Three-layer problems and the Generalized Pareto distribution

Dr. Michael Fackler

Independent Actuary, Munich, Germany

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About the speaker

- Full actuary (DAV), self-employed
- Studied Math at Univ. Munich, Pisa, Oldenburg
- Started actuarial career at Rome
- 10 years with leading reinsurers
- 10+ years as consulting actuary
- Specialized in: non-life reinsurance pricing, dealing with scarce data

Situation

Tail modelling, e.g. for layer pricing, Solvency

- Very scarce loss data
- Helpful information possibly from different sources, e.g. **your portfolio** vs **market benchmark**
- Models not fully specified
- Only easily accessible data bits:

frequencies at thresholds / risk premiums of layers

MTPL Example

Task: Pricing of layers from **1** up to **20** (million USD)

- A dozen large losses from your portfolio enable you to quote the layer **2 xs 1**, risk premium: **1.04**

For the whole market someone quoted the layer **5 xs 5**, risk premium: **3**

- Your portfolio supposedly has average exposure, market share is 8%, thus your risk premium for this «market» layer would be: **0.24**

For higher layers you don’t have market quotations or don’t believe them

- Maximum desired payback period for large events (politically set): **200 years**

General approach

- Be **modest**: no best-fit ambitions, a **good-enough** model is fine (*satisfice*, don't *optimize*)
- Use Collective Model of Risk Theory
- Try to find frequency / severity that reproduce given data bits (essentially a moment matching variant)

Three-layer problems

Given input:

- Risk premiums for 3 layers
- Frequencies for 3 thresholds
- Mixed cases

Heuristics: frequency at threshold = *risk rate on line* of very thin layer

$$RRoL = \frac{risk\ premium}{limit}$$

MTPL Example

Formulate as (mixed) three-layer problem:

- layer **2 xs 1**: *RRoL* = **52%**
- layer **5 xs 5**: *RRoL* = **4.8%**
- threshold **20**: *freq.* = **0.5%**

Theorem

For 3 **disjoint** layers with RRoL's $r_1 > r_2 > r_3 > 0$

the problem can be solved:

by a **unique** GPD tail severity $\mathrm{P}(X > x | X > s) = \left(\left(1 + \xi \frac{x-s}{\sigma}\right)^{+}\right)^{-\frac{1}{\xi}}$

together with a (unique) frequency
at the attachment point $s \geq 0$ of the lowest layer

- Works also with thresholds or mixed input
- Top layer may be unlimited

Remarks

- Easy to find numerically
- Special case: 1 layer with risk premium, layer loss frequency, and total layer loss frequency
- Single-parameter Pareto solves analogous **2-layer** problems
- GPD solves many real-world **4-layer** problems approximately, piecewise GPD exactly
- Results yield **model-building recipes** for a variety of scarce-data situations

MTPL Example

$s = 1$ (million USD)

- $\lambda = 1.09$
- $\xi = 0.41$ $(\alpha = 2.44)$
- $\sigma = 0.96$

Model risk

… must be high with scarce data, however:

- **Major uncertainty** is expected loss – and possibly the loss count model
- Higher moments of the severity often don’t add much further uncertainty, in particular for layers in the middle of a program
- The GPD is a choice, but a good one, both in **practical** and **statistical** sense: other severities are less handy and will often produce very similar output

Parameter-free inequality

Limited layer: limit *c*, layer loss severity *Z*, $f \geq r \geq g \geq 0$

with loss frequency *f*, total loss frequency *g*, RRoL *r*

$$1 - \frac{f-r}{f-g}\frac{r-g}{r} \leq \frac{\mathrm{E}(Z^2)}{c\,\mathrm{E}(Z)} \leq 1$$

- Interval is narrow for heavy tailed severity
- Narrower interval for concave cdf

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Conclusion

The building of models by solving three-layer problems is powerful and, in case of very scarce data, an excellent trade-off between statistical ambition and the need to get things done.

Thanks for joining this talk.

Feedback welcome, now or later.

michael_fackler@web.de