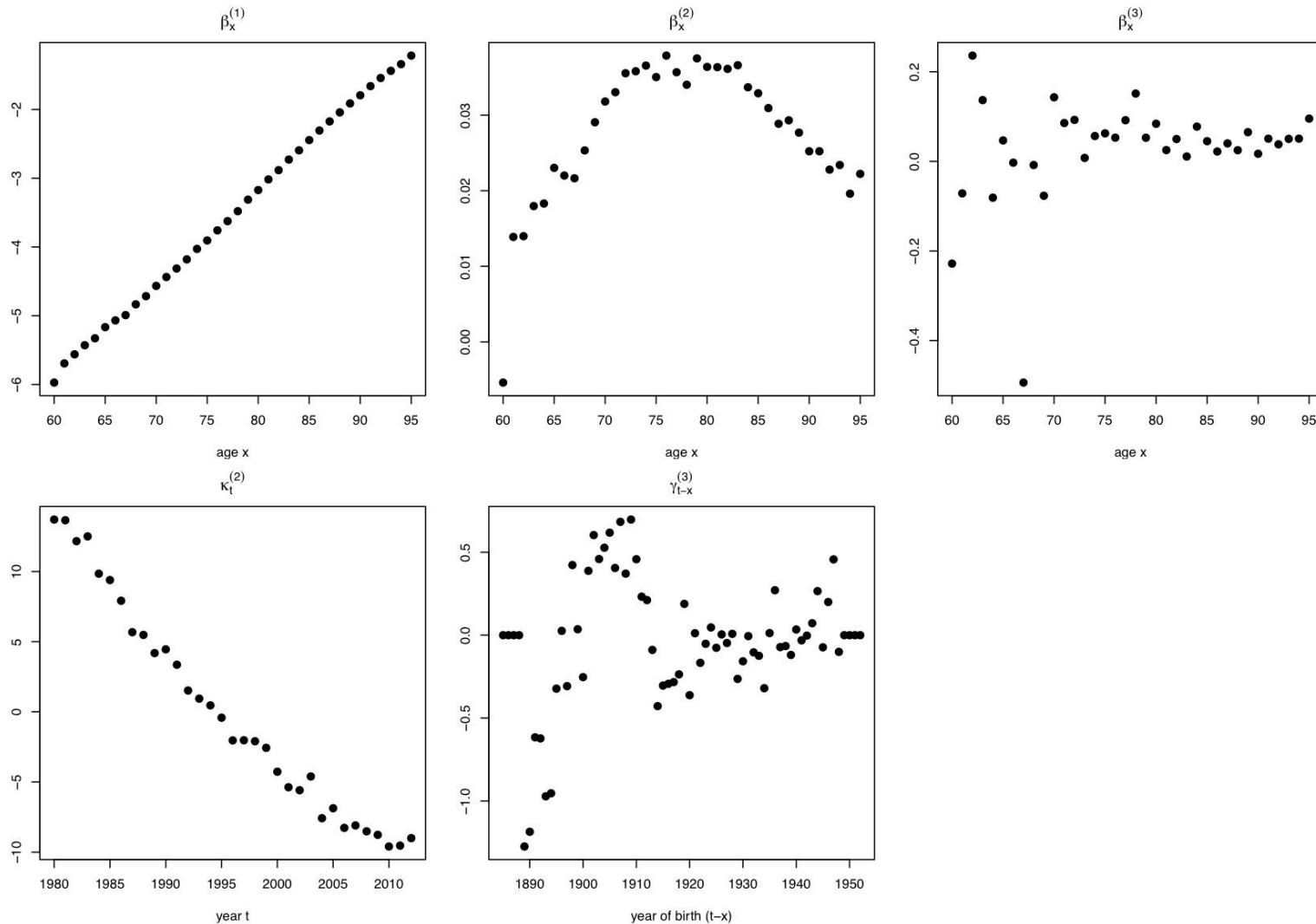
Parametri del modello RH: **Inps dip.**

Femmine



Parametri del modello RH: **Inps aut.**

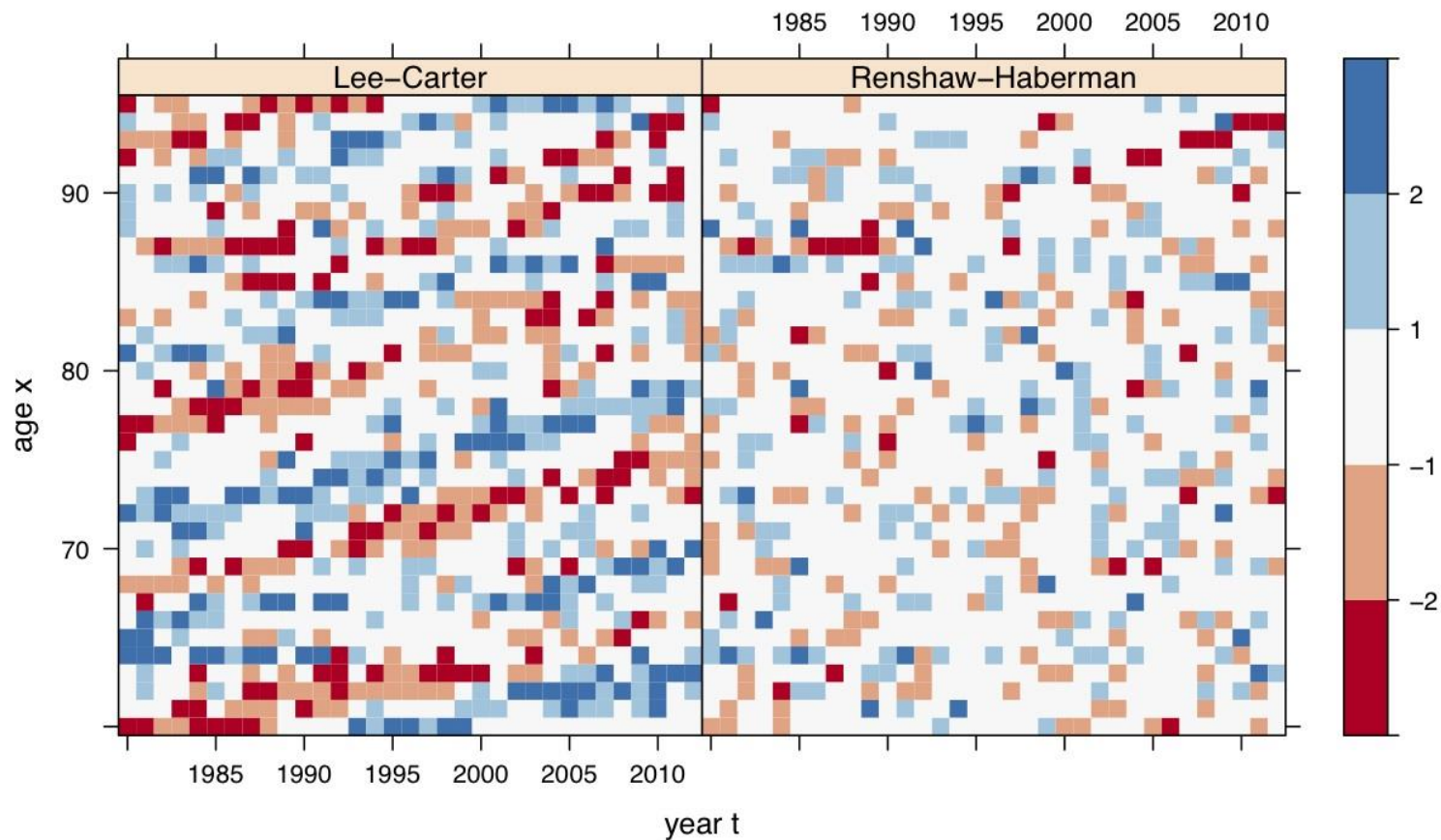
Femmine



Confronto tra i modelli: residui standardizzati

Maschi

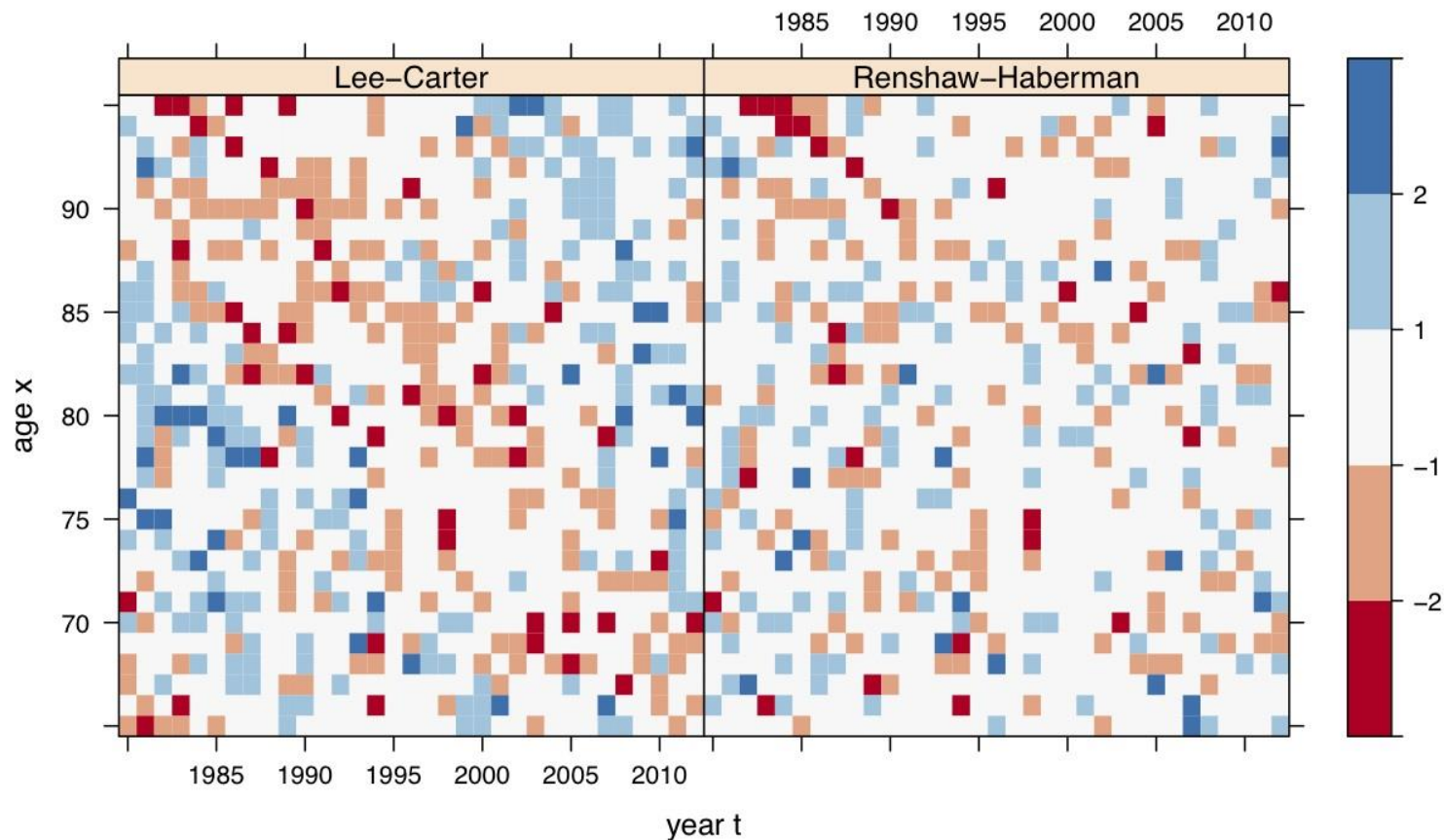
INPS dipendenti



Confronto tra i modelli: residui standardizzati

Maschi

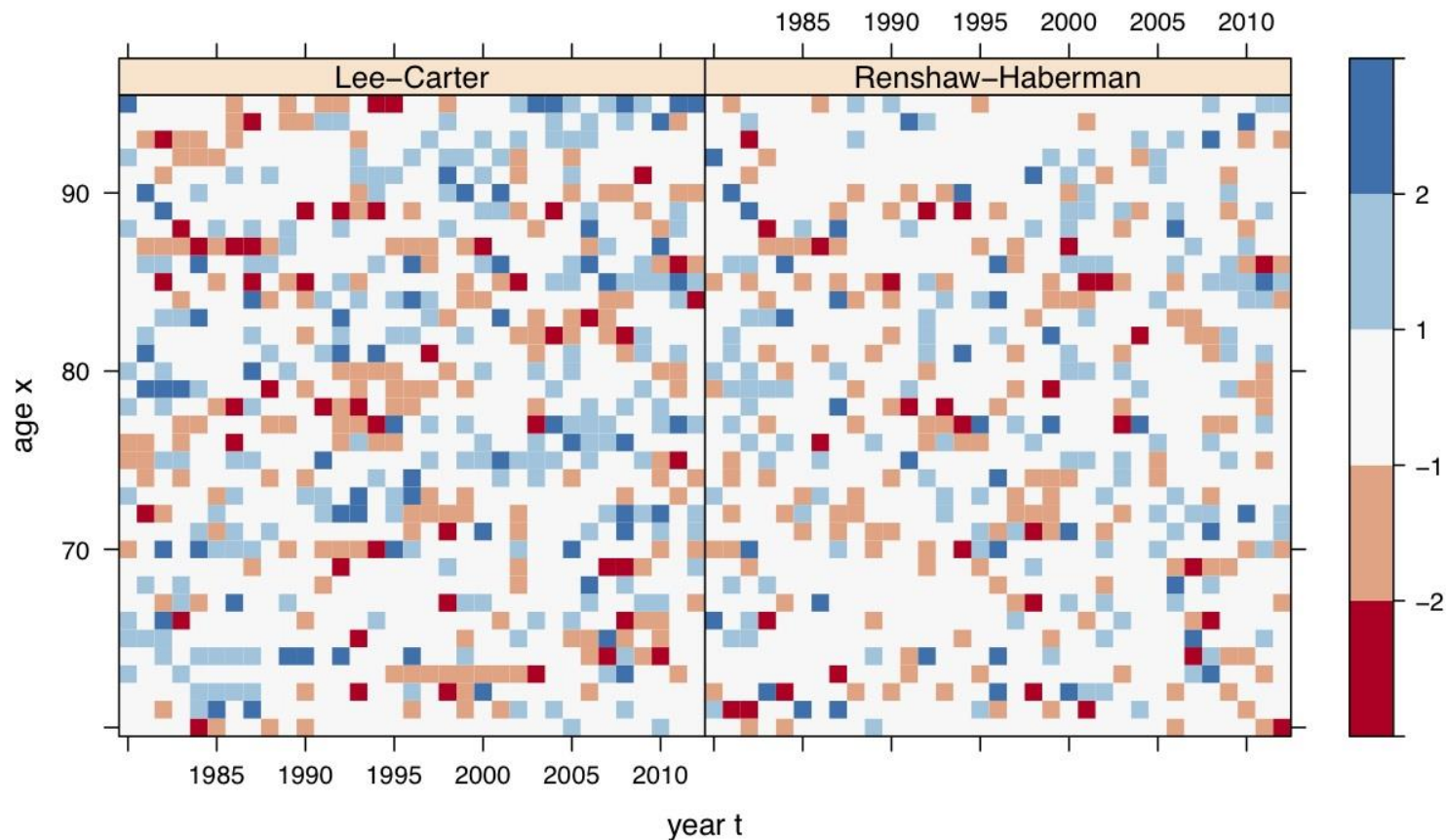
INPS autonomi



Confronto tra i modelli: residui standardizzati

Femmine

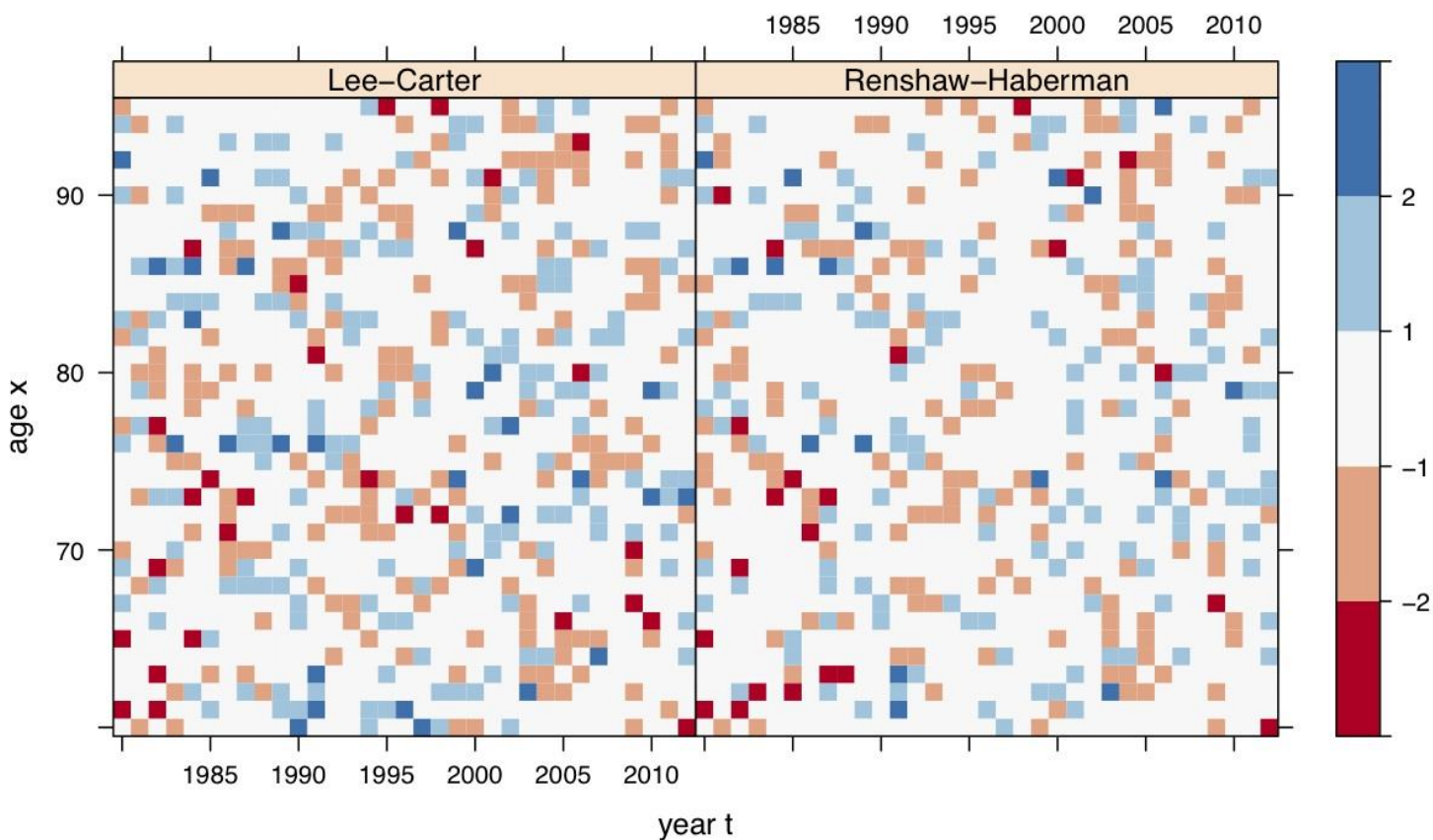
INPS dipendenti



Confronto tra i modelli: residui standardizzati

Femmine

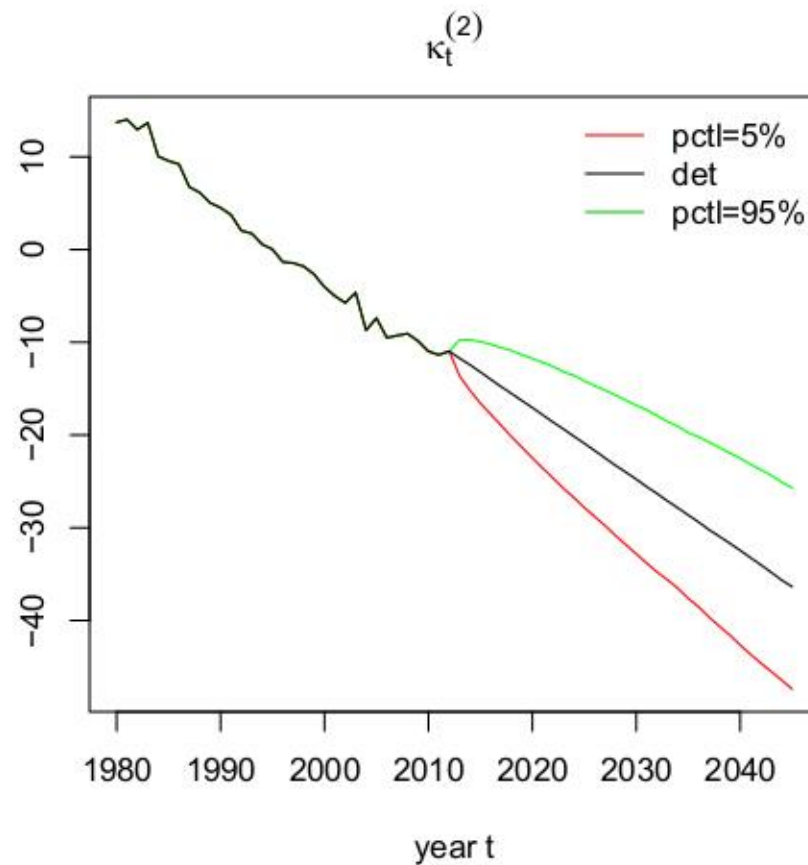
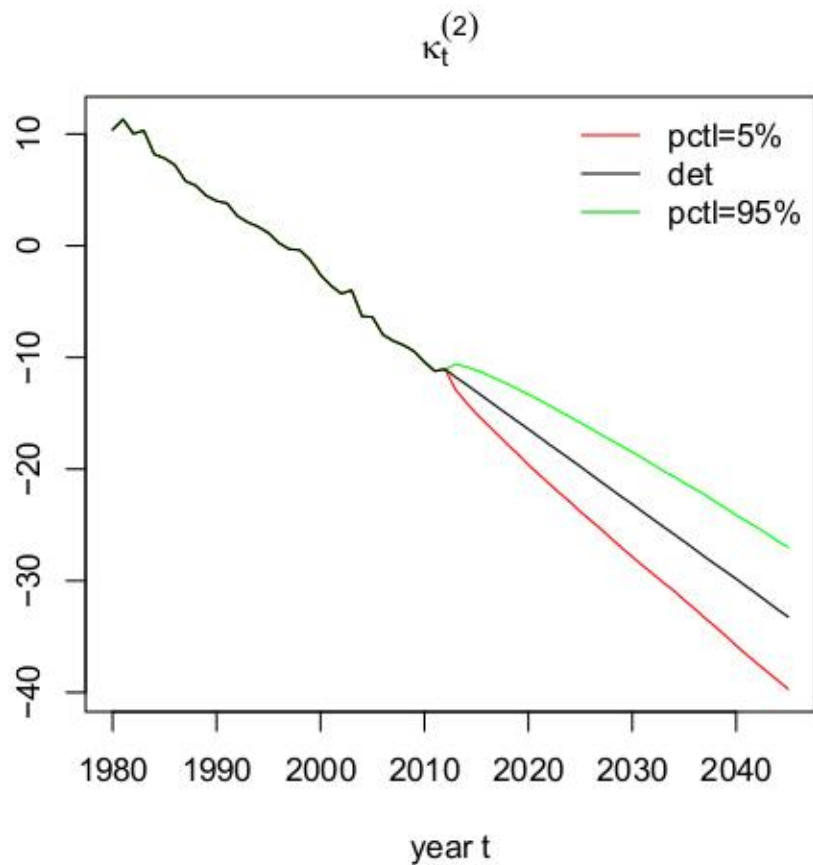
INPS autonomi



Modello Lee-Carter: $\kappa_t^{(2)}$, 1980-2045 - Inps dip.

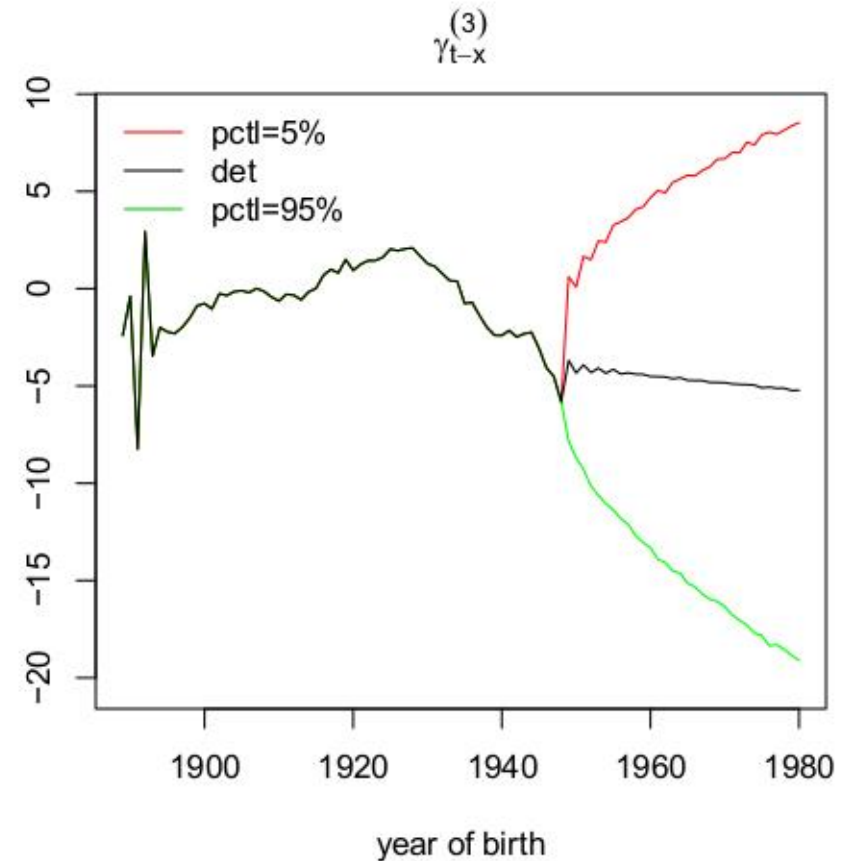
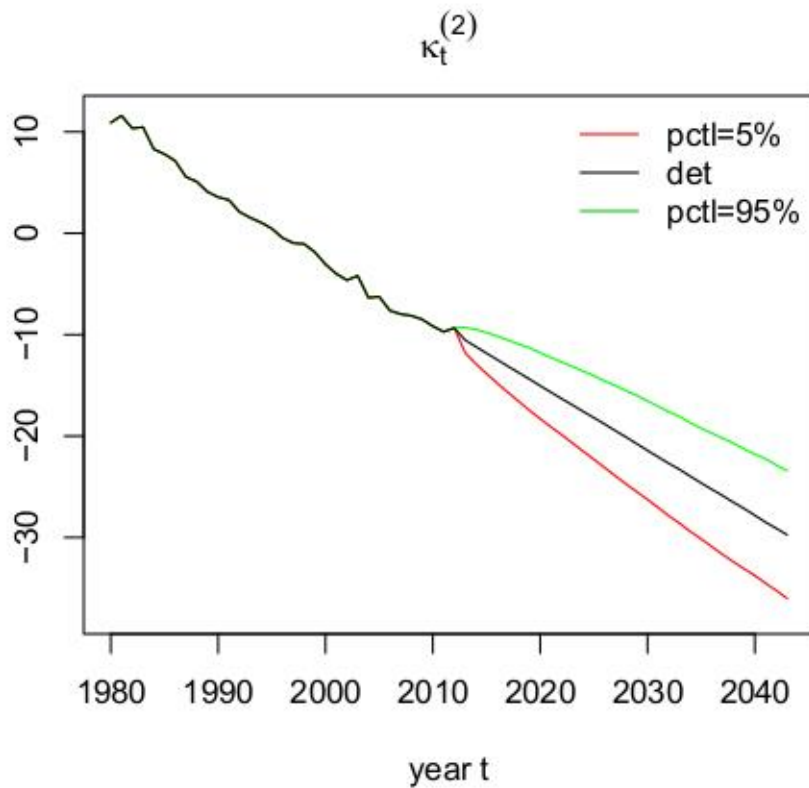
Maschi

Femmine



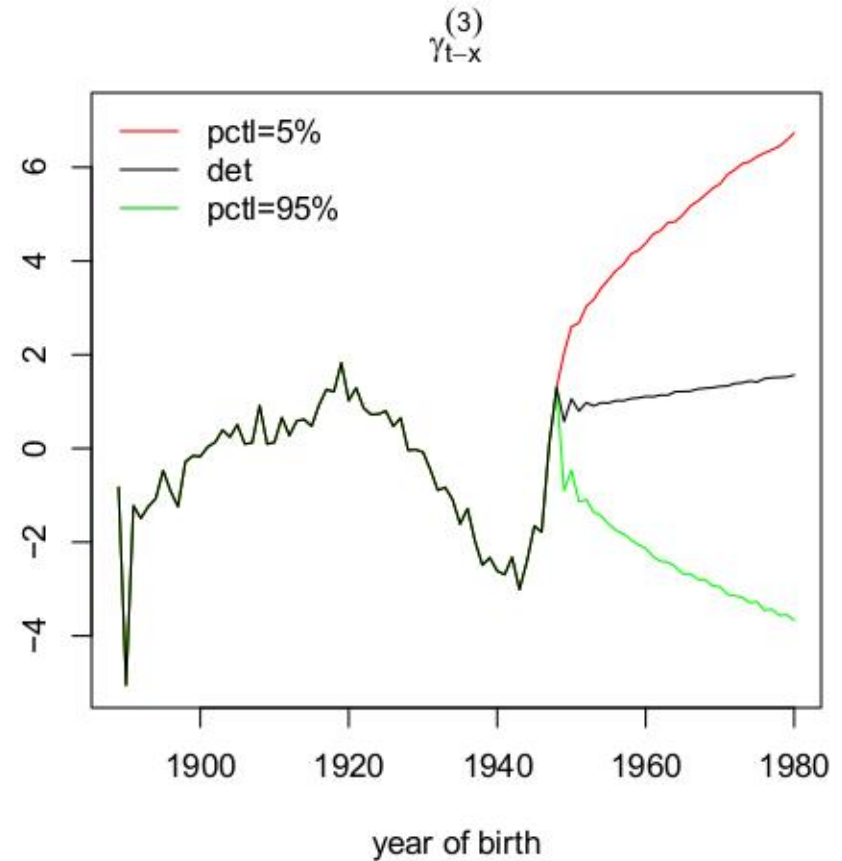
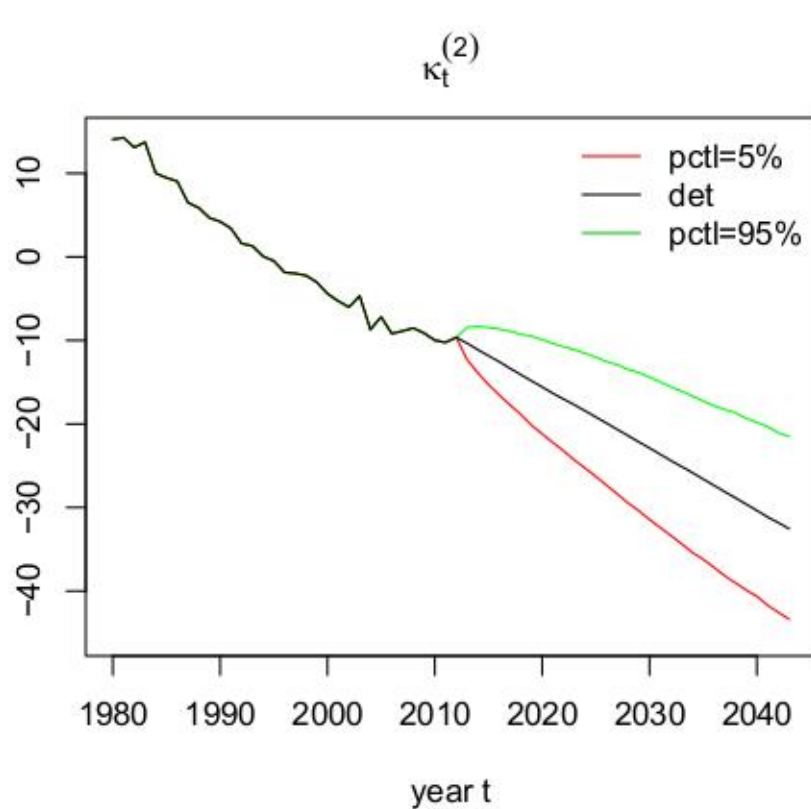
Modello RH: $k_t^{(2)}$, $\gamma_{t-x}^{(3)}$, 1980-2045 - Inps dip.

Maschi

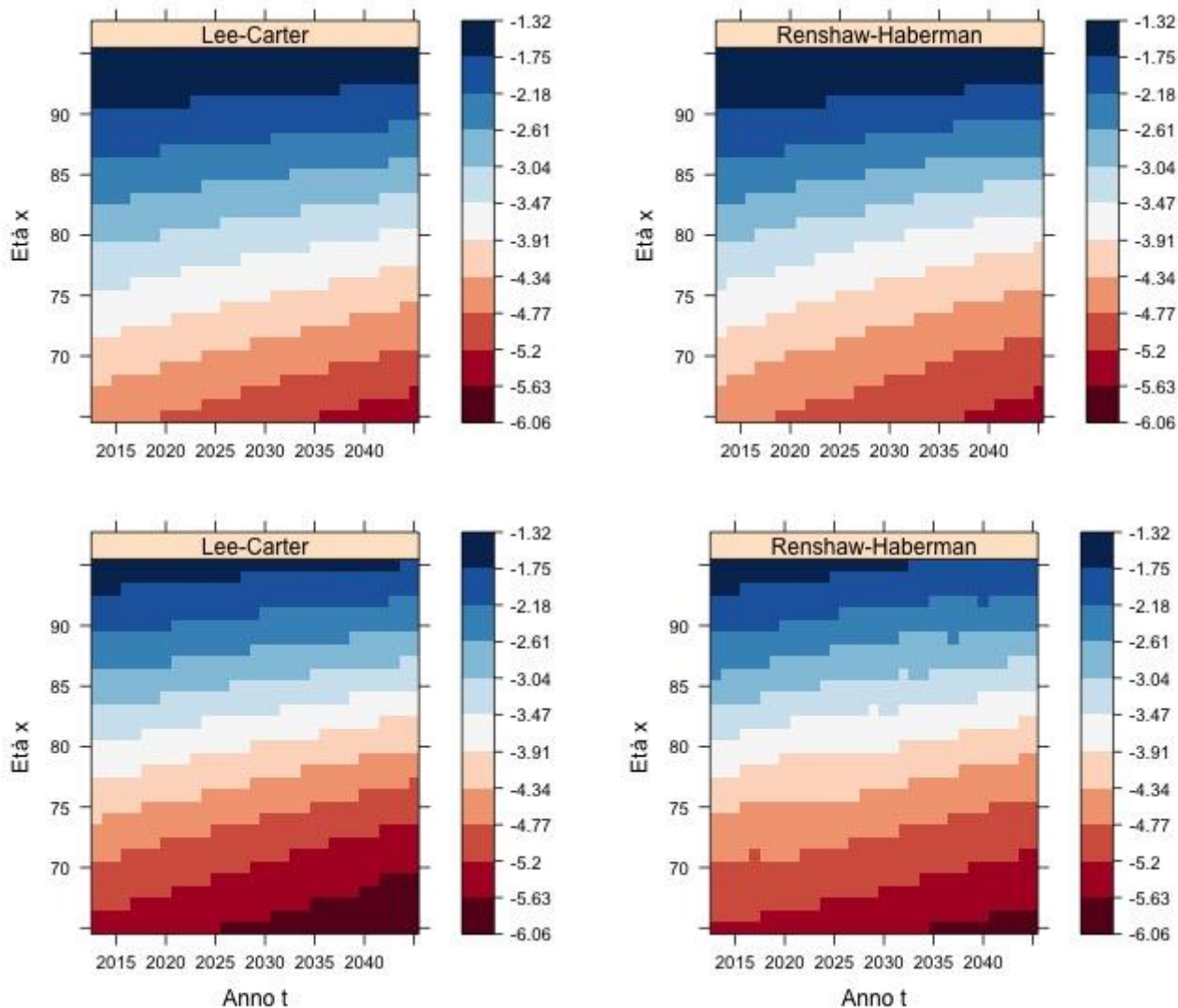


Modello RH: $k_t^{(2)}$, $\gamma_{t-x}^{(3)}$, 1980-2045 - Inps dip.

Femmine



Risultati delle proiezioni: $\log(q_{x,t})$ - Inps dip.

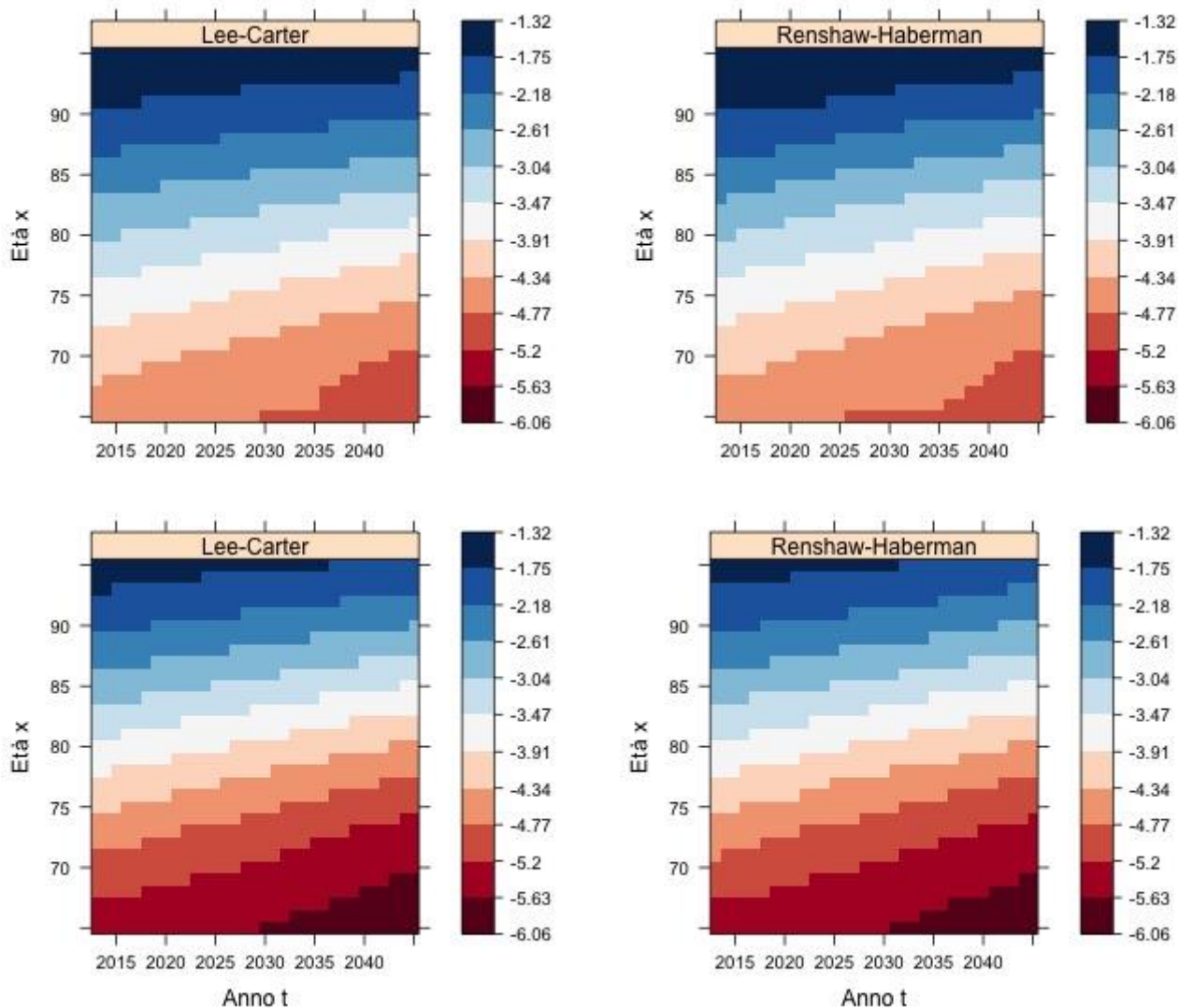


Scenario
centrale

Maschi

Femmine

Risultati delle proiezioni: $\log(q_{x,t})$ - Inps aut.



Scenario
centrale

Maschi

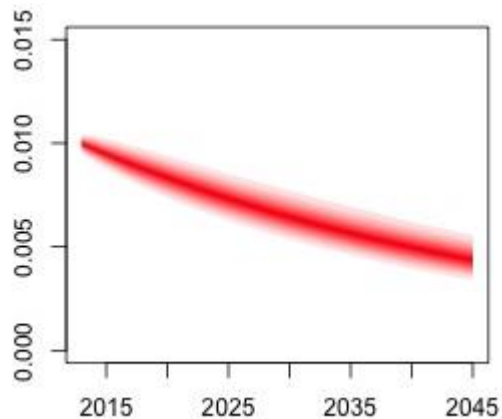
Femmine

Probabilità di morte $q(x,t)$: proiezioni Lee-Carter

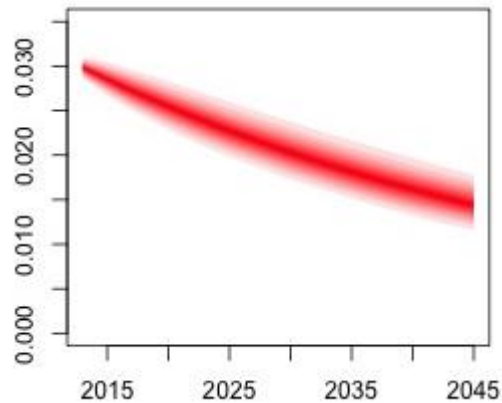
Maschi

dipendenti

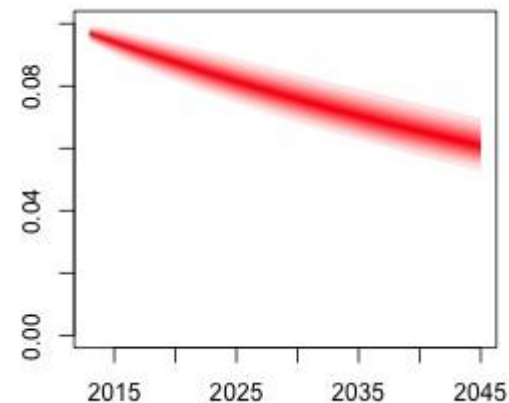
65 anni



75 anni

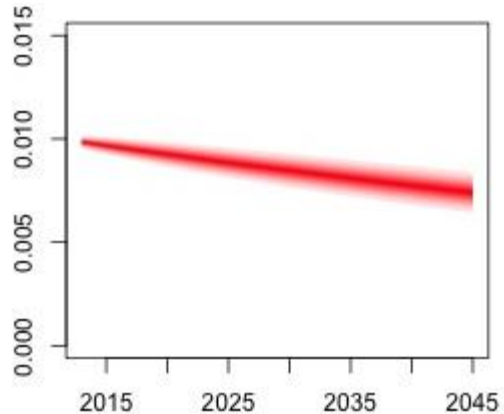


85 anni

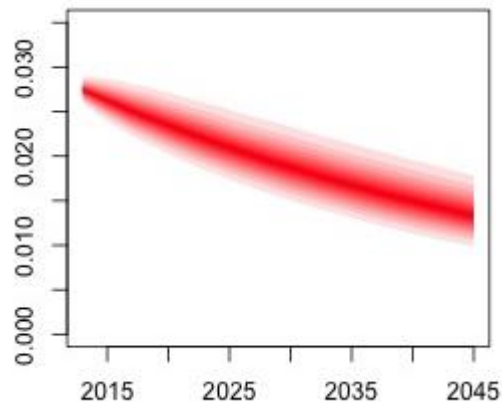


autonomi

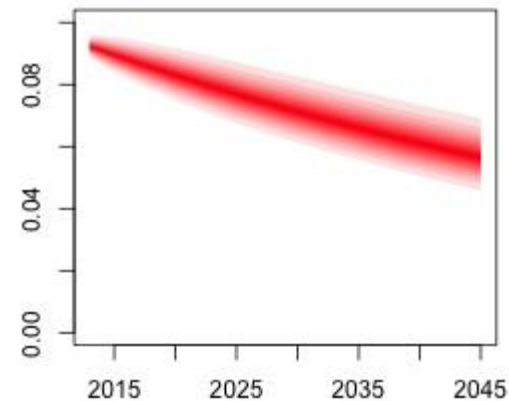
65 anni



75 anni



85 anni

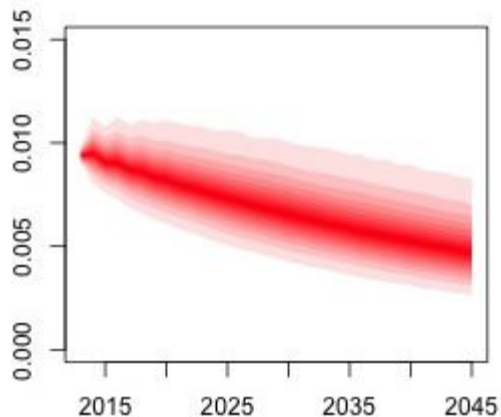


Probabilità di morte $q(x,t)$: proiezioni RH

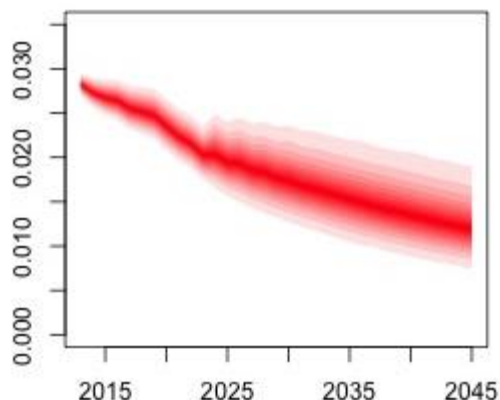
Maschi

dipendenti

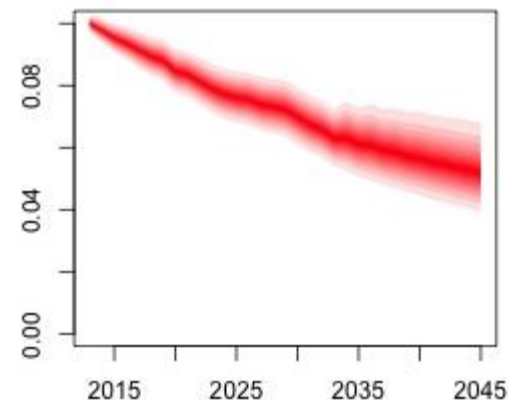
65 anni



75 anni

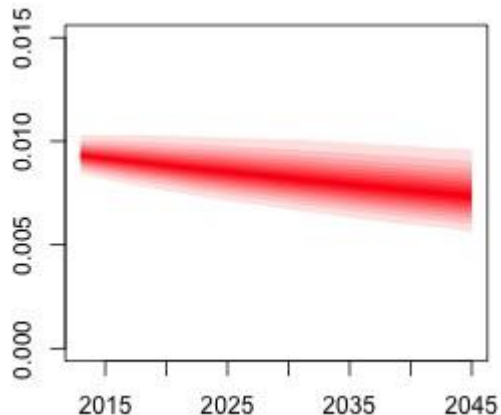


85 anni

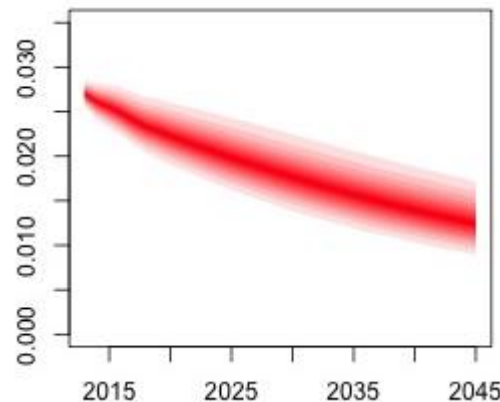


autonomi

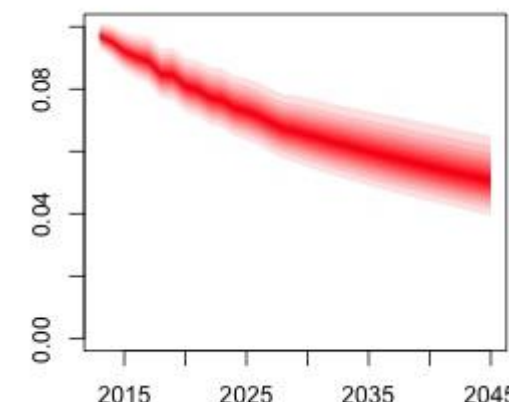
65 anni



75 anni



85 anni

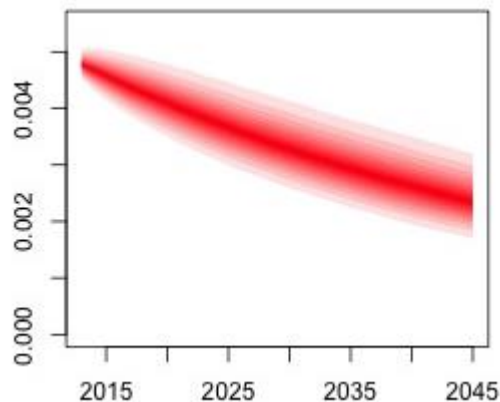


Probabilità di morte $q(x,t)$: proiezioni Lee-Carter

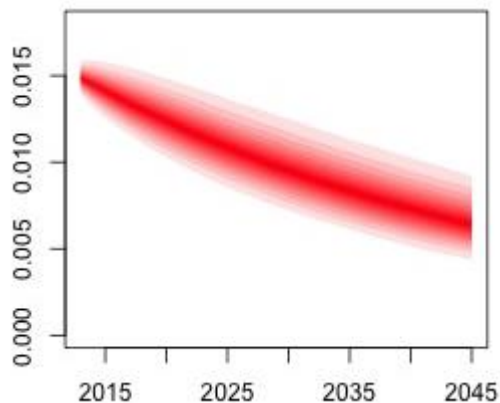
Femmine

dipendenti

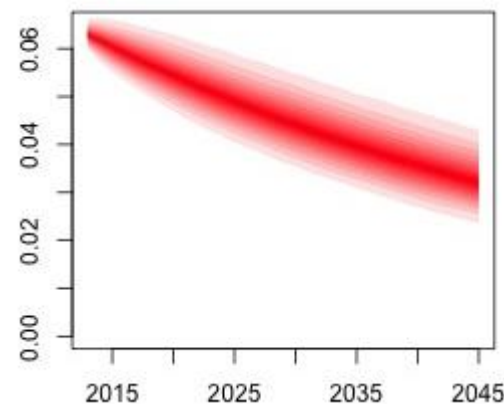
65 anni



75 anni

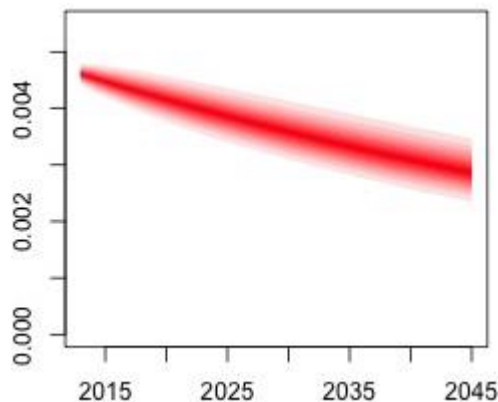


85 anni

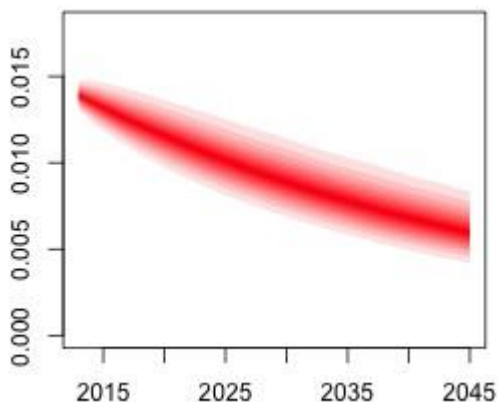


autonomi

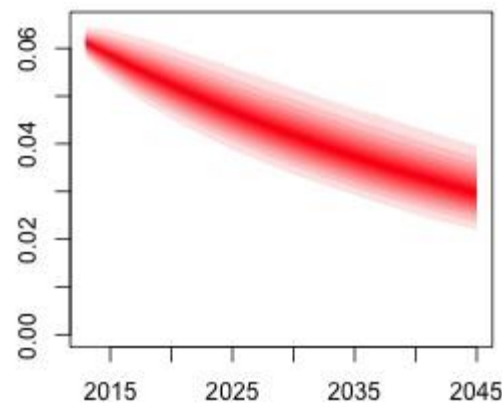
65 anni



75 anni



85 anni

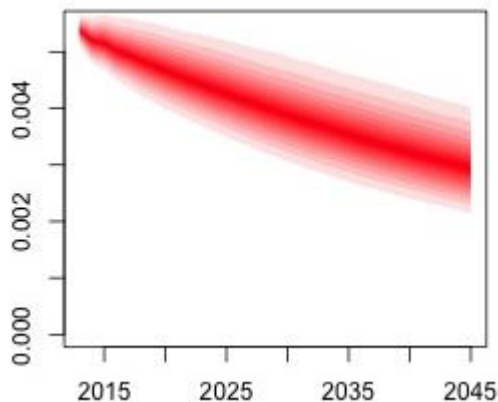


Probabilità di morte $q(x,t)$: proiezioni RH

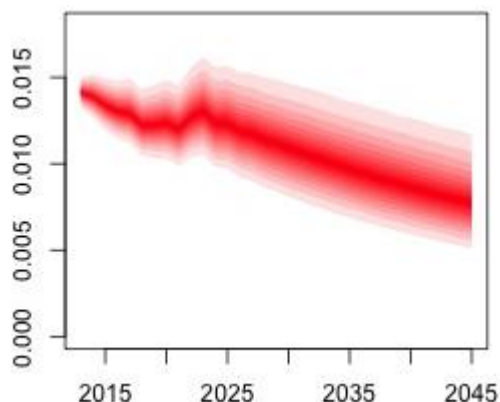
Femmine

dipendenti

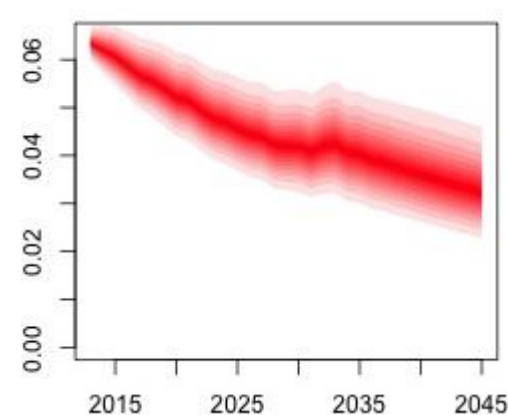
65 anni



75 anni

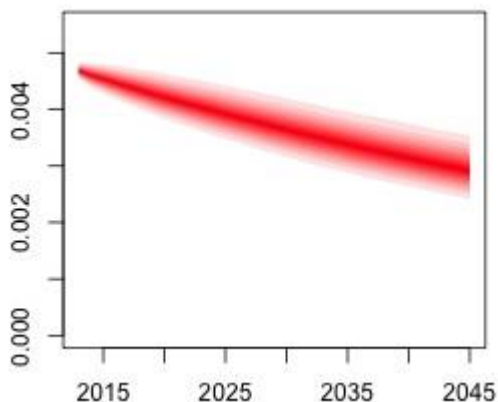


85 anni

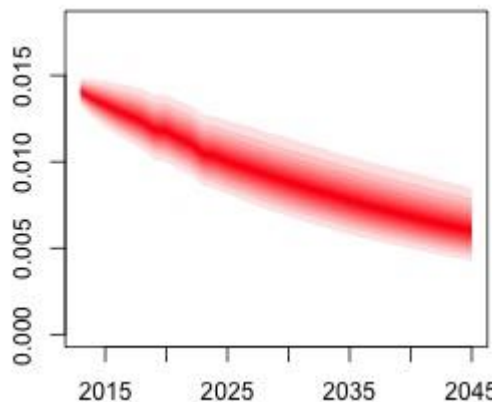


autonomi

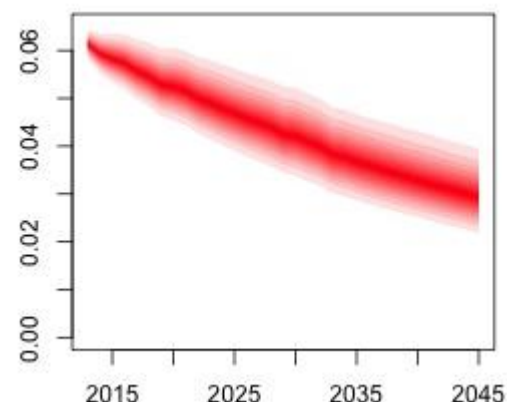
65 anni



75 anni



85 anni



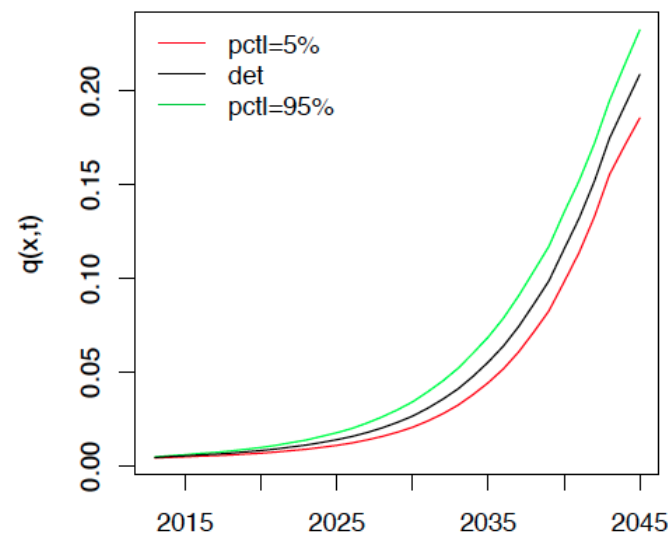
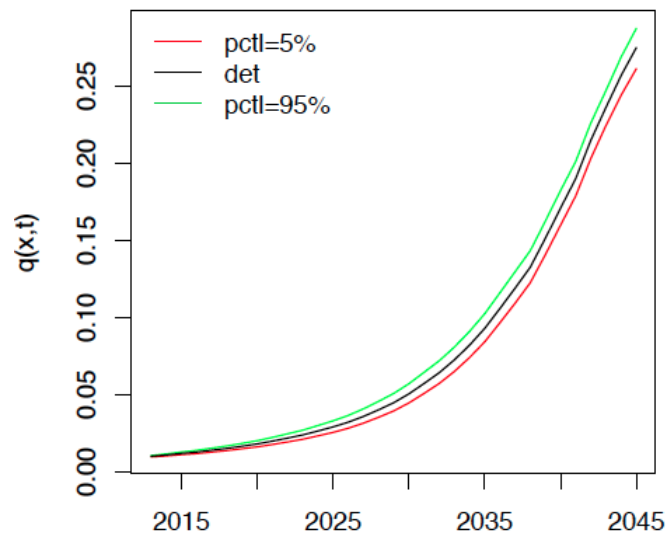
Modello Lee-Carter: $q(65,t)$, anni 2013-2045

Maschi

Femmine

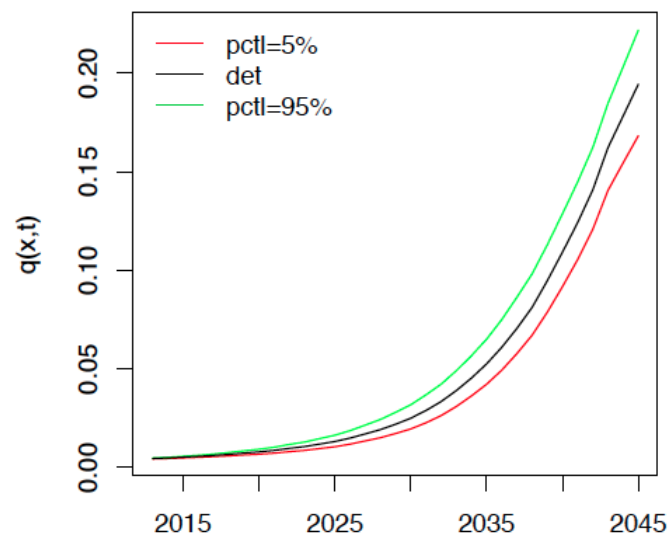
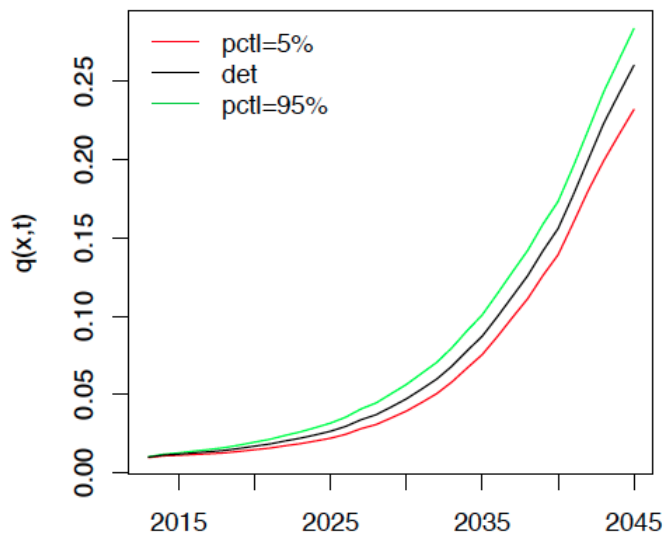
dipendenti

dipendenti



autonomi

autonomi



Modello RH: $q(65,t)$, anni 2013-2045

Maschi

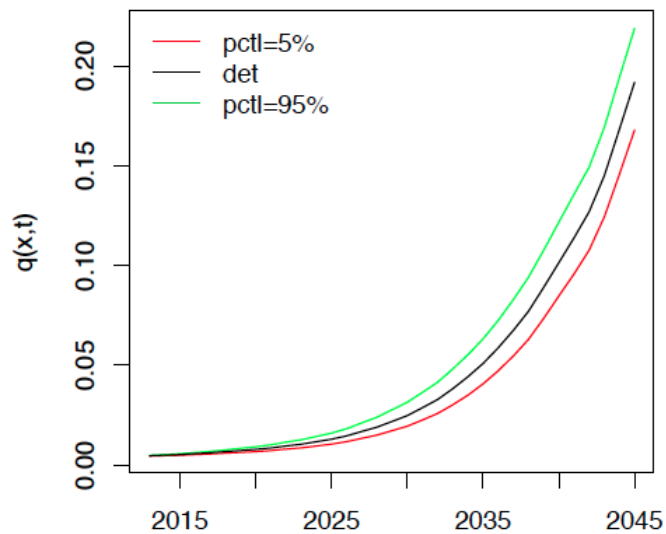
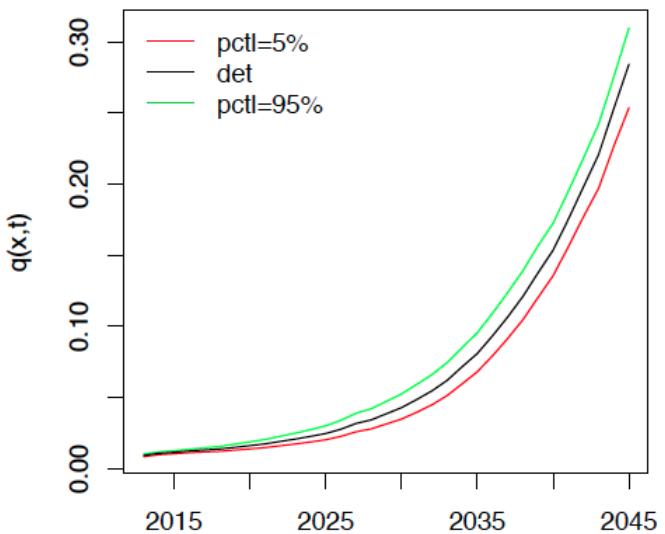
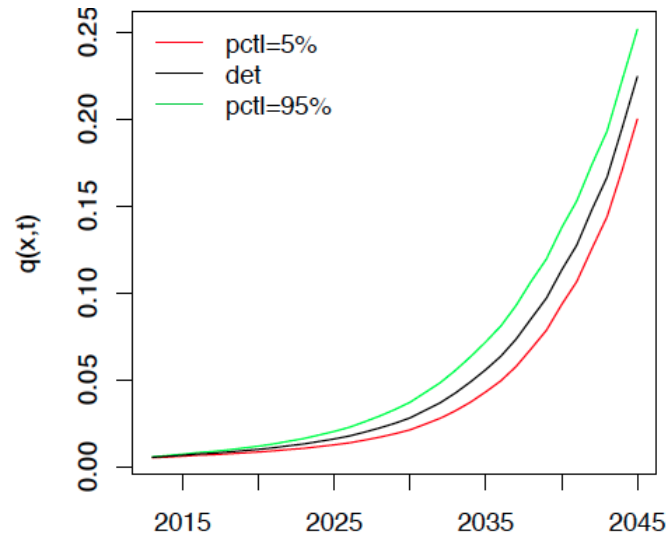
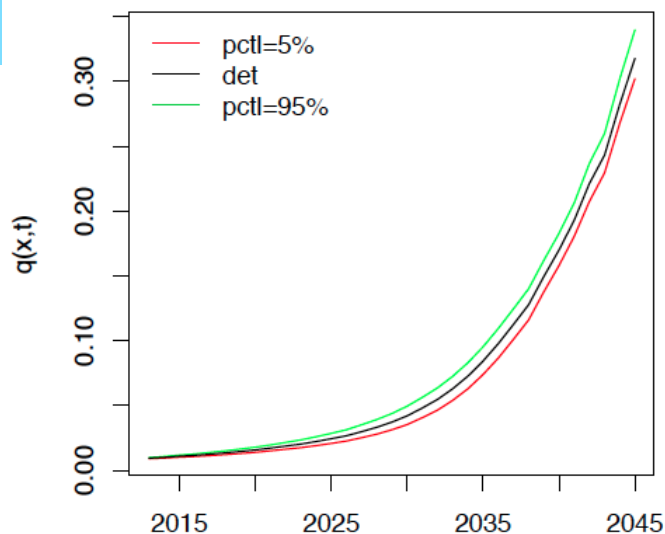
Femmine

dipendenti

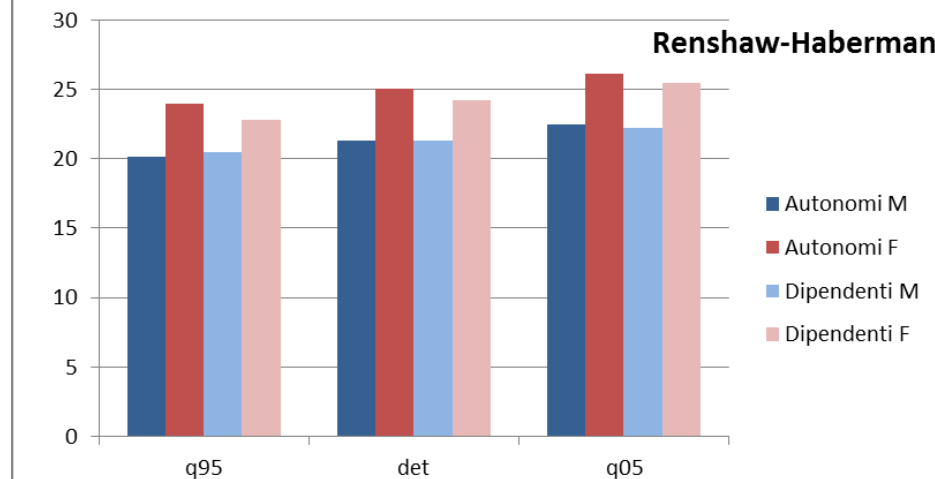
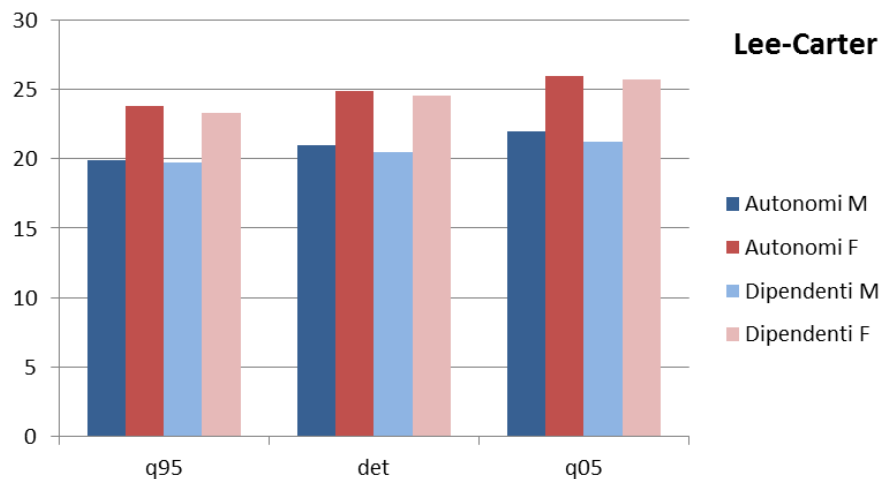
dipendenti

autonomi

autonomi



Annualità vitalizie anticipate (mensili) a tasso d'interesse 0%



Popolazione e sesso	Lee-Carter	Renshaw-Haberman
Autonomi – M	9.6%	10.6%
Autonomi – F	8.9%	8.8%
Dipendenti – M	7.2%	8.1%
Dipendenti – F	9.5%	10.9%

Indice di variabilità:

$$\frac{Q(q_{x,t})_{5\%} - Q(q_{x,t})_{95\%}}{Q(q_{x,t})_{50\%}}$$

GRAZIE PER L'ATTENZIONE

